Concept of p value

Mr. Atish Tangawade

Assistant Professor
Department of Statistics
Vivekanand College, Kolhapur
(Empowered Autonomous)

Introduction:

Nowadays use of p-value is becoming more and more popular because of the following two reasons:

1. Most of the statistical software provides p-value rather than critical value.
2. p-value provides more information compared to critical value as far as rejection or do not rejection of H_o.

Definition

Definition of p Value

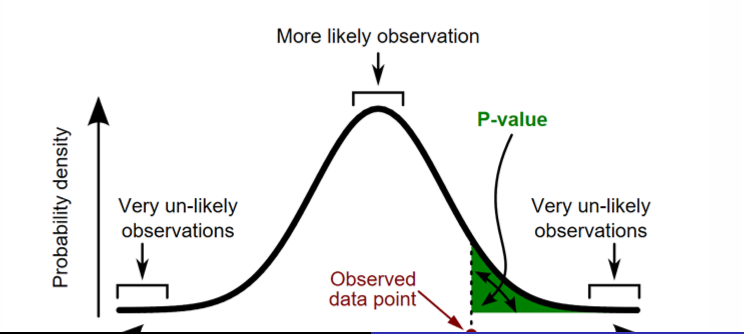
The p-value is the probability of obtaining a test statistic equal to or more extreme (in the direction of supporting H_1) than the actual value obtained when null hypothesis is true.

Interpretation:

The p-value also depends on the type of the test. If test is one-tailed then the p value is defined as:

For right-tailed test:

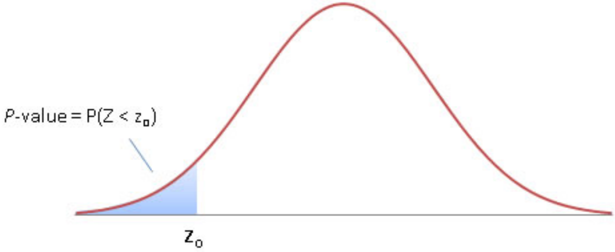
p-value $=$ P[Test Statistic (T) $\geq$ observed value of the test statistic]



Interpretation:

For left-tailed test:

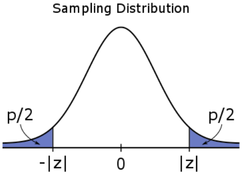
p-value $=$ P[Test Statistic (T) $\leq$ observed value of the test statistic]



Interpretation:

For two-tailed test:Symmetric Sampling Distribution

p-value $= 2P[T \geq |\text{observed value of the test statistic}|]$



For two-tailed test: General Case

p-value $= 2\ \min\{P[T \leq t], P[T \geq t]\}$
where t is observed value of test statistic

Discussion about p value

Example1. One roll of a pair of dice

- H_o : dice are fair & Test is one tailed(assume)
- Test statistic is $T=$ "The sum of the rolled numbers"
- Here, sample space S=$\{(1,1), (1,2),...,(6,6)\}$ so $n(S) = 36$
  Let outcome of this random experiment is both dice show 6 yielding a test statistic T$=$ 12.

Test Statistic(t)	2	3	...	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	...	$\frac{1}{36}$

  So, $p-value = P[T \geq 12] = \frac{1}{36}$
  assume a significance level $\alpha = 0.05$
- Researcher would deem this result significant and would reject the hypothesis that the dice are fair.

Impact of Alternative hypothesis:

Example2. Flips a coin five times in a row

- H_o : coin is fair
- Test statistic is $T=$ "Total number of heads" Let $\alpha = 0.05$
- Here, sample space S={(HHHHH),...,(TTTTT)} so $n(S) = 32$
  Let outcome of this random experiment is heads each time (HHHHH) yielding a test statistic T= 5.
  In a one-tailed test, p-value $= \frac{1}{2^5} = 1/32 \approx 0.03$
  Decision: Reject the hypothesis that the coin is fair.
  In a two-tailed test, p-value $= 2\frac{1}{2^5} = 2/32 \approx 0.06$,
  DecisionFail to reject H_0.

Sample size dependence:

Suppose a researcher flips a coin some arbitrary number of times n and H_0 : The coin is fair and $\alpha = 0.05$
Let test statistic(T) = The total number of heads.
Suppose the researcher observes heads for each flip,
$T = n$ & $p - Value = \frac{2}{2^n}$.
If $n = 5$, the $p - Value = \frac{2}{2^5} = 0.0625$ So, $p > \alpha$
But if $n = 10$, the $p - Value = \frac{2}{2^5} \approx 0.002$ So, $p < \alpha$
This demonstrates that in interpreting p-values, one must also know the sample size, which complicates the analysis.

Impact of Test Statistic:

Let us consider, random experiment of flipping a coin 10 times and it resulted in {HTHTHTHTHT},

H_0 : Coin is fair and alternative is two sided

Case1: Test Statistic(T):Total number of heads, So $T = 5$
Since, 5 is expected value of T implies p Value $= 1$
We do not reject H_0

Case2: Test Statistic(T):Number of alternations, So $T = 9$
p Value $= \frac{2}{2^9} = 0.0039$
Since, $p < \alpha$ We reject H_0
This example demonstrates that the p-value depends completely on the test statistic used.

Misunderstanding about p Values:

1. The p-value is not the probability that the null hypothesis is true, nor is it the probability that the alternative hypothesis is false – it is not connected to either of these.
2. The p-value is not the probability of falsely rejecting the null hypothesis.
3. The p-value is not the probability that replicating the experiment would yield the same conclusion.
4. The significance level, such as 0.05, is not determined by the p-value.


thank
you