"Dissemination of Education for Knowledge, Science and Culture"
-Shikshanmaharshi Dr. Bapuji Salunkhe

Shri Swami Vivekanand Shikshan Sanstha, Kolhapur

Vivekanand College, Kolhapur

(Empowered Autonomous)

Department of Physics

Notice

Date : 02-12-2023

All the students of M.Sc. I are informed that **Open Book Test** on Topic **"Fourier Series"** will be conducted on 4th December 2023 from 3.00 PM to 4.00 PM at Department of Physics. Attendance is mandatory.

Teacher Incharge ShindeAV
(Mr. A.V. Shinde)

sslatthe

Dr. S. S. Latthe

HEAD
DEPARTMENT OF PHYSICS
VIVEKANAND COLLEGE, KOLHAPUR
(EMPOWERED AUTONOMOUS)

"Dissemination of Education for Knowledge, Science and Culture"
-Shikshanmaharshi Dr. Bapuji Salunkhe

Shri Swami Vivekanand Shikshan Sanstha, Kolhapur

Vivekanand College, Kolhapur (Autonomous)

Department of Physics

M.Sc. Part- I

Fourier Series

Open Book Test

Date: 04/12/2023 | **Total Marks: 20**

Day: - Monday | **Time: - 3pm to 4pm**

Instructions: -

1) All questions are compulsory.
2) Each question carries 5 marks

Q.1) Derive the relation for Fourier coefficients a_0, a_n and b_n.

Q.2) Find the Fourier expansion of following function

$F(x) = x \quad 0 < x < 2\pi$

Q.3) Give Detailed Fourier analysis of square wave

Q.4) Derive the Fourier coefficients for complex form of Fourier series

"Dissemination of Education for Knowledge, Science and Culture"
-Shikshanmaharshi Dr. Bapuji Salunkhe

Shri Swami Vivekanand Shikshan Sanstha, Kolhapur

Vivekanand College, Kolhapur (Autonomous)

Department of Physics

M.Sc. I

Open Book Test

Fourier Series

Attendance Sheet

Date : 04/12/2023

Roll. No.	Name of Candidate	Sign
1302	Dhiraj. P. Bhingardeve	Bhingardeve
1307	Mahek S Jamadar	Samu
1313	Vedaja A. Yadav	Yadav
1308	Anjali B. Kamble.	Kamble
1304	Devkule Nishigandha. Nishikant	Devkul
1309	Akash B Kavathekar	Akash

Teacher Incharge.......ShindeAV.......

(Mr. A.V. Shinde)

VIVEKANAND COLLEGE KOLHAPUR (Empowered Autonomous) ESTD JUNE 1964

"Dissemination of Education for Knowledge, Science and Culture"
-Shikshanmaharshi Dr. Bapuji Salunkhe

Shri Swami Vivekanand Shikshan Sanstha, Kolhapur

Vivekanand College, Kolhapur (Autonomous)

Department of Physics

M.Sc. I

Fourier Series

Open Book Test

Result

Date : 04-12-2023

Roll. No.	Marks
1301	-
1302	17
1303	-
1304	17
1305	-
1306	-
1307	18
1308	19
1309	16
1310	-
1311	-
1312	-
1313	18

Teacher Incharge.......ShindeAV.......
(Mr. A.V. Shinde)

SBLatthe
HEAD
DEPARTMENT OF PHYSICS
VIVEKANAND COLLEGE, KOLHAPUR
(EMPOWERED AUTONOMOUS)

VIVEKANAND COLLEGE KOLHAPUR (Empowered Autonomous) ESTD JUNE 1964

Name - Anjali Bhagwan Kamble.

॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥
– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

34540

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor	

Suppliment No. : 1

Roll No. : 1308

Class : M.Sc I

Subject : Fourier Series.

Test / Tutorial No. : open book test.

Div. : -

19/20

Q.1. → We know that fourier expansion of function $f(x)$ is given

$$f(x) = \frac{a_0}{2} + a_1\cos x + a_2\cos 2x + a_3\cos 3x + \dots$$
$$+ b_1\sin x + b_2\sin 2x + b_3\sin 3x \quad \text{——— (1)}$$

1) To find a_0,

Integrating above equation on both sides from $x = 0$ to

$$\int_0^{2\pi} f(x)\,dx = \frac{a_0}{2}\int_0^{2\pi} dx + a_1\int_0^{2\pi}\cos x\,dx + a_2\int_0^{2\pi}\cos 2x\,dx + \dots$$
$$+ b_1\int_0^{2\pi}\sin x\,dx + b_2\int_0^{2\pi}\sin 2x\,dx + \dots$$

4

$$\int_0^{2\pi} f(x)\,dx = \frac{a_0}{2}\left[x\right]_0^{2\pi}$$
$$= \frac{a_0}{2}\,2\pi$$

To find a_n,

Multiply each side of equation (1) by $\cos nx$ and integrate from $x = 0$ to $x = 2\pi$

$$\int_0^{2\pi} f(x).\cos nx\,dx = \cancel{a_0}\,\frac{a_0}{2}\int_0^{2\pi}\cos nx\,dx + a_1\int_0^{2\pi}\cos nx\cos x\,dx +$$
$$a_n\int_0^{2\pi}\cos^2 nx\,dx + b_1\int_0^{2\pi}\sin x\cos nx\,dx + b_2\int_0^{2\pi}\sin 2x\cos$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x) \cdot \cos nx \, dx$$

To find b_n,

Multiply each side of equation (1) by $\sin nx$ and integrate from $x = 0$ to 2π.

$$\int_0^{2\pi} f(x) \sin nx \, dx = \frac{a_0}{2}\int_0^{2\pi} \sin nx + a_1 \int_0^{2\pi} \sin nx \cos x \, dx + \ldots$$

$$+ b_1 \int_0^{2\pi} \sin x \sin nx + b_2 \int_0^{2\pi} \sin nx \sin 2x + \ldots b_n \int_0^{2\pi} \sin^2 n$$

$$\int_0^{2\pi} f(x) \sin nx \, dx = b_n \pi$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x) \sin nx \, dx$$

Q.2 We know that Fourier series is given as,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \quad \text{——} ①$$

Let, $a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$

$$= \frac{1}{\pi}\int_0^{2\pi} x\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_0^{2\pi} \quad \cancel{\text{——} ②}$$

$$= \frac{1}{\pi}\left[\frac{4\pi}{2}\right]$$

$$= 2\pi \quad \text{——} ②$$

Let, $a_n = \frac{1}{\pi}\int_0^{2\pi} x \cos nx\,dx$

$$= \frac{1}{\pi}\left[x\frac{\sin nx}{n} - \int 1\cdot\frac{\sin nx}{n}\,dx\right]_0^{2\pi}$$

5

$$= \frac{1}{\pi}\left[x\frac{\sin nx}{n} + \frac{\cos nx}{n^2}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{2\pi\sin n2\pi}{n} + \frac{\cos n2\pi}{\pi^2} - 0 - \frac{\cos 0}{n^2}\right]$$

$$= \frac{1}{\pi}\left[0 + \frac{1}{n^2} - \frac{1}{n^2}\right]$$

$$= 0 \quad \text{——} ③$$

$b_n = \frac{1}{\pi}\int_0^{2\pi} x\cdot\sin nx\,dx$

$$= \frac{1}{\pi}\left[\frac{-x\cos nx}{n} + \int\frac{\cos nx\,dx}{n}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{-x\cos nx}{n} + \frac{\sin nx}{n^2}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{-2\pi}{n} + 0 - (0+0)\right]$$

$$= \frac{-2}{n} \quad \text{——} (4)$$

eq^n (1) becomes,

$$f(x) = \frac{2\pi}{2} + 0 + \sum_{n=1}^{\infty} (-2/n) \sin nx$$

$$= \pi - \left(2\sin x + \sin 2x + \frac{2}{3}\sin 3x + \dots\right)$$

Name :- Anjali Bhagwan Kamble.

॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥

– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

34567

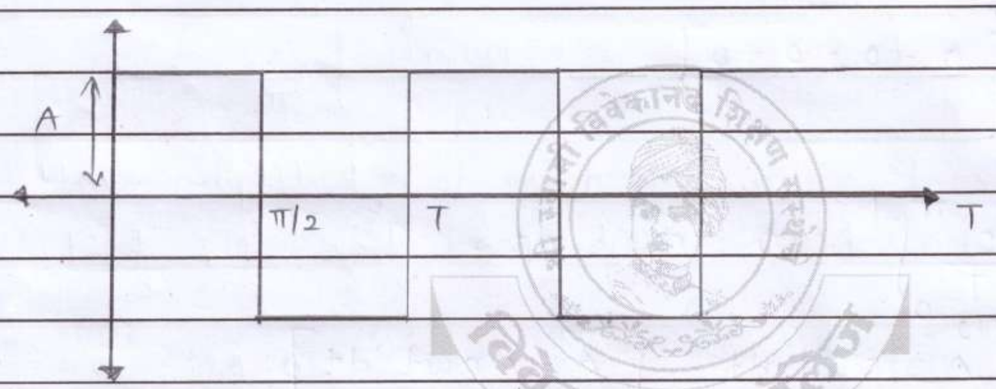
Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Suppliment No. : 2	Signature of Supervisor
Roll No. : 1308	Subject : Fourier Series.
Class : M.Sc I	Test / Tutorial No. : Open book test.
	Div. :

Q.3.



Here, $f(t) = A \quad ; \quad 0 < t < T/2$

$\qquad\quad = -A \quad ; \quad T/2 < t < T$

for,

$$a_0 = \frac{2}{T}\int_0^T f(t)\,dt$$

$$= \frac{2}{T}\left[\int_0^{T/2} f(t)\,dt + \int_{T/2}^{T} f(t)\,dt\right]$$

5

$$\cancel{= \frac{2A}{T}\left[T/2 - (T - T/2)\right]}$$

$$= \frac{2}{T}\left[\int_0^{T/2} A\,dt + \int_0^{T/2} -A\,dt\right]$$

$$= \frac{2}{T}\left[A\left[t\right]_0^{T/2} - A\left[t\right]_{T/2}^{T}\right]$$

$$= \frac{2A}{T}\left(T/2 - T + T/2\right)$$

$$= 0$$

for,

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\,dt \ldots \quad \text{where } \omega = \frac{2\pi}{t}$$

$$= \frac{2}{T}\left[\int_0^{T/2} A\cos n\omega t\,dt - \int_{T/2}^{T} (A)\cos n\omega t\,dt\right]$$

$$= \frac{2A}{T}\left[\left\{\frac{\sin n\omega t}{n\omega}\right\}_0^{T/2} - \left\{\frac{\sin n\omega t}{n\omega}\right\}_{T/2}^{T}\right]$$

$$= \frac{2A}{n\omega T}\left[\left\{\sin n\frac{2\pi}{T}\frac{T}{2} - 0\right\} - \left\{\sin\frac{2\pi}{T}T - \sin n\frac{2\pi}{T}\frac{T}{2}\right\}\right]$$

$$= \frac{2A}{n\omega T}\left[\sin n\pi - 0 - \sin 2n\pi + \sin n\pi\right]$$

$$= \frac{2A}{n\omega t}\left[0 - 0 - 0 - 0\right]$$

$$= 0.$$

for $b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\,dt.$

$$= \frac{2}{T}\left[\int_0^{T/2} A\sin \omega t\,dt - \int_{T/2}^{T} A\sin n\omega t\,dt\right]$$

$$= \frac{2A}{T}\left\{\left[-\frac{\cos n\omega t}{n\omega}\right]_0^{T/2} + \left[\frac{\cos n\omega t}{n\omega}\right]_{T/2}^{T}\right\}$$

$$= \frac{2A}{n\omega T}\left[-\cos n\frac{2\pi}{T}\cdot\frac{T}{2} + \cos 0\right] + \left[\cos n\frac{2\pi}{T}\cdot T - \cos n\frac{2\pi}{T}\right.$$

$$= \frac{2A}{n\frac{2\pi}{T}T}\left[-\cos n\pi + 1 + \cos 2\pi n - \cos n\pi\right]$$

$$= \frac{A}{n\pi}\left[-2\cos n\pi + 1 + \cos 2\pi n\right]$$

$$= \frac{A}{n\pi}\left[2(-1) + 1 + 1\right]$$

$$= \frac{A}{n\pi}\left[-2(-1)^n + 2\right]$$

$$b_n = \frac{2A}{n\pi}\left[1 - (-1)^n\right]$$

$= \frac{4A}{n\pi}$ — for n = odd integer.

$= 0$ — for n = even integer.

we have fourier expansion.

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \sin n\omega t.$$

$$= 0 + 0 + \sum_{n=1}^{\infty} \frac{4A}{n\pi} \sin n\omega t$$

for n = odd integer

$$f(t) = \frac{4A}{\pi} \sin \omega t + \frac{4A}{3\pi} \sin 3\omega t + \frac{4A}{5\pi} \sin 5\omega t + \dots$$

$$= \frac{4A}{\pi}\left[\sin \omega t + \frac{1}{3} \sin 3\omega t + \frac{1}{5} \sin 5\omega t + \dots\right]$$

Here amplitude of each sin wave decreases as $1 : \frac{1}{3} : \frac{1}{5}$ and frequency of each in wave increases as $1 : 3 : 5$.

Q.4.

fourier series of function $f(x)$ of period $2l$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$$

$$f(x) = \frac{a_0}{2} + a_1 \cos \frac{\pi x}{L} + a_2 \cos \frac{2\pi x}{L} + a_3 \cos \frac{3\pi x}{L} + a_n \cos \frac{n\pi x}{L}$$

$$+ b_1 \sin \frac{\pi x}{L} + b_2 \sin \frac{2\pi x}{L} + b_3 \sin \frac{3\pi x}{L} + \dots b_n \sin \frac{n\pi x}{L}$$

$$= \frac{a_0}{2} + a_1 \frac{e^{i(\pi x/l)} + e^{-i\pi x/l}}{2} + a_2 \frac{e^{i2\pi x/l} + e^{-i2\pi x/l}}{2}$$

$$a_n \frac{e^{in\pi x/l} + e^{-in\pi x/l}}{2} + b_1 \left[\frac{e^{i\pi x/l} - e^{-i\pi x/l}}{2i}\right] +$$

5

$$b_2 \frac{e^{i2\pi x/l}}{2i} e^{-i2\pi x/l} + \dots b_n \frac{e^{in\pi x/l} - e^{-in\pi x/l}}{2i}$$

$$= \frac{a_0}{2} + \frac{1}{2}\left\{(a_1 - ib_1) e^{i\pi x/l} + (a_1 + ib_1) e^{-i\pi x/l} + (a_2 - ib_2) e^{i2\pi x/l} + (a_2 + ib_2) e^{-i2\pi x/l} + \dots\right\}$$

$$= \frac{a_0}{2} + \left\{\frac{(a_1 - ib_1}{2} e^{i\pi x/l} + (a_2 - ib_2) e^{2i\pi x/l} + \dots \frac{(a_1 + ib_1)}{2} e^{-i\pi x/l} + \frac{(a_2 + ib_2)}{2} e^{-2i\pi x/l}\right\}$$

$$= \frac{a_0}{2} + \left\{c_1 e^{i\pi x/l} + c_2 e^{2i\pi x/l} + \dots + c_{-1} e^{-i\pi x/l} + c_{-2} e^{-2i\pi x/l}\right.$$

$$= c_0 + \frac{1}{2}\left\{\sum_{n=1}^{\infty} c_n e^{ni\pi x/l} + \sum_{n=1}^{\infty} c_{-n} e^{-ni\pi x/l}\right\}$$

where, $c_0 = \frac{a_0}{2} = \frac{1}{2l}\int_0^{2l} f(x)\, dx$

Name :- Anjali Bhagwan kamble.

॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥

– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

34571

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor

Suppliment No. : 3

Roll No. : 1308

Class :

Subject : Fourier Series.

Test / Tutorial No. : Open book test.

Div. :

$$c_n = \frac{1}{2}(a_n - i b_n)$$

$$= \frac{1}{2}\left\{\left(\frac{1}{l}\int_0^{2l} f(x)\cdot\cos\frac{n\pi x}{l}\,dx\right) - i\left(\frac{1}{l}\int_0^{2l} f(x)\sin\frac{n\pi x}{l}\right.\right.$$

$$= \frac{1}{2l}\int_0^{2l} f(x)\left(\cos\frac{n\pi x}{l} - i\sin\frac{n\pi x}{l}\right)dx$$

$$c_n = \frac{1}{2l}\int_0^{2l} f(x)\, e^{-in\pi x/l}\,dx$$

$$c_{-n} = \frac{1}{2}(a_n + i b_n)$$

$$= \frac{1}{2}\left\{\left(\frac{1}{l}\int_0^{2l} f(x)\cdot\cos\frac{n\pi x}{l}\,dx\right) + i\left(\frac{1}{l}\int_0^{2l} f(x)\sin\frac{n\pi x}{l}\,dx\right)\right\}$$

$$= \frac{1}{2l}\int_0^{2l} f(x)\left(\cos\frac{n\pi x}{l} + i\sin\frac{n\pi x}{l}\,dx\right)$$

$$c_{-n} = \frac{1}{2l}\int_0^{2l} f(x)\, e^{in\pi x/l}\,dx$$

Mahek S. Jamadar.

॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥
– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

34541

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor	

Suppliment No. : 1

Roll No. : 1307

Class : M.Sc. I

Subject : Fourier Series

Test / Tutorial No. : Open book Test.

Div. :

18/20

Q1] find the fourier coefficients a_n, b_n, a_0.

Ans → We know that, fourier expansion of function $f(x)$ is given as $f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + \cdots + b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + \ldots$ — (1)

① To find a_0 :-

Integrating above equation on both sides from $x=0$ to $x=2\pi$

$$\int_0^{2\pi} f(x)\,dx = \frac{a_0}{2}\int_0^{2\pi} dx + a_1\int_0^{2\pi}\cos x\,dx + a_2\int_0^{2\pi}\cos 2x\,dx + \cdots + b_1\int_0^{2\pi}\sin x\,dx + b_2\int_0^{2\pi}\sin 2x\,dx + \ldots$$

4

$$= \int_0^{2\pi} f(x)\,dx = \frac{a_0}{2}\left[x\right]_0^{2\pi}$$

$$= \frac{a_0}{\cancel{2}}\,\cancel{2}\pi$$

$$\therefore a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$$

② To find a_n :-

Multiply each side of eqn ① by $\cos nx$ and

and integrate from $x=0$ to $x=2\pi$

$$\int_0^{2\pi} f(x)\cos nx\,dx = \frac{a_0}{2}\int_0^{2\pi}\cos nx\,dx + a_1\int_0^{2\pi}\cos nx\cos x\,dx$$
$$+ \cdots a_n\int_0^{2\pi}\cos^2 nx\,dx +$$
$$b_1\int_0^{2\pi}\sin x\cos nx\,dx + b_2\int_0^{2\pi}\sin 2x\cos nx\,dx + \cdots$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx.$$

③ To find b_n :-

Multiply each side of eqn ① by $\sin nx$ and integrate from $x=0$ to $x=2\pi$

$$\int_0^{2\pi} f(x)\sin nx\,dx = \frac{a_0}{2}\int_0^{2\pi}\cos nx\,dx + a_1\int_0^{2\pi}\cos x\sin nx\,dx$$
$$+ b_1\int_0^{2\pi}\sin x\sin nx\,dx + b_2\int_0^{2\pi}\sin 2x$$
$$\sin nx\,dx + \cdots + b_n\int_0^{2\pi}\sin nx\cdot\sin nx\,dx$$
$$= b_n\pi$$

$$\therefore b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx.$$

2] We know that fourier series is given as,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \quad \text{---} (1)$$

Let $a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$

$$= \frac{1}{\pi}\int_0^{2\pi} x\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{4\pi^2}{2}\right]$$

$$a_0 = 2\pi \quad \text{---} (2)$$

Let $a_n = \frac{1}{\pi}\int_0^{2\pi} x\cos nx\,dx$

$$= \frac{1}{\pi}\left[x\frac{\sin nx}{n} - \int 1 . \frac{\sin nx}{n}\,dx\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{x\sin nx}{n} + \frac{\cos nx}{n^2}\right]_0^{2\pi}$$

4

$$= \frac{1}{\pi}\left[\frac{2\pi \sin n2\pi}{n} + \frac{\cos n2\pi}{n^2} - 0 - \frac{\cos 0}{n^2}\right]$$

$$= \frac{1}{\pi}\left[0 + \frac{1}{n^2} - \frac{1}{n^2}\right] = 0 \quad \text{---} (3)$$

Now, $b_n = \frac{1}{\pi}\int_0^{2\pi} x\sin nx\,dx$

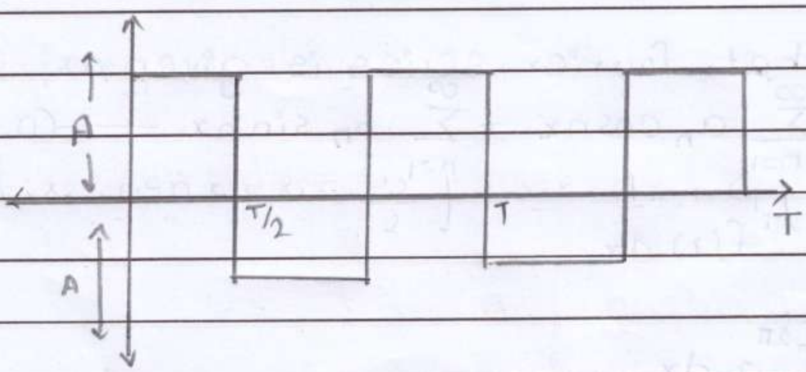
$$= \frac{1}{\pi}\left[\frac{-x\cos nx}{n} + \int \frac{\cos nx}{n}\,dx\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{-x\cos nx}{n} + \frac{\sin nx}{n^2}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{-2\pi}{n} + 0 - (0+0)\right]$$

$$= \frac{-2}{n} \quad \text{---} (4)$$

→ 3]



Here, $f(t) = A \quad ; \quad 0 < t < T/2$

$\quad\quad = -A \quad ; \quad T/2 < t < T$

for,

$$a_0 = \frac{2}{T}\int_0^T f(t)\,dt$$

$$= \frac{2}{T}\left[\int_0^{T/2} f(t)\,dt + \int_{T/2}^{T} f(t)\,dt\right]$$

$$= \frac{2}{T}\left[\int_0^{T/2} A\,dt + \int_{T/2}^{T}(-A)\,dt\right]$$

$$= \frac{2A}{T}\left[\int_0^{T/2} dt - \int_{T/2}^{T} dt\right]$$

$$= \frac{2A}{T}\left[T/2 - T + T/2\right]$$

5

$$= 0.$$

For, $a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\,dt$

[where ω = ang. freq. $= \frac{2\pi}{T}$]

$$= \frac{2}{T}\left[\int_0^{T/2} A\cos n\omega t\,dt - \int_{T/2}^{T} A\cos n\omega t\,dt\right]$$

$$= \frac{2A}{T}\left\{\left[\frac{\sin n\omega t}{n\omega}\right]_0^{T/2} - \left[\frac{\sin n\omega t}{n\omega}\right]_{T/2}^{T}\right\}$$

$$= \frac{2A}{n\omega T}\left\{\left[\sin n\frac{2\pi}{T}\cdot\frac{T}{2} - 0\right] - \left[\sin n\frac{2\pi}{T}T - \sin\frac{2\pi}{T}\cdot\frac{T}{2}\right]\right\}$$

$$= \frac{2A}{n\omega T}\left[\sin n\pi - 0 - \sin 2n\pi + \sin n\pi\right]$$

$$= \frac{2A}{n\omega T}[0] = \underline{\underline{0}}$$

Mahek Jamadar

॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥
– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

34565

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor

Suppliment No. : 2

Subject : Fourier series

Roll No. : 1307

Test / Tutorial No. :

Class : M.Sc. I

Div. :

for, $b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\, dt$

$$= \frac{2}{T}\left[\int_0^{T/2} A\sin n\omega t\, dt - \int_{T/2}^{T} A\sin n\omega t\, dt\right]$$

$$= \frac{2A}{T}\left\{\left[\frac{-\cos n\omega t}{n\omega}\right]_0^{T/2} + \left[\frac{\cos n\omega t}{n\omega}\right]_{T/2}^{T}\right\}$$

$$= \frac{2A}{n\omega T}\left[-\cos n\cdot\frac{2\pi}{T}\cdot\frac{T}{2} + \cos 0\right] + \left[\cos n\frac{2\pi}{T}\cdot T - \cos n\, 2\pi/_T \cdot T/_2\right]$$

$$= \frac{2A}{n\frac{2\pi}{T}\cdot T}\left[-\cos n\pi + 1 + \cos 2\pi n - \cos n\pi\right]$$

$$= \frac{A}{n\pi}\left[-2\cos\pi + 1 + \cos 2\pi n\right]$$

$$= \frac{A}{n\pi}\left[-2(-1)^n + 1 + 1\right] = \frac{A}{n\pi}\left[-2(-1)^n + 2\right]$$

$$b_n = \frac{2A}{n\pi}\left[1 - (-1)^n\right]$$

$$= \frac{4A}{n\pi} \text{ — for odd integer}$$

$b_n = 0$ —— for even integer.

We know that, fourier expansion is given as

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \sin n\omega t$$

$$= 0 + 0 + \sum_{n=1}^{\infty} \frac{4A}{n\pi} \sin n\omega t$$

for n= odd integer

$$f(t) = \frac{4A}{\pi} \sin \omega t + \frac{4A}{3\pi} \sin 3\omega t + \frac{4A}{5\pi} \sin 5\omega t + \ldots$$

$$= \frac{4A}{\pi}\left[\sin \omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \ldots\right]$$

Here amplitude of each sine wave decreases as $1 : \frac{1}{3} : \frac{1}{5} \ldots$ and frequency of each sine wave increases as $1 : 3 : 5$.

→ 4] Fourier series of a function f(x) of period 2l is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$$

$$= \frac{a_0}{2} + a_1 \cos \frac{\pi x}{l} + a_2 \cos \frac{2\pi x}{l} + a_3 \cos \frac{3\pi x}{l} + \ldots + a_n \cos \frac{n\pi x}{l} + b_1 \sin \frac{\pi x}{l} + b_2 \sin \frac{2\pi x}{l} + b_3 \sin \frac{3\pi x}{l} + \ldots + b_n \sin \frac{n\pi x}{l}$$

5

$$= \frac{a_0}{2} + a_1 \frac{e^{\frac{i\pi x}{l}} + e^{-i\pi x/l}}{2} + a_2 \frac{e^{i2\pi x/l} + e^{-i2\pi x/l}}{2} + \ldots + b_1 \frac{e^{i\pi x/l} - e^{-i\pi x/l}}{2i} + b_2 \frac{e^{i2x\pi/l} - e^{-i2\pi x/l}}{2i} + \ldots$$

$$= \frac{a_0}{2} + \frac{1}{2}\left\{ (a_1 - ib_1) e^{i\pi x/l} + (a_1 + ib_1) e^{-\pi i x/l} + (a_2 - ib_2) e^{i2\pi x/l} + \frac{(a_2 + ib_2)}{2} e^{-i2\pi x/l} + \ldots \right\}$$

$$= \frac{a_0}{2} + \frac{1}{2}\left\{ \frac{(a_1 - ib_1)}{2} e^{i\pi x/l} + \frac{(a_2 - ib_2)}{2} e^{i2\pi x/l} + \ldots + \frac{(a_1 + ib_1)}{2} e^{-i\pi x/l} + \frac{(a_2 + ib_2)}{2} e^{-2i\pi x/l} + \ldots \right\}$$

$$= \frac{a_0}{2} + \left\{ c_1 e^{i\pi x/l} + c_2 e^{2\pi i x/l} + \dots + c_{-1} e^{-i\pi x/l} + c_{-2} e^{-2i\pi x/l} + \dots \right\}$$

$$= c_0 + \frac{1}{2} \left\{ \sum_{n=1}^{\infty} c_n e^{\pi n i x/l} + \sum_{n=1}^{\infty} c_n e^{-n i \pi x/l} \right\}$$

where, $c_0 = \frac{a_0}{2} = \frac{1}{2l} \int_0^{2l} f(x)\,dx$

$$c_n = \frac{1}{2}(a_n - ib_n) = \frac{1}{2}\left\{ \left(\frac{1}{l} \int_0^{2l} f(x) \cdot \cos\frac{n\pi x}{l}\,dx \right) - i \left(\frac{1}{l} \int_0^{2l} f(x) \sin\frac{n\pi x}{l}\,dx \right) \right\}$$

$$= \frac{1}{2l} \int_0^{2l} f(x) \left(\cos\frac{n\pi x}{l} - i \sin\frac{n\pi x}{l} \right) dx.$$

$$c_n = \frac{1}{2l} \int_0^{2l} f(x)\, e^{-in\pi x/l}\,dx$$

$$c_{-n} = \frac{1}{2}(a_n + ib_n) = \frac{1}{2}\left\{ \left(\frac{1}{l} \int_0^{2l} f(x) \cos\frac{n\pi x}{l}\,dx \right) + i \left(\frac{1}{l} \int_0^{2l} f(x) \sin\frac{n\pi x}{l}\,dx \right) \right\}$$

$$= \frac{1}{2l} \int_0^{2l} f(x) \left(\cos\frac{n\pi x}{l} + i \sin\frac{n\pi x}{l} \right) dx$$

$$c_{-n} = \frac{1}{2l} \int_0^{2l} f(x)\, e^{in\pi x/l}\,dx.$$

Hence, fourier series in complex form is given as,

$$f(x) = c_0 + \left\{ \sum_{n=1}^{\infty} c_n e^{in\pi x/l} + \sum_{n=1}^{\infty} c_{-n} e^{-i\pi n x/l} \right\}$$

AKASH BHIMRAO KAVATHEKAR

।। ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ।।

– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

34544

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor

Suppliment No. : 1

Roll No. : 1309

Class : MSc-I

Subject : forier Series

Test / Tutorial No. : open book test

Div. :

16/20

① We know that forier series of $f(x)$.

$$f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + \ldots$$

$$+ b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + \ldots$$

① To find a_0 :-

Integrated above eq^n on both side from $x = 0$, $x = 2\pi$

4

$$\int f(x)\,dx = \frac{a_0}{2}\int_0^{2\pi} dx + a_1 \int_0^{2\pi} \cos x\,dx + a_2 \int_0^{2\pi} \cos$$

$$\ldots + b_1 \int_0^{2\pi} \sin x\,dx + b_2 \int_0^{2\pi} \sin 2x\,dx + \ldots$$

$$= \frac{a_0}{2}(x)\Big|_0^{2x}$$

$$= \frac{a_0}{2} \times 2\pi$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$$

② To find a_n :-

multiplying each side of eqⁿ ① by $\cos nx$ and integrated from $x=0$, $x=2\pi$

$$\int_0^{2\pi} f(x)\cdot\cos nx\,dx = \frac{a_0}{2}\int_0^{2\pi}\cos nx\,dx + a_1\int_0^{2\pi}\cos nx\cos x$$

$$+ \cdots + a_n\int_0^{2\pi}\cos^2 nx\,dx$$

$$+ b_1\int_0^{2\pi}\sin x\cos nx\,dx + b_2\int_0^{2\pi}\sin 2x\cos nx$$

$$+ \cdots$$

$$\therefore a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$

③ To find b_n :-

multiplying each side of eqⁿ ① by $\sin nx$ and integrated from $x=0$, $x=2\pi$.

$$\int_0^{2\pi} f(x)\sin nx\,dx = \frac{a_0}{2}\int_0^{2\pi}\sin nx\,dx + a_1\int_0^{2\pi}\sin nx.\cos x\,dx$$

$$+ \cdots$$

$$+ b_1\int_0^{2\pi}\sin x.\sin nx\,dx +$$

$$b_2\int_0^{2\pi}\sin nx\cdot\sin 2x\,dx + \cdots$$

$$+ b_n\int\sin^2 nx\,dx$$

$$\int_0^{2\pi} f(x) \sin nx \, dx = b_n \pi$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin nx \, dx$$

(2) given

$f(x) = x$. $\quad 0 < x < 2\pi$

Now. we know that,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \quad \text{———} (1)$$

let, $$a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x) \, dx$$

$$= \frac{1}{\pi} \int_0^{2\pi} x \, dx$$

4

$$= \frac{1}{\pi} \left[\frac{x^2}{2} \right]_0^{2\pi}$$

$$= \frac{1}{\pi} \left(\frac{4\pi^2}{2} \right)$$

$$\boxed{a_0 = 2\pi}$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} x \cdot \cos nx \, dx$$

$$= \frac{1}{\pi} \int_0^{2\pi} x \cdot \cos nx \, dx$$

$$= \frac{1}{\pi}\left[x \cdot \frac{\cos nx}{n} - \int 1 \cdot \frac{\sin nx}{n}\,dx\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[x \cdot \frac{\cos nx}{n} - \frac{\cos nx}{n^2}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[2\pi \frac{\sin n2\pi}{n} + \frac{\cos n2\pi}{n^2} - 0 - \frac{\cos 0}{n^2}\right]$$

$$= \frac{1}{\pi}\left[0 + \frac{1}{n^2} - \frac{1}{n^2}\right]$$

$$= 0 \quad \text{—— (3)}$$

$$b_n = \frac{1}{\pi}\int f(x) \sin nx \, dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} x \cdot \sin nx \, dx$$

$$= \frac{1}{\pi}\left[\left(-\frac{x \cos nx}{n}\right) + \int \frac{\cos nx}{n}\,dx\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[-\frac{x \cos nx}{n} + \frac{\sin nx}{n^2}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[-\frac{2\pi}{n} + 0 - (0+0)\right]$$

$$= \frac{-2}{n} \quad \text{—— (4)}$$

॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥

– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

34550

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor	

Suppliment No. : 2

Roll No. : 1309

Class :

Subject :

Test / Tutorial No. :

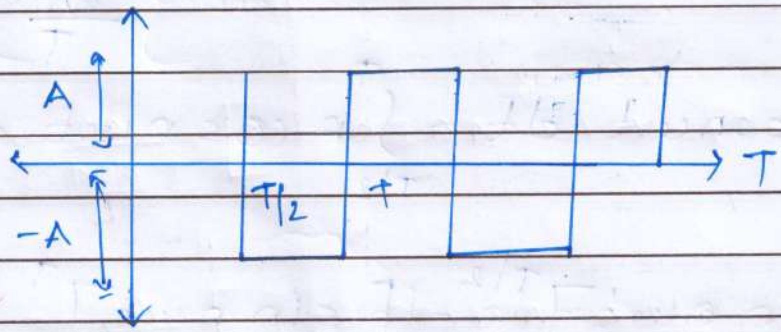
Div. :

Now eq^n ① become

$$f(x) = \frac{2\pi}{2} + 0 + \sum_{n=1}^{\infty} \left(-\frac{2}{n}\right) \sin x$$

$$\boxed{f(x) = \pi - \left(2\sin x + \sin 2x + \frac{2}{3}\sin 3x + \cdots\right.}$$

③



Here. $f(t) = A \quad ; \; 0 < t < t/2$

$= -A \quad ; \; T/2 < t < T$

for,

$$a_0 = \frac{2}{T}\int_0^T f(t)\, dt$$

$$= \frac{2}{T}\left[\int_0^{T/2} f(t)\,dt + \int_{T/2}^{T} f(t)\,dt\right]$$

$$= \frac{2}{T}\left[\int_0^{T/2} A\,dt + \int_{T/2}^{T} (-A)\,dt\right]$$

$$= \frac{2A}{T}\left[\int_0^{T/2} dt - \int_{T/2}^{T} dt\right]$$

$$= 0$$

for,

4

$$a_n = \frac{2}{T}\int_0^T f(t) \cos n\omega t \; dt .$$

[where ω - angular freq. $\frac{2\pi}{T}$]

$$= \frac{2}{T}\left[\int_0^{T/2} A \cos n\omega t \; dt - \int_{T/2}^{T} A \cos n\omega t . dt\right]$$

$$= \frac{2A}{T}\left\{\left[\frac{\sin n\omega t}{n\omega}\right]_0^{T/2} - \left[\frac{\sin n\omega t}{n\omega}\right]_{T/2}^{T}\right\}$$

$$= \frac{2A}{n\omega T}\left\{\left[\sin n \frac{2\pi}{T} \frac{T}{2} - 0\right] - \left[\sin n\frac{2\pi}{T} . T - \sin n\frac{2\pi}{T} . \frac{T}{2}\right]\right.$$

$$= \frac{2A}{n\omega T}\left[\sin n\pi - 0 - \sin 2n\pi + \sin n\pi\right]$$

$$= \frac{2A}{n\omega T}[0] = \underline{\underline{0}}$$

<u>for</u> $b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\, dt$

$$= \frac{2}{T}\left[\int_0^{T/2} A\sin n\omega t\, dt - \int_{T/2}^{T} A\sin n\omega t\, dt\right]$$

$$= \frac{2A}{T}\left\{\left[-\frac{\cos n\omega t}{n\omega}\right]_0^{T/2} + \left[\frac{\cos n\omega t}{n\omega}\right]_{T/2}^{T}\right\}$$

$$= \frac{2A}{n\omega T}\left[-\cos n\frac{2\pi}{T}\cdot\frac{T}{2} + \cos 0\right] + \left[\cos n\frac{2\pi}{T}\cdot T - \cos n\frac{2\pi}{T}\cdot\frac{T}{2}\right]$$

$$= \frac{2A}{n\frac{2\pi}{T}T}\left[-\cos n\pi + 1 + \cos 2\pi n - \cos n\pi\right]$$

$$= \frac{A}{n\pi}\left[-2\cos n\pi + 1 + \cos 2\pi n\right]$$

$$= \frac{A}{n\pi}\left[-2(1)^n + 1 + 1\right] \Rightarrow \frac{A}{n\pi}\left[-2(-1)^n + 2\right]$$

$$b_n = \frac{2A}{n\pi}\left[1 - (-1)^n\right]$$

$b_n = \frac{4A}{n\pi}$ ——— for odd integer

$b_n = 0$ ——— for even integer

We know that.

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \sin n\omega t$$

$$= 0 + 0 + \sum_{n=1}^{\infty} \frac{4A}{n\pi} \sin n\omega t$$

for odd integer

$$f(t) = \frac{4A}{\pi} \sin\omega t + \frac{4A}{3\pi} \sin 3\omega t + \frac{4A}{5\pi} \sin 5\omega t$$

$$= \frac{4A}{\pi} \left[\sin\omega t + \frac{1}{3} \sin 3\omega t + \frac{1}{5} \sin 5\omega t \ldots \right]$$

Here, amplitude of each sine wave decreases as $1 : \frac{1}{3} : \frac{1}{5} \ldots$ and frequency of each wave increases $1:3:5$ ...

(4) fourier series in complex form.

forier series of a fun^n $f(x)$ of period $2l$

$$f(x) = \frac{a_0}{2} \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

$$= \frac{a_0}{2} + a_1 \cos\frac{\pi x}{l} + a_2 \cos\frac{2\pi x}{l} + a_3 \cos\frac{3\pi x}{l} + \ldots + a_n \cos\frac{n\pi x}{l} + \ldots$$

॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥
– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

34551

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Suppliment No. : 3		Signature of Supervisor	
Roll No. : 1309		Subject :	
Class :		Test / Tutorial No. :	
		Div. :	

$$+ b_1 \sin\frac{\pi x}{l} + b_2 \sin\frac{2\pi x}{l} + b_3 \sin\frac{3\pi x}{l} + \cdots + b_n \sin\frac{n\pi x}{l}$$

$$= \frac{a_0}{2} + a_1 \frac{e^{\frac{i\pi x}{l}} + e^{\frac{i\pi x}{l}}}{2} + a_2 \frac{e^{\frac{i2\pi x}{l}} + e^{\frac{-i2\pi x}{l}}}{2} + \cdots$$

$$+ b_1 \frac{e^{\frac{i\pi x}{l}} - e^{\frac{-\pi i x}{l}}}{2i} + b_2 \frac{e^{\frac{i2x\pi}{l}} - e^{\frac{-i2\pi x}{l}}}{2i} + \cdots$$

4

$$= \frac{a_0}{2} + \frac{1}{2}\Big\{ (a_1 - ib_1) e^{i\pi x/l} + (a_1 + ib_1) e^{\frac{-\pi i x}{l}} + (a_2 - ib_2) e^{\frac{i2\pi x}{l}} + (a_2 + ib_2) e^{-i2\pi}$$

$$= \frac{a_0}{2} + \Big\{ c_1 e^{i\pi x/l} + c_2 e^{2\pi i x/l} + \cdots + c_{-1} e^{-i\pi x/l} + c_{-2} e^{-2i\pi x/l} + \cdots \Big\}$$

$$= c_0 + \frac{1}{2}\Big\{ \sum_{n=1}^{\infty} c_n e^{n\pi i x/l} + \sum_{n=1}^{\infty} c_{-n} e^{-n i\pi x/l}$$

Where,

$$c_0 = \frac{a_0}{2} = \frac{1}{2l}\int_0^{2l} f(x)\,dx$$

$$c_n = \frac{1}{2}(a_n - ib_n) = \frac{1}{2}\left\{\left(\frac{1}{l}\int_0^{2l} f(x)\cos\frac{n\pi x}{l}\,dx\right)\right.$$

$$\left. - i\left(\frac{1}{l}\int_0^{2l} f(x)\sin\frac{n\pi x}{l}\,dx\right)\right\}$$

$$= \frac{1}{2l}\int_0^{2l} f(x)\left(\cos\frac{n\pi x}{l} - i\sin\frac{n\pi x}{l}\right)$$

$$c_n = \frac{1}{2l}\int_0^{2l} f(x)\,e^{-in\pi x/l}\,dx$$

$$c_{-n} = \frac{1}{2}(a_n + ib_n) = \frac{1}{2}\left\{\left(\frac{1}{l}\int_0^{2l} f(x)\cos\frac{n\pi x}{l}\,dx\right)\right.$$

$$\left. i\left(\frac{1}{l}\int_0^{2l} f(x)\sin\frac{n\pi x}{l}\right)\right\}$$

$$= \frac{1}{2l}\int_0^{2l} f(x)\left(\cos\frac{n\pi x}{l} + i\sin\frac{n\pi x}{l}\right)dx$$

$$c_{-n} = \frac{1}{2l}\int_0^{2l} f(x)\,e^{in x\pi/l}\,dx$$

Hence

fourier series in complex form of given

$$f(x) = c_0 + \left[\sum_{n=1}^{\infty} c_n e^{i\pi n x/l} + \sum_{n=1}^{\infty} c_{-n} e^{-in\pi x/l}\right]$$

Name: Vedaja Ajay Yadav.

॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥
– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

34542

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor

Suppliment No. : 1

Roll No. : 1313

Class : M.Sc I

Subject : Fourier Series

Test / Tutorial No. : Open Book Test

Div. :

$\frac{18}{20}$

Q.1.

→ We know that, Fourier expansion of function $f(x)$ is given as,

$$f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + \dots + b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + \dots \quad \text{--- (1)}$$

(1) To find a_0 :-

Integrating above equation on both sides from $x=0$ to $x=2\pi$

$$\int_0^{2\pi} f(x)\,dx = \frac{a_0}{2}\int_0^{2\pi} dx + a_1 \int_0^{2\pi} \cos x\,dx + a_2 \int_0^{2\pi} \cos 2x\,dx + \dots + b_1 \int_0^{2\pi} \sin x\,dx + b_2 \int_0^{2\pi} \sin 2x\,dx + \dots$$

4

$$\int_0^{2\pi} f(x)\,dx = \frac{a_0}{2}\left[x\right]_0^{2\pi}$$

$$\frac{a_0 \cdot 2\pi}{2}$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx.$$

(2) To find a_n :

Multiply each side of eq^n (1) by $\cos nx$ and integrate from $x=0$ to $x=2\pi$

$$\int_0^{2\pi} f(x)\cos nx\,dx = \frac{a_0}{2}\int_0^{2\pi}\cos nx\,dx + a_1\int_0^{2\pi}\cos nx\cos x\,dx + \dots + a_n\int_0^{2\pi}\cos^2 nx$$

$$+ b_1\int_0^{2\pi}\sin x\cos nx\,dx + b_2\int_0^{2\pi}\sin 2x\cos nx\,dx + \dots$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$

③ To find b_n :-

Multiply each side of eq^n ① by $\sin nx$ and integrate from $x=0$ to $x=2\pi$

$$\int_0^{2\pi} f(x)\sin nx\,dx = \frac{a_0}{2}\int_0^{2\pi}\cos nx\,dx + a_1\int_0^{2\pi}\cos x\sin nx\,dx + \dots +$$

$$b_1\int_0^{2\pi}\sin x\sin nx\,dx + b_2\int_0^{2\pi}\sin 2x\sin nx\,dx + \dots$$

$$+ b_n\int_0^{2\pi}\sin nx\cdot\sin nx\,dx$$

$$= b_n\pi$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx.$$

Q.2.

(I) $f(x) = x \quad , 0 < x < 2\pi$

→ We know that, Fourier series is given as,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \quad \text{— (1)}$$

Let,

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\, dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} x\, dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{4\pi^2}{2}\right]$$

$$a_0 = 2\pi \quad \text{— (2)}$$

Let,

$$a_n = \frac{1}{\pi}\int_0^{2\pi} x \cos nx\, dx$$

$$= \frac{1}{\pi}\left[x\frac{\sin nx}{n} - \int 1 \cdot \frac{\sin nx}{n}\, dx\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{x \sin nx}{n} + \frac{\cos nx}{n^2}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{2\pi \sin n2\pi}{n} + \frac{\cos n2\pi}{n^2} - 0 - \frac{\cos 0}{n^2}\right]$$

$$= \frac{1}{\pi}\left[0 + \frac{1}{n^2} - \frac{1}{n^2}\right] = 0 \quad \text{— (3)}$$

Now,

$$bn = \frac{1}{\pi}\int_0^{2\pi} x \sin nx \, dx$$

$$= \frac{1}{\pi}\left[\frac{-x \cos nx}{n} + \int \frac{\cos nx}{n} dx\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{-x \cos nx}{n} + \frac{\sin nx}{n^2}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{-2\pi}{n} + 0 - (0+0)\right]$$

$$= \frac{-2}{n} \quad \text{—— (4)}$$

$\therefore$ Eq^n (1) becomes,

$$f(x) = \frac{2\pi}{2} + 0 + \sum_{n=1}^{\infty}\left(\frac{-2}{n}\right)\sin nx$$

$$= \pi - \left(2\sin x + \sin 2x + \frac{2}{3}\sin 3x + \dots\right)$$

4

Name: Vedaja Ajay Yadav

॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥

– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

34564

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor	

Suppliment No. : 2

Roll No. : 1313

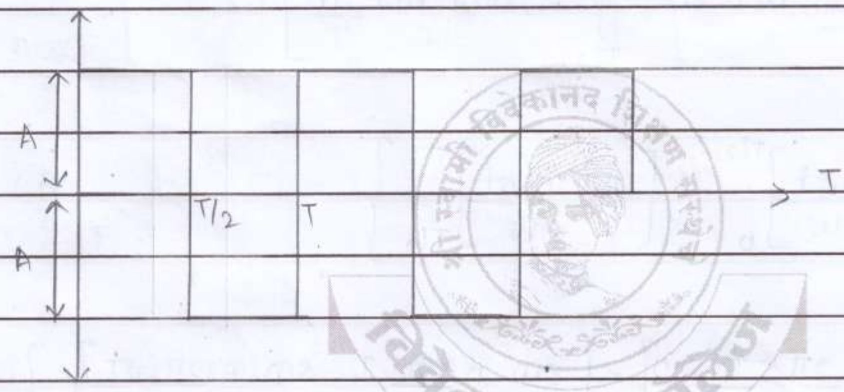
Class : M.Sc I

Subject : Fourier Series

Test / Tutorial No. : Open Book Test

Div. :

Q-3.



Here,

$$f(t) = A \quad ; \quad 0 < t < T/2$$
$$= -A \quad ; \quad T/2 < t < T$$

For,

$$a_0 = \frac{2}{T}\int_0^T f(t)\,dt$$

$$= \frac{2}{T}\left[\int_0^{T/2} f(t)\,dt + \int_{T/2}^{T} f(t)\,dt\right]$$

$$= \frac{2}{T}\left[\int_0^{T/2} A\,dt + \int_{T/2}^{T} f(t)\,dt\right]$$

$$= \frac{2}{T}\left[\int_0^{T/2} A\,dt + \int_{T/2}^{T} (-A)\,dt\right]$$

$$= \frac{2A}{T}\left[\int_0^{T/2} dt - \int_{T/2}^{T} dt\right]$$

$$= \frac{2A}{T}\left[T/2 - T + T/2\right]$$

$$= 0$$

For,

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t \; dt$$

$\left[\text{where } \omega = \text{angular frequency} = \frac{2\pi}{T}\right]$

$$= \frac{2}{T}\left[\int_0^{T/2} A\cos n\omega t \; dt - \int_{T/2}^{T} A\cos n\omega t \; dt\right]$$

$$= \frac{2A}{T}\left\{\left[\frac{\sin n\omega t}{n\omega}\right]_0^{T/2} - \left[\frac{\sin n\omega t}{n\omega}\right]_{T/2}^{T}\right\}$$

$$= \frac{2A}{n\omega T}\left\{\left[\sin n\frac{2\pi}{T}\cdot\frac{T}{2} - 0\right] - \left[\sin n\frac{2\pi}{T}\cdot T - \sin n\frac{2\pi}{T}\cdot\frac{T}{2}\right]\right\}$$

$$= \frac{2A}{n\omega T}\left[\sin n\pi - 0 - \sin 2n\pi + \sin n\pi\right]$$

$$= \frac{2A}{n\omega T}[0]$$

$$= 0$$

For,

$$b_n = \frac{2}{T}\int_0^T f(t) \sin n\omega t \, dt$$

$$= \frac{2}{T}\left[\int_0^{T/2} A \sin n\omega t \, dt - \int_{T/2}^{T} A \sin n\omega t \, dt\right]$$

$$= \frac{2A}{T}\left\{\left[\frac{-\cos n\omega t}{n\omega}\right]_0^{T/2} + \left[\frac{\cos n\omega t}{n\omega}\right]_{T/2}^{T}\right\}$$

$$= \frac{2A}{n\omega T}\left[-\cos n \cdot \frac{2\pi}{T} \cdot \frac{T}{2} + \cos 0\right] + \left[\cos n \frac{2\pi}{T} \cdot T - \cos n \frac{2\pi}{T} \cdot \frac{T}{2}\right]$$

$$= \frac{2A}{n \frac{2\pi}{T} \cdot T}\left[-\cos n\pi + 1 + \cos 2\pi n - \cos n\pi\right]$$

$$= \frac{A}{n\pi}\left[-2\cos n\pi + 1 + \cos 2\pi n\right]$$

$$= \frac{A}{n\pi}\left[-2(-1)^n + 1 + 1\right] = \frac{A}{n\pi}\left[-2(-1)^n + 2\right]$$

$$b_n = \frac{2A}{n\pi}\left[1 - (-1)^n\right]$$

$$b_n = \frac{4A}{n\pi} \text{ — for odd integer} \qquad b_n = 0 \text{ — for even integer}$$

We know that, Fourier expansion is given as,

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \sin n\omega t$$

$$= \frac{2A}{T}\left[\int_0^{T/2} dt - \int_{T/2}^{T} dt\right]$$

$$= \frac{2A}{T}\left[T/2 - T + T/2\right]$$

$$= 0$$

For,

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t \, dt$$

[where ω = angular frequency = $\frac{2\pi}{T}$]

$$= \frac{2}{T}\left[\int_0^{T/2} A\cos n\omega t \, dt - \int_{T/2}^{T} A\cos n\omega t \, dt\right]$$

$$= \frac{2A}{T}\left\{\left[\frac{\sin n\omega t}{n\omega}\right]_0^{T/2} - \left[\frac{\sin n\omega t}{n\omega}\right]_{T/2}^{T}\right\}$$

$$= \frac{2A}{n\omega T}\left\{\left[\sin n\frac{2\pi}{T}\cdot\frac{T}{2} - 0\right] - \left[\sin n\frac{2\pi}{T}\cdot T - \sin n\frac{2\pi}{T}\cdot\frac{T}{2}\right]\right\}$$

$$= \frac{2A}{n\omega T}\left[\sin n\pi - 0 - \sin 2n\pi + \sin n\pi\right]$$

$$= \frac{2A}{n\omega T}[0]$$

$$= 0$$

$$= 0 + 0 + \sum_{n=1}^{\infty} \frac{4A}{n\pi} \sin n\omega t$$

for n = odd integer

$$f(t) = \frac{4A}{\pi} \sin \omega t + \frac{4A}{3\pi} \sin 3\omega t + \frac{4A}{5\pi} \sin 5\omega t + \cdots$$

$$= \frac{4A}{\pi} \left[\sin \omega t + \frac{1}{3} \sin 3\omega t + \frac{1}{5} \sin 5\omega t + \cdots \right]$$

Here amplitude of each sine wave decreases as $1 : \frac{1}{3} : \frac{1}{5} : \ldots$ and frequency of each sine wave increases as $1 : 3 : 5 : \ldots$

34569

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor	

Suppliment No. :

Roll No. :

Class :

Subject :

Test / Tutorial No. :

Div. :

Q.4.

Fourier series of a function f(x) of period 2l is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

$$= \frac{a_0}{2} + a_1\cos\frac{\pi x}{l} + a_2\cos\frac{2\pi x}{l} + a_3\cos\frac{3\pi x}{l} + \dots + a_n\cos\frac{n\pi x}{l}$$

$$b_1\sin\frac{\pi x}{l} + b_2\sin\frac{2\pi x}{l} + b_3\sin\frac{3\pi x}{l} + \dots + b_n\sin\frac{n\pi x}{l}$$

5

$$= \frac{a_0}{2} + a_1\frac{e^{\frac{i\pi x}{l}} + e^{\frac{i\pi x}{l}}}{2} + a_2\, e^{\frac{i2\pi x}{l}} + e^{\frac{-i2\pi x}{l}} + \dots +$$

$$b_1\frac{e^{\frac{i\pi x}{l}} - e^{\frac{-\pi i x}{l}}}{2i} + b_2\frac{e^{\frac{i2x\pi}{l}} - e^{\frac{-i2\pi x}{l}}}{2i} + \dots$$

$$= \frac{a_0}{2} + \frac{1}{2}\Big\{(a_1 - ib_1)e^{i\pi x/l} + (a_1+ib_1)e^{\frac{-i\pi x}{l}} + (a_2 - ib_2)e^{\frac{i2\pi x}{l}} + (a_2+ib_2)e^{\frac{-i2\pi x}{l}} + \dots\Big\}$$

$$= \frac{a_0}{2} + \frac{1}{2}\Big\{\frac{(a_1-ib_1)}{2}e^{i\pi x/l} + \frac{(a_1 \pm ib_2)}{2}e^{i2\pi x/l} + \dots + \frac{(a_1+ib_1)}{2}e^{\frac{-i\pi x}{l}} + \frac{(a_2+ib_2)}{2}e^{\frac{-2i\pi x}{l}} + \dots\Big\}$$