Vivekanand College, Kolhapur. (Autonomous)
Department of Physics
Internal Examination Notice
2019-20

Date:11 September 2019

All students of class B.Sc. I, B.Sc. II and B.Sc. III are hereby noticed that the first term internal evaluation examination is scheduled as per following time table.

Nature of question paper:

For B.Sc. I : Long answer question (Any one from given two questions) for 10 marks

Short answer question (Any two from given four questions) for 10 marks

For B.Sc. II : Long answer question (Any one from given two questions) for 10 marks

Short answer question (Any two from given four questions) for 10 marks

For B.Sc. II (Astro) : Long answer question (Any one from given two questions) for 10 marks

Short answer question (Any two from given four questions) for 10 marks

For B.Sc. III : Long answer question (Any one from given two questions) for 10 marks

Short answer question (Any two from given four questions) for 10 marks

Internal Evaluation Examination 2019-20.
SEM I, SEM III and SEM V
Time Table

Sr. No.	Class	Paper	Date	Time
1.	B.Sc. I	Paper I	23/09/2019	11:00 am to 12:00 pm
2.	B.Sc. II	Paper III	23/09/2019	11:00 am to 12:00 pm
3.	B.Sc. II (Astrophysics)	Paper I	25/09/2019	11:00 am to 12:00 pm
4.	B.Sc. III	Paper V (section I)	26/09/2019	11:00 am to 12:00 pm
		Paper V (section II)		01:00 am to 2:00 pm
		Paper VI (section I)	27/09/2019	11:00 am to 12:00 pm
		Paper VI (section II)		01:00 am to 2:00 pm

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HOD
Head of the
Department of Physics
Vivekanand College, Kolhapur.

"Dissemination of Education for Knowledge, Science and Culture"
- Shikshanmaharshi Dr. Bapuji Salunkhe

Shri Swami Vivekanand Shikshan Sanstha's

Vivekanand College, Kolhapur

(Autonomous)

Department of Physics

Internal exam (2019-20)

B.Sc.III Sem V

Attendance Sheet

Roll No.	Name Of The Student	Signature			
8001	Chougule Abhijeet Bajirao				
8002	Dalvi tejas chetan				
8003	Dinde Akash Sadashiv				
8004	Gaikwad Suraj Dhananjay				
8005	Ghosalkar Pranav Shankar				
8006	Harshad Sitaram Katroot				
8007	Jadhav Pratiksha Harish				
8008	Joshi Sourabh Kiran				
8009	Kamble Prasad Vilas				
8010	Kumbhar Prathmesh Mallikarjun	P.M.Kumbhar	P.M.Kumbhar	P.M.Kumbhar	P.M.Kumbhar
8011	Kumbhar Jayvant Rajaram	YRK	YRK	YRK	YRK
8012	Manasi Vinayak Kulkarni				
8013	Manasi Kahnderao Jagadale				
8014	Nalavade Ankita Amar				
8015	Paranjape Anish Shriram	AP	AP	AP	AP
8016	Patil Amruta Bhuigonda	A.Patil	A.Patil	A.Patil	A.Patil
8017	Patil Sujata Anandrao				
8018	Patil Jeevan Maruti				
8019	Patil Tejaswini Krishna				
8020	Paul Jonathan Sanjay				
8021	Potdar Aishwarya Sharad				
8022	Radhika Baburao Shinde				
8023	Ragini Jayprakash Benake				
8024	Sandhya Sudhakar Dingane				
8025	Sawant Rohit Ramchandra				
8026	Sourabh Vijay Ghatage				
8027	Sujit Dinkar Katale	S.P.K	S.P.K.	S.P.K.	S.P.K
8028	Swaranjali Sanjay Shinde				
8029	Swarupa Baburao Dhavale				
8030	Tanvi Vikas Mohite				
8031	Tibile Rohan Arjun				
8032	Tushar Arvind Patil				

8033	Yogita Vishnu Zirenge	Yogita	Yogita	Yogita	Yogita

Internal Examinar.......................................

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Vivekananda College Kolhapur (Autonomous).

Department of Physics: Internal examination2019-20

B.Sc. III Semester V

Subject: Atomic and Molecular Spectra, Astronomy and Astrophysics

Marks: 20 (Each question carry one mark) **Time : 20 min**

Q.1 Attempt any ONE **(10)**

1. Explain the principle of electron-synchrotron with special reference to two-step acceleration.
2. Discuss the principle of proton-synchrotron with a special reference to two step acceleration.

Q.2 Attempt any TWO **(10)**

1. What are nucleons? Explain their intrinsic properties.
2. What is the shape and size of nucleus?
3. Discuss different methods used to measure nuclear radius.

।। ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ।।
– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

27461

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor

Suppliment No. :

Roll No. : 8032

Class : BSc-III

Subject : Elements of Modern physics

Test / Tutorial No. : Internal Examination (2022-23)

Div. :

12/20

Q1.

1] When two or more number of electrons from one atom may transfer to another atom then ____ bond is formed

→ ii) ionic.

2] In the classical Expression for energy of rotating diatomic molecule, there is no restriction on the value of ____

→ ii) L.

3] In rotational spectra, the selection rule for transition is ____.

→ iii) $\Delta J = \pm 1$.

4] The spectral lines found on low frequency side of exciting lines are known as

→ i) Stokes lines.

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5) In experimental set up of Raman Effect, cylinder 'c' is filled with saturated solution of ______

→ i) Sodium Nitrate.

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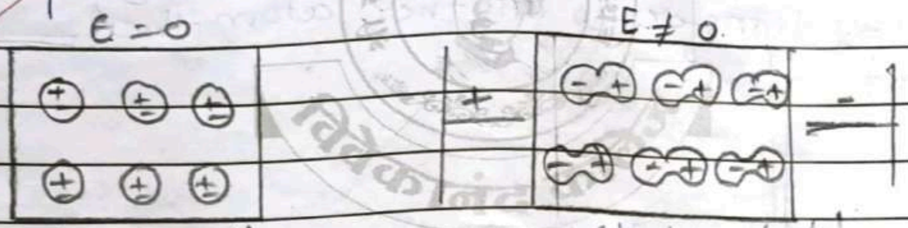
Q3.

2]

→ Classical Theory of Raman Effect:

When an electric field is applied ~~to~~ on a molecule. the polarization takes place of the molecule & the induced ~~ionic~~ dipole moment is produced.



No electric field applied

Electric field applied

The magnetude of Induced dipole moment (p) is directly proportional to the strength of applied electric field i.e. $p \propto E$.

$$\therefore \quad p = \alpha E \quad \text{——— (1)}$$

where α is called as polarizibility

The strength of the applied electric field is given by,

$$E = E_0 \sin 2\pi\nu_0 t \quad \text{——— (2)}$$

using eq (2) in eq (1).

$$p = \alpha (E_0 \sin 2\pi\nu_0 t) \quad \text{——— (3)}$$

If the molecule starts excutiving the vibrational motion in the presence of the external electric field.

$$\alpha = \alpha_0 + \beta \sin 2\nu_{vib} t \rightarrow \text{———} (4)$$

$\therefore \alpha_0$ = eq^n polarbizibility
β = Range of polarizibility
ν = vibrating molecule

using eq (4) in eq (3).

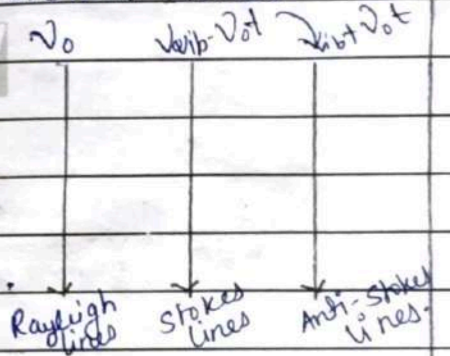
$$p = (\alpha_0 + \beta \sin 2\nu_{vib} t) \cdot \epsilon_0 \sin 2\nu_0 t$$

$$p = \alpha_0 E_0 \sin 2\nu_0 t \; \frac{\beta}{2} \left(2 \sin(\nu_{vib}) - \sin(\nu_0 t)\right)$$

$$= \alpha_0 \epsilon_0 \sin 2\nu_0 t \; \frac{\beta}{2} \left\{ \cos(\nu_{vib} - \nu_0 t) - \cos(\nu_{vib} + \nu_0 t) \right\}$$

$$= \alpha_0 \epsilon_0 \sin 2\nu_0 t \; \frac{\beta}{2} \cos(\nu_{vib} - \nu_0) 2t - \cos(\nu_{vib} + \nu_0) 2t$$

from above eq we come to known that there are three types:



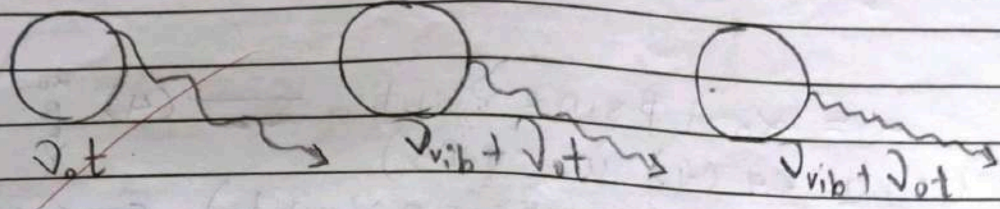
1] $\nu_0 t$ = Rayleigh lines
2] $\nu_{vib} - \nu_0 t$ = Stokes lines
3] $\nu_{vib} + \nu_0 t$ = Anti-stokes lines.

• Limitation of classical Theory of Raman effect.

1] The stokes and Anti-stoke lines are emitted from the same molecules. But it is not ture because they have different intensities.

Q2.

I) Quantum Theory of Raman Effect.



The ~~molecules~~ atoms in the molecules. are stable

02

Name:- Sapana Raviranjan Singh.

Signature of Supervisor

Suppliment No. :

Roll No. : 8015

Class : B.Sc. III Physics

Subject : Physics

Test / Tutorial No. : Internal Exam

Div. :

18/20

Q. 1)

1) When two or more number of electrons from one atom may transfer to another atom then 2) ionic bond is formed.

2) In the classical ~~et.~~ expression for energy of rotating diatomic molecule, there is no restriction on the value of 2) L

3) In rotational spectra, the selection rule for transition is 3) $\Delta J = \pm 1$

4) The spectral lines found on low frequency side of exciting lines are known as 1) Stokes lines.

5) In experimental set up of Raman Effect, cylinder 'C' is filled with saturated solution of 1) Sodium Nitrate.

04

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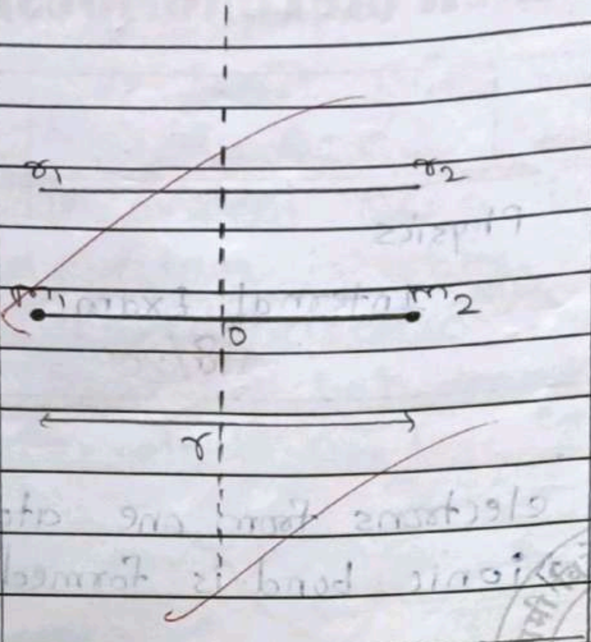
Q 2) Diatomic molecule as rigid rotator and its rotational energy states.

→ Diatomic molecule means arrangement of two atoms in the molecule which ~~is~~ can be simillar or dissimillar.



Consider a molecule as rigid rotator moving in its rotational energy states with angular velocity ω & I be the moment of Inertia.

Let m_1 & m_2 be the masses of the molecule with center 'O' perpendicular to the plane of mass m_1 & m_2. r_1 & r_2 are the distance from perpendicular axis 'O' of mass m_1 & m_2.

Hence it is given by

$$m_1 r_1 = m_2 r_2 \quad \text{--- (1)}$$

As r is the distance between m_1 & m_2, then it is given by $r = r_1 + r_2$

$$\therefore r_1 = r - r_2 \quad \text{--- (2)}$$

Substituting the value of r_1 in eq^n (1) we get,

$$m_1(r - r_2) = m_2 r_2$$

$$\cancel{m_1 r +}$$

$$m_1 r - m_1 r_2 = m_2 r_2$$

$$m_1 r = m_2 r_2 + m_1 r_2$$

$$m_1 r = r_2(m_1 + m_2)$$

$$\therefore r_2 = \frac{m_1 r}{(m_1 + m_2)}$$

$$\therefore \frac{m_1 r}{(m_1+m_2)} = r_2$$

Simillarly for r_1 we get,

$$r_1 = \frac{m_2 r}{(m_1 + m_2)}$$

Q2) 2) The M.I of molecule as rigid rotator ~~& its~~ is given by

$$I = m_1 r_1^2 + m_2 r_2^2$$

substituting the value of r_1 & r_2 we get,

$$I = m_1\left(\frac{m_2 r}{m_1+m_2}\right)^2 + m_2\left(\frac{m_1 r}{m_1+m_2}\right)^2$$

$$= \frac{m_1 m_2}{m_1+m_2} r^2 \left(\frac{m_1+m_2}{m_1+m_2}\right)$$

$$\therefore I = \frac{m_1 m_2}{m_1+m_2} r^2$$

$$\therefore I = \mu r^2 \quad \text{—— (3)}$$

Where $\mu = \frac{m_1 m_2}{m_1+m_2}$

In classical expression for energy of rotating diatomic molecule, there is no restriction on the value of L.

$\therefore L = I\omega$

$\therefore$ Kinetic Energy of the di molecule is given by

$$E = \frac{1}{2}\frac{I^2\omega^2}{I}$$

$$\therefore E = \frac{1}{2}\frac{L^2}{I}$$

Substituting the value of I from eq^n (3) we get,

$$\therefore E = \frac{1}{2}\frac{L^2}{\mu r^2} \quad \text{—— (4)}$$

08

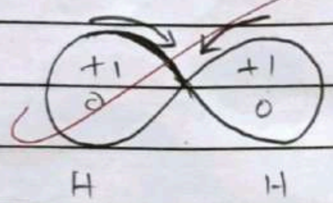
$\therefore$ Equation (4) gives the energy state of diatomic Molecule as rigid rotator.

Q3) 1) Types of bonds.

A molecule is a stable arrangements ~~of~~ two or more atoms. There are 3 types of bonds.

1) Covalent Bond.
2) Ionic Bond
& 3) No Bond.

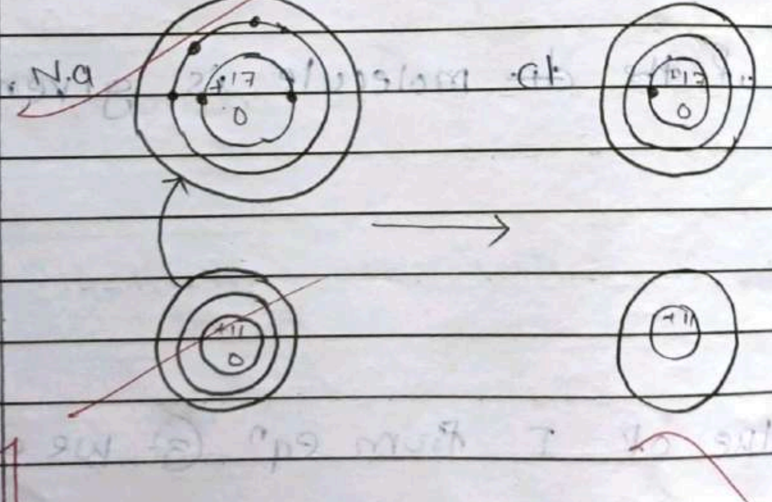
1) Covalent Bond formation:- When one or more pair of electron is shared from two atoms, then the shared electrons spends more time in the avarage between their parent nuclei, then the covalent Bond is formed.



Covalent Bonding

The electron spends more time between the atoms rather the moving elsewhere. The fig shows covalent Bond of H_2 molecules. This in the example of covalent Bond.

2) Ionic Bond formation:- When two or more number of electrons from one atom may transfer to another atom then ionic bond is formed.



॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥
– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

27453

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Name:- Aaryan Patil

Signature of Supervisor	

Suppliment No. :

Roll No. : 8012

Class : BSc - III

Subject : Physics Elements of modern Physics

Test / Tutorial No. : Internal exam

15/20

Div. :

Q1.

1) (ii) ~~ionic~~

2) (ii) L

3) (iv) $\Delta J = \pm 1/2$

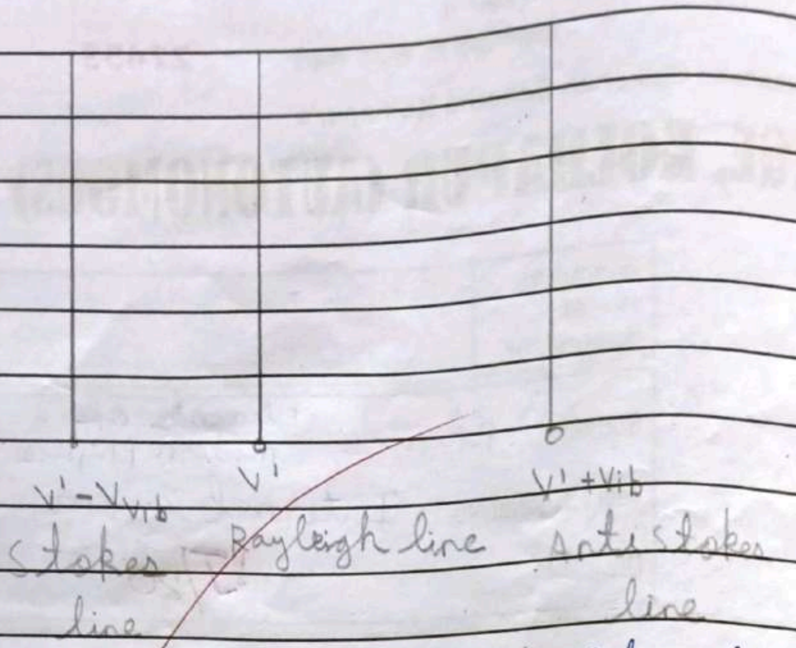
4) (i) Stokes lines

5) (i) Sodium Nitrate

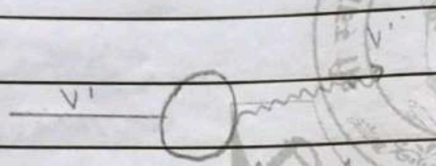
04

Q2

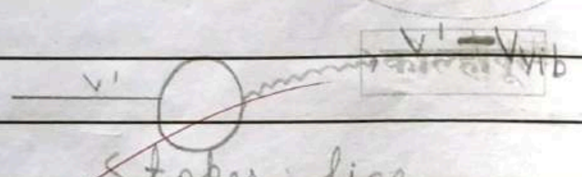
1) In the quantum theory of Raman effect, the three visible lines are present i.e. Rayleigh line, the stokes line and the antistokes line.
The Rayligh line is denoted by ν', the Stokes line is denoted by $\nu' - \nu_{vib}$, antistokes line is denoted by $\nu' + \nu_{vib}$



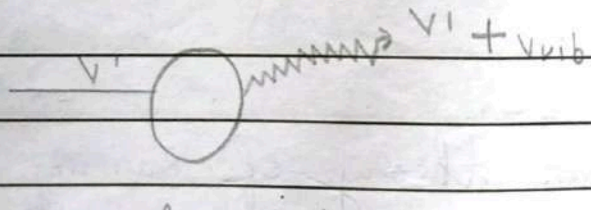
Here, The three visible lines are on the course of Rayleigh line, Stokes line and Anti Stokes line.



Rayleigh lines



Stokes line



Anti Stokes line

In the Rayleigh lines, the energy of the molecules is not transferred and hence no change takes place in the breaking of bonds.

In the Stokes line, the energy of the molecules is transffered but the energy

which is being transffered is in negative. In The antistokes line, the energy which is transffered is in the positive and hence breaking of bonds takes place.
There are a few applications of Raman effect.

1. It is used to describe the molecular structure of the molecules.

2. It is used to calculate the rotational and the vibrational energies of the molecules

3. It is used to describe the composition of the molecule.

4. It is used for the real arrangement of atoms in the molecule.

5. It is used to predict the spin quantum properties of the molecule.

06

Q3.

1) On the basis of bond formation, there are four types of bond formations which takes place in the molecule.
1. Ionic bond
2. Covalent bond
3. Van der walls
4. No bonds

(i) Ionic bond :- Its formation takes place when one or more electrons from the atom of one molecule is shared to the atom of another molecule is called as an ionic bond. Hence ionic bond is strong in nature.

(ii) Covalent bond :- Heat is required in order to supply external energy for the bond transfer to take place. In Covalent bond, the atoms are shared in between two atoms of two molecules.
ex:- Na^+ and Cl^- ions form NaCl molecule which is complex in nature.

(iii) Van der Waal's bond :- The bond which is formed in between two atoms of two molecules using strong Van der Waal's forces are called as Van der Waal's bond. These Van der Waal's forces may be strong or weak. These forces of attraction depend on the heat supplied.

(iv) No bond :- When sometimes during a reaction, the ion exchange does not take place, so there is sometimes no bond formation in between the two atoms of the two molecules. Hence in such situations, no bond is formed. So sometimes no bond is formed during ion exchange in between the atoms.

08

Vivekananda College Kolhapur (Autonomous).
Department of Physics: Internal examination 2019-20
B.Sc. III Semester V
Subject: Classical mechanics

Marks: 20 (Each question carry one mark) Time : 20 min

Q.1 Attempt any ONE **(10)**

1. Obtain an expression for the divergence of vector field in orthogonal curvilinear co-ordinate system. Extend the above formula in spherical polar co-ordinate system.
2. Obtain an expression for the curl of vector field in orthogonal curvilinear co-ordinates.

Q.2 Attempt any TWO **(10)**

1. Obtain an expression for gradient of a scalar field in orthogonal curvilinear co-ordinate system.
2. Obtain Laplacian operator in orthogonal curvilinear co-ordinate. Extend the result in cylindrical co-ordinates.
3. Describe spherical polar co-ordinate system.

Name:- Girish Chandrakant Mane. 20/20

Class - Bsc - III Div - B Roll No - ~~7243~~ 8031

Internal Exam

VIVEKANAND COLLEGE, KOLHAPUR.

Q.1 Write the correct alternative.

1) Mass is the measure of inertia in linear motion.

2) Acceleration of a body rolling down in an inclined plane is independent of mass of the body.

3) Force in rotational motion is analogous to torque in translation motion.

4) Moment of inertia of a spherical shell about its diameter is $\frac{2}{3} mR^2$.

Q.2 Answer of the following question.

1) Explain analogy of rotational motion with translation motion.

→ 1) It is noticed that there is an analogy between varies physical quantity in rotational & translitional motion.

2) In rotational motion about a fixed axies, the moment of inertia (I) is analogy us to the mass (m) in linear motion. But mass is the linear motion is the major of inertia of the body.

3) There for, moment of inertia is also regarded as the rotational inertia.

4) The moment of inertia of the body is case of, rotational motion plays, the same roll as the mass of body in translatary motion.

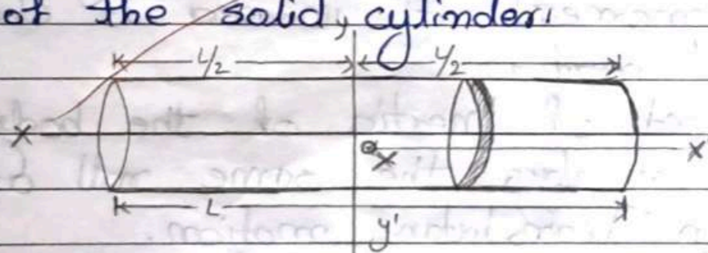
5) In case of translatary motion the inertia of the body depends totaly upon its mass, but in case of rotational motion on the moment of inertia of the body, ~~and on~~ not only depends on the mass of body but also on distribution of mass about the given axis of rotation.

6) The analogy between varius physicaly quantities in two types of motion.

Transiational motion	Rotational motion
1) Mass = m	Moment of inertia = I
2) Displacement = s or x	Angular displacement = θ
3) Velocity = v	Angular velocity = ω
4) Accelaration = a	Angular accelatation = α
5) force = F = ma	Torque = T = Iα
6) Linear Momentum = P = mv	Angularmomentum = L = Iω
7) kinetic energy = $\frac{1}{2}mv^2$	Rotational K.E. = $\frac{1}{2}I\omega^2$
8) Work done = F.s.	Work done = I.Q.

Q.2 Derive and expression of M.I of a solid cylinder about its axis of symmetry.

→ 1) Let M be the mass, R be the radius & L be the length of the solid, cylinder.



2) The mass per unit length of the solid cylinder is M/L, Let yy' be the axis passing through its center O and perpendicular to its own axis xx' as shown in fig.

3) To find its moment of inertia imagin that the cylinder to be made up of large no. of thin disc lat, ~~...~~ us ~~cylind~~ consider one of usch disc' that a distance x from o, the thikness of the disc is dx obusly, mass of the disc is $(m/L)\,dx$

4) There fore, moment of inertia of disc about it diameter = Mass of disc $\times \frac{(\text{radius})^2}{4}$

$$= \frac{M}{L}\,dx \cdot \frac{R^2}{4}$$

According to the principal of parallel axis a moment of inertia of the disc about the axis.

$$YY' = \frac{m}{L}\,dx \cdot \frac{R^2}{4} + \frac{M}{L}\,dx \cdot x^2$$

5) The moment of inertia of the ~~..~~ whole cylinder abowd the axis YY' can be obtained by intergration the abowe eq^n between the limits $x=0$, to $L/2$ and multiplying the result by 2

6) Therefore the moment of inertia of the cylinder about the axis YY' is,

$$I = \int dI = 2\int_0^{L/2} \left(m/L \; R^2/4 \, dx + m/L \; x^2\, dx \right)$$

$$= 2\frac{m}{L}\left[\frac{R^2}{4}x + \frac{x^3}{3}\right]^{L/2}$$

$$= 2\frac{m}{L}\left(\frac{R^2}{4} + \frac{L^3}{8\times 3}\right]$$

$$= 2\frac{m}{L}\left[\frac{R^2 \cdot L}{8} + \frac{L^3}{24}\right]$$

$$I = m\left[\frac{R^2}{4} + \frac{L^2}{12}\right]$$

This is the required expression.

Name - Darshan shivaji Naik.
Class - B Sc. III Div = B. Roll No. 8033 ~~7235~~, Physics
sem - I.

INTERNAL EXAMINATION

16/20

PHYSICS

Q.1. Explain analogy of rotational motion with translational motion.

→ i) It is noticed that there is an analogy between various physical quantities in rotational and, translational motion.

ii) In rotational motion about a fixed axis, the moment of inertia (I) is analogues to mass (m) in linear motion. But mass is the linear motion is the measure of inertia of the body.

iii) Therefore, moment of inertia is also regarded as the rotational motion.

iv) The moment of inertia of the body in case of rotational motion plays the same role as the body in translatory motion.

v) In case of translatory motion, the inertia of the body depends totally upon its mass, but in case of rotational motion the moment of inertia of the body not only depends on the mass of the body but also on distribution of mass about the given axis of rotation.

vi) The analogy between various physical quantities in two types of motion.

Sr. No	Translational Motion	Rotational Motion.
1	Mass = m	Moment of inertia = I
2	Displacement = S or x	Angular displacement. θ
3.	Velocity = v	Angular velocity = ω

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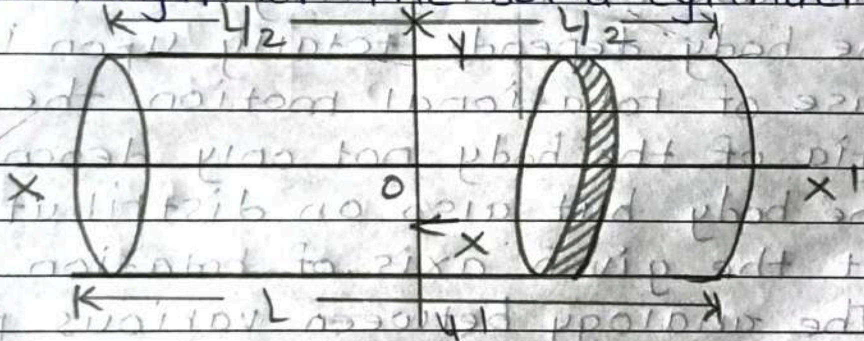
4.	Accelaration = a	Angular Accelaration = α.
5.	Force = F = ma.	Torque = I = $I\alpha$.
6.	Linear Momentum = P = mv	Angular momentum = L = $I\omega$.
7.	kinetic energy = $\frac{1}{2} mv^2$	Rotational K.E = $\frac{1}{2} I\omega^2$
8.	work done = f.s.	work done = I.θ.

2) Derive an expression for M.I of a solid cylinder about its axis of symmetry

→ 1) Let M be the mass, R be the radius and L be the length of the solid cylinder.



2) The mass per unit length of the solid cylinder is M/L. Let YY' be the axis passing through its centre O and perpendicular to its own axis XX' as shown in fig.

3) To find its moment of inertia, imagine that the cylinder to be made up of large number of thin discs. Let us consider one of such disc at a distance x from O. The thickness of the disc is dx. obviously, mass of the disc is $(M/L)\,dx$.

4) Therefore, moment of inertia of the disc about its diameter = mass of disc × $\frac{(\text{radius})^2}{4}$.

$$= \frac{m}{L}\,dx \cdot \frac{R^2}{4}.$$

4) According to the principle of parallel axes a moment of inertia of the disc about the axis YY' = $\frac{m}{L}dx\,\frac{R^2}{4} + \frac{m}{L}dx.\,x^2.$

5) The moment of inertia of the whole cylinder about the axis YY' can be obtained by integrating the equation between the limits $x = 0$ to $L/2$ and multiplying the result by 2.

6) Therefore, the moment of inertia of the cylinder about the axis YY' is,

$$I = \int dI = 2\int_0^{l/2} \left(m/L\,R^4/4\,dx + m/L\,x^2 dx\right)$$

$$= 2\,\frac{m}{L}\left[\frac{R^4}{4}x + \frac{x^3}{3}\right]_0^{L/2}$$

$$= 2\,\frac{m}{L}\left[\frac{R^4}{4}\cdot\frac{L}{2} + \frac{L^3}{8\times 3}\right]$$

$$= 2\,\frac{m}{L}\left[\frac{R^4 . L}{8} + \frac{L^3}{24}\right]$$

$$I = m\left[\frac{R^4}{4} + \frac{L^2}{12}\right]$$

This is the required expression.

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class - B.sc - III Div - B
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Sub :- Physiecs (Internal).
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Kolhapur.

20/20

Q1 Write the correct alternative.

1) Mass is the measure of inertia in linear motion.

2) Acceleration of a body rolling down in an inclined plane in independent of Mass of the body.

3) Force in rotational motion in analogus to torque in translation motion.

4) moment of inertia of a spherical shell about its diameter is $\frac{2}{3}MR^2$.

4

Q2 Answer the following question.

1) Explain analogy of rotatinal motion with translational motion.

→ ① It is noticed that there is an analogy between various physical quantities in rotational and translational motion.
② In rotational motion about a fixed axis, the moment of inertia (I) is analogus to the mass (m) in linear motion. But mass is the linear motion is the measure of inertia of the body.
③ Therefore moment of inertia is also regarded as the rotational inertia.
④ The moment of inertia of the body in case of rotational motion plays the same role as the mass of the body in translatory motion.

P.T.O

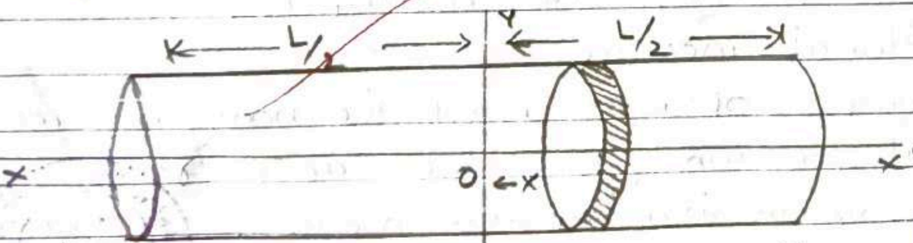
⑤ In case of translatory motion, the inertia of the body depends totally upon its mass, but in case of rotational motion the moment of inertia of the body not only depends on the mass of the body but also an distribution of mass about the given axis of rotation.

⑥ The analogy between various physical quantities in two types of motion.

Sr. No	Translational Motion	Rotational Motion.
1	Mass = m	Moment of intertia - I
2	Displacement = S or x	Angular displacement - θ
3	velocity = v	Angular velocity = ω
4	Accelaration = a	Angular Accelaration = α
5	Force = F - ma	Torque = T - $I\alpha$
6	work done = F.S	work done = T. θ
7	Linear Momentum = P = mv	Angular Momentum = L = $I\omega$
8	kinetic energy = $\frac{1}{2} mv^2$	Rotational K.E = $\frac{1}{2} I\omega^2$

2. Derive an expression for M.I of a Solid cylinder about its axis of Symmetry.

→ ① Let M be the mass, R be the radius and L be the length of the Solid Cylinder.



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② The mass per unit length of the solid cylinder is M/L. Let YY' be the axis passing through its centre O and perpendicular to its own axis XX as shown in Fig.

③ To find its moment of inertia, imagine that the cylinder to be made up of large number of thin discs. Let us consider one of such disc at a distance x from O. The thickness of the disc is dx. Obviously, mass of the disc is $(M/L)\,dx$.

④ Therefore, moment of inertia of the disc about its diameter = mass of disc × $\frac{(\text{radius})^2}{4}$

$$= \frac{M}{L} dx \cdot \frac{R^2}{4}$$

④ According to the principle of parallel axis a moment of inertia of the disc about the axis YY' $= \frac{M}{L} dx \frac{R^2}{4} + \frac{M}{L} dx \cdot x^2$

⑤ The moment of inertia of the whole cylinder about the axis YY' can be obtained by integrating the above equation between the limits $x = 0$ to $L/2$ and multiplying the result by 2.

⑥ Therefore, the moment of inertia of the cylinder about the axis YY' is,

$$I = \int dI = 2\int_0^{L/2} \left(\frac{M}{L}\frac{R^4}{4} dx + \frac{M}{L} x^2 dx\right)$$

$$= 2\frac{M}{L}\left[\frac{R^4}{4}x + \frac{x^3}{3}\right]_0^{L/2}$$

$$= 2\frac{M}{L}\left[\frac{R^4}{4}\cdot\frac{L}{2} + \frac{L^3}{8\times 3}\right]$$

$$= 2\frac{M}{L}\left[\frac{R^4 \cdot L}{8} + \frac{L^3}{24}\right]$$

$$I = M\left[\frac{R^4}{4} + \frac{L^2}{12}\right]$$

This is the required expression.

Vivekananda College Kolhapur (Autonomous).
Department of Physics: Internal examination 2019-20
B.Sc. III Semester V
Subject: Mathematical Physics

Marks: 20 (Each question carry one mark) **Time : 20 min**

Q.1 Attempt any ONE **(10)**

1. Discuss Hamilton variational principle.
2. Derive Hamilton's canonical equation of motion from variational principle.

Q.2 Attempt any TWO **(10)**

1. State equivalence of Lagrange's and Newton's equations.
2. Write a note on degree of freedom and constraints.
3. What is relation between H and L?

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– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

34051

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

20/20

Signature of Supervisor:

Suppliment No. :

Subject : mathematical physics

Roll No. : 8031

Test / Tutorial No. :

Class : BSC-III

Div. :

Q. 1.

Ans :- The divergence of vector function may be obtained from Gauss divergence theorem. Gauss divergence theorem in Vector analysis is given by

$$\iiint_V (\nabla \cdot A)\, dv = \iint_S \vec{A}\, ds$$

where $\vec{A} = \vec{A_1}\vec{u_1} + \vec{A_2}\vec{u_2} + \vec{A_3}\vec{u_3}$

By the mean value theorem for integrals, we write

$$\overline{\operatorname{div}\vec{A}} \iiint_V dv = \iint_S \bar{s}\, ds$$

or $$\overline{\operatorname{div}\vec{A}} = \lim_{dv \to 0} \frac{\iint_S \vec{A}\, ds}{\iiint_V dv}$$

Now consider the volume element Δv of infinitesimal parallelopiped having edges h_1du_1, h_2du_2 & h_3du_3.

Let us find first component of $\iint_S \vec{A}\, ds$ along u_1 as

Let A_1 be the energy per unit area entering the parallelopiped at the face OBHC having area $h_2 h_3 du_2 du_3$. The energy energy leaves the parallelopiped at the face AKGT having the same area $h_2 h_3 du_2 du_3$. energy entering is supposed to be -ve & that leaving is the Therefore we can write.

Energy entering through OBHC = $-A_1 h_2 h_3 du_2 du_3$.

Energy leaving through AKGT = $A_1 h_2 h_3 du_2 du_3 + \frac{\partial}{\partial u_1}(A_1 h_2 h_3) du_1 du_2 du_3$

The net energy outgoing is the sum of energy entering & energy leaving is

$$\left[\iint_s \vec{A}\, ds\right]_{u_1} = \frac{\partial}{\partial u_1}(A_1 h_2 h_3 du_2 du_3) du_1$$

$$= \frac{\partial}{\partial u_1}(A_1 h_2 h_3) du_1 du_2 du_3$$

Similarly, component along u_2 & u_3 can be written as

10

$$\left[\iint_s \vec{A}\, ds\right]_{u_2} = \frac{\partial}{\partial u_2}(A_2 h_3 h_1) du_1 du_2 du_3$$

&

$$\left[\iint_s \vec{A}\, ds\right]_{u_3} = \frac{\partial}{\partial u_3}(A_3 h_1 h_2) du_1 du_2 du_3$$

The contribution from all six faces of ΔV is

$$\left[\frac{\partial}{\partial u_1}(\vec{A} h_2 h_3) + \frac{\partial}{\partial u_2}(\vec{A} h_1 h_3) + \frac{\partial}{\partial u_3}(\vec{A} h_1 h_2)\right] du_1 du_2 du_3$$

Volume of parallelopiped is

$\Delta V = \iiint dv = h_1 h_2 h_3 du_1 du_2 du_3$ & taking the limit as du_1, du_2, du_3 approach zero, we find from equation,

$$\nabla \cdot \bar{A} = \frac{\left[\frac{\partial}{\partial u_1}(\vec{A}_1 h_2 h_3) + \frac{\partial}{\partial u_2}(A_2 h_1 h_3) + \frac{\partial}{\partial u_3}(A_3 h_1 h_2)\right] du_1 du_2 du_3}{h_1 h_2 h_3 du_1 du_2 du_3}$$

$$\therefore \nabla \cdot \bar{A} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}(\vec{A} h_2 h_3) + \frac{\partial}{\partial u_2}(\bar{A}_2 h_1 h_3) + \frac{\partial}{\partial u_3}(\vec{A} h_1 h_2)\right]$$

This is the required eq^n for divergence of a vector field.

Q. 2

Ans- We know that

$$\text{div } \vec{A} = \vec{\nabla} \cdot \vec{A} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}(\bar{A} h_2 h_3) + \frac{\partial}{\partial u_2}(\bar{A} h_1 h_3) + \frac{\partial}{\partial u_3}(\bar{A} h_1 h_2)\right]$$

Let $\vec{A} = \vec{\nabla} \cdot F$, where f is a scalar field, substituting,

$$\text{div } \vec{A} = \vec{\nabla} \cdot \vec{\nabla} f = \nabla^2 f$$

$$= \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}\left(h_2 h_3 (\vec{\nabla} f)_{u_1}\right) + \frac{\partial}{\partial u_2}\left(h_3 h_1 (\vec{\nabla} F)_{u_2}\right) + \frac{\partial}{\partial u_3}\left(h_1 h_2 (\vec{\nabla} F)_{u_3}\right)\right]$$

$$\nabla^2 f = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}\left(\frac{h_2 h_3}{h_1}\frac{\partial f}{\partial u_1}\right) + \frac{\partial}{\partial u_2}\left(\frac{h_1 h_3}{h_2}\frac{\partial f}{\partial u_2}\right) + \frac{\partial}{\partial u_3}\left(\frac{h_1 h_2}{h_3}\frac{\partial f}{\partial u_3}\right)\right]$$

The Laplacian operator ∇^2 in orthogonal curvilinear co-ordinate is given by.

$$\nabla^2 = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}\left[\frac{h_2 h_3}{h_1}\frac{\partial}{\partial u_1}\right] + \frac{\partial}{\partial u_2}\left[\frac{h_3 h_1}{h_2}\frac{\partial}{\partial u_2}\right] + \frac{\partial}{\partial u_3}\left(\frac{h_1 h_2}{h_3}\frac{\partial}{\partial u_3}\right)\right]$$

Q. 3

Ans Let us consider $\phi(u_1, u_2, u_3)$ a scalar function. The gradient of the scalar field ϕ in the direction of u_1-axis can be written as.

$$(\text{grad } \phi)_{u_1} = (\nabla \phi)_{u_1} = \lim_{\delta u_1 \to 0} \frac{\phi(B) - \phi(A)}{AB}$$

Where $\phi(B)$ & $\phi(A)$ are the values of scalar function ϕ at B & A separated by distance $AB = h_1 du_1$. The quantity $[\phi(B) - \phi(A)]$ may be taken as increase in ϕ on travelling a distance $AB = h_1 du_1$ this may be written as $\partial\phi$ for the limiting case where $\delta u_1 \to 0$

$$\therefore (\text{grad } \phi)_{u_1} = (\nabla\phi)_{u_1} = \lim_{\delta u_1 \to 0} \left[\frac{\delta\phi}{h_1 du_1}\right]$$

$$(\nabla\phi)_{u_1} = \frac{1}{h_1}\frac{\partial\phi}{\partial u_1}$$

Similarly, the component of gradient of ϕ in the directions u_2 & u_3 axes are

$$(\text{grad } \phi)_{u_2} = (\nabla\phi)_{u_2} = \frac{1}{h_2}\frac{\partial\phi}{\partial u_2}$$

$$\& \quad (\text{grad } \phi)_{u_3} = (\nabla\phi)_{u_3} = \frac{1}{h_3}\frac{\partial\phi}{\partial u_3}$$

If $\hat{u}_1, \hat{u}_2, \hat{u}_3$ are unit vectors along the u_1, u_2, u_3 directions respectively, then we can write.

$$\text{grad } \phi = \nabla\phi = \frac{\hat{u}_1}{h_1}\frac{\partial\phi}{\partial u_1} + \frac{\hat{u}_2}{h_2}\frac{\partial\phi}{\partial u_2} + \frac{\hat{u}_3}{h_3}\frac{\partial\phi}{\partial u_3}$$

$$\text{or} \quad \nabla\phi = \left(\frac{\hat{u}_1}{h_1}\frac{\partial}{\partial u_1} + \frac{\hat{u}_2}{h_2}\frac{\partial}{\partial u_2} + \frac{\hat{u}_3}{h_3}\frac{\partial}{\partial u_3}\right)\phi$$

It gives the del operator $(\vec{\nabla})$ in orthogonal curvilinear co-ordinates

$$\vec{\nabla} = \frac{\hat{u}_1}{h_1}\frac{\partial}{\partial u_1} + \frac{\hat{u}_2}{h_2}\frac{\partial}{\partial u_2} + \frac{\hat{u}_3}{h_3}\frac{\partial}{\partial u_3}$$

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VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

$\frac{20}{20}$

Signature of Supervisor	
Subject :	Mathematical physics
Test / Tutorial No. :	
Div. :	

Suppliment No. :

Roll No. : 8026

Class : B.Sc-III

Q.1
Ans:

the divergence of vector function may be obtained from gauss divergence theorem. Gauss divergence theorem in vector analysis is given by

$$\iiint_V (\nabla\cdot\vec{A})\,dv = \iint_S \vec{A}\,ds$$

where $\vec{A} = \vec{A_1}\hat{u_1} + \vec{A_2}\hat{u_2} + A_2\hat{u_3}$

$$\text{div}\,\vec{A} \iiint_V dv = \iint_S \bar{s}\,ds$$

or $$\text{div}\,\vec{A} = \lim_{dv\to 0} \frac{\iint_S \vec{A}\,ds}{\iiint_V dv}$$

Let us find first component of $\iint_S \vec{A}\,ds$ along u_1 as,

Energy entering through OBHC $= -A_1 h_2 h_3\, du_2\, du_3$.

Energy leaving through AKGJ $= A_1 h_2 h_3\, du_2\, du_3 + \frac{\partial}{\partial u_1}(A_1 h_2 h_3)\, du_1 du_2 du_3$

$$\left[\iint_S \vec{A}\, ds\right]_{u_1} = \frac{\partial}{\partial u_1}\left(A_1 h_2 h_3\, du_2\, du_3\right) du_1$$

$$= \frac{\partial}{\partial u_1}\left(A_1 h_2 h_3\right) du_1 du_2 du_3$$

$$\left[\iint_S \vec{A}\, ds\right]_{u_2} = \frac{\partial}{\partial u_2}\left(A_2 h_3 h_1\right) du_1 du_2 du_3$$

$$\& \left[\iint_S \vec{A}\, ds\right]_{u_3} = \frac{\partial}{\partial u_3}\left(A_3 h_1 h_2\right) du_1 du_2 du_3$$

$$\left[\frac{\partial}{\partial u_1}\left(\vec{A} h_2 h_3 + \frac{\partial}{\partial u_2}\left(\vec{A} h_1 h_3 + \frac{\partial}{\partial u_3}\left(\vec{A}_3 h_1 h_2\right)\right]\right.\right. du_1 du_2 du_3$$

$$\nabla V = \iiint_V dv = h_1 h_2 h_3\, du_1 du_2 du_3$$

$$\nabla \cdot \bar{A} = \frac{\left[\frac{\partial}{\partial u_1}\left(\vec{A}_1 h_2 h_3\right) + \frac{\partial}{\partial u_2}\left(\vec{A} h_1 h_3\right) + \frac{\partial}{\partial u_3}\left(\vec{A} h_1 h_2\right)\right] du_1 du_2 du_3}{h_1 h_2 h_3\, du_1 du_2 du_3}$$

$$\therefore \nabla \vec{A} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}\left(\vec{A}_1 h_2 h_3\right) + \frac{\partial}{\partial u_2}\left(\vec{A}_2 h_1 h_3\right) + \frac{\partial}{\partial u_3}\left(\vec{A}_3 h_1 h_2\right)\right.$$

Q.2

Ans: We know that,

$$\text{div}\vec{A} = \vec{\nabla}\cdot\vec{A} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}\left(\bar{A}_1h_2h_3\right) + \frac{\partial}{\partial u_2}\left(\bar{A}_2h_1h_3\right) + \frac{\partial}{\partial u_3}\left(\bar{A}_3h_1h_2\right)\right]$$

Let $\vec{A} = \vec{\nabla}\cdot f$.

$$\text{div}\,\vec{A} = \vec{\nabla}\cdot\vec{\nabla}f = \nabla^2 f$$

$$= \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}\left(h_2h_3(\vec{\nabla}f)_{u_1}\right) + \frac{\partial}{\partial u_2}\left(h_3h_1(\vec{\nabla}f)_{u_2}\right) + \frac{\partial}{\partial u_3}\left(h_1h_2(\vec{\nabla}f)_{u_3}\right)\right]$$

$$\nabla^2 f = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}\left(\frac{h_2h_3}{h_1}\frac{\partial f}{\partial u_1}\right) + \frac{\partial}{\partial u_2}\left(\frac{h_1h_3}{h_2}\frac{\partial f}{\partial u_2}\right) + \frac{\partial}{\partial u_3}\left(\frac{h_1h_2}{h_3}\frac{\partial f}{\partial u_3}\right)\right]$$

$$\therefore \nabla^2 = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}\left(\frac{h_2h_3}{h_1}\frac{\partial}{\partial u_1}\right) + \frac{\partial}{\partial u_2}\left(\frac{h_3h_1}{h_2}\frac{\partial}{\partial u_2}\right) + \frac{\partial}{\partial u_3}\left(\frac{h_1h_2}{h_3}\frac{\partial}{\partial u_3}\right)\right]$$

Q.3.

Ans:

$$(\text{grad}\,\phi)_{u_1} = (\nabla\phi)_{u_1} = \lim_{\delta u_1 \to 0}\frac{\phi(B) - \phi(A)}{AB}$$

$$\therefore (\text{grad}\,\phi)_{u_1} = (\nabla\phi)_{u_1} = \lim_{\delta u_1 \to 0}\left[\frac{\delta\phi}{h_1\,du_1}\right]$$

$$(\nabla\phi)_{u_1} = \frac{1}{h_1}\frac{\partial\phi}{\partial u_1}$$

$$(\text{grad}\ \phi)_{u_2} = (\nabla\phi)_{u_2} = \frac{1}{h_2}\frac{\partial\phi}{\partial u_2}$$

& $$(\text{grad}\ \phi)_{u_3} = (\nabla\phi)_{u_3} = \frac{1}{h_3}\frac{\partial\phi}{\partial u_3}$$

$$\text{grad}\ \phi = \nabla\phi = \frac{\hat{u}_1}{h_1}\frac{\partial\phi}{\partial u_1} + \frac{\hat{u}_2}{h_2}\frac{\partial\phi}{\partial u_2} + \frac{\hat{u}_3}{h_3}\frac{\partial\phi}{\partial u_3}$$

or

$$\nabla\phi \left[\frac{\hat{u}_1}{h_1}\frac{\partial}{\partial u_1} + \frac{\hat{u}_2}{h_2}\frac{\partial}{\partial u_2} + \frac{\hat{u}_3}{h_3}\frac{\partial}{\partial u_3}\right]\phi$$

$$\therefore \vec{\nabla} = \frac{\hat{u}_1}{h_1}\frac{\partial}{\partial u_1} + \frac{\hat{u}_2}{h_2}\frac{\partial}{\partial u_2} + \frac{\hat{u}_3}{h_3}\frac{\partial}{\partial u_3}$$

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VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

19/20

Signature of Supervisor	
Subject :	Mathematical physics
Test / Tutorial No. :	
Div. :	

Suppliment No. :

Roll No. : 8019

Class : BSC - III

Q.1

Ans→ The divergence of vector function may be obtained from Gauss divergence theorem. Gauss divergence theorem in vector analysis is given by

$$\iiint_V (\nabla \cdot \vec{A})\, dv = \iint_S \vec{A}\, ds$$

where

$$\vec{A} = A_1 \hat{u}_1 + A_2 \hat{u}_2 + A_3 \hat{u}_3$$

By the mean value theorem for integrals, we write L.H.S

$$\overline{\mathrm{div}\, \vec{A}} \iiint_V dv = \iint_S \bar{s}\, ds$$

or

$$\mathrm{div}\, \vec{A} = \lim_{dv \to 0} \frac{\iint_S \vec{A}\, ds}{\iiint_V dv}$$

Now consider the volume element Δv of infinitesimal parallelopiped having edges $h_1 du_1$, $h_2 du_2$ & $h_3 du_3$.

Let us find first component of $\iint_S \vec{A}\, ds$ along u_1 as

Energy entering through OBHC $= -A_1 h_2 h_3\, du_2\, du_3$

Energy leaving through AKGJ $= A_1 h_2 h_3\, du_2\, du_3 + \frac{\partial}{\partial u_1}(A_1 h_2 h_3)\, du_1\, du_2\, du_3$

$$\left[\iint_S \vec{A}\, ds\right]_{u_1} = \frac{\partial}{\partial u_1}(A_1 h_2 h_3\, du_2\, du_3)\, du_1$$

$$= \frac{\partial}{\partial u_1}(A_1 h_2 h_3)\, du_1\, du_2\, du_3$$

$$\left[\iint_S \vec{A}\, ds\right]_{u_2} = \frac{\partial}{\partial u_2}(A_2 h_3 h_1)\, du_1\, du_2\, du_3$$

& $$\left[\iint_S \vec{A}\, ds\right]_{u_3} = \frac{\partial}{\partial u_3}(A_3 h_1 h_2)\, du_1\, du_2\, du_3$$

$$\left[\frac{\partial}{\partial u_1}(\vec{A}_1 h_2 h_3) + \frac{\partial}{\partial u_2}(\vec{A}_2 h_1 h_3) + \frac{\partial}{\partial u_3}(\vec{A}_3 h_1 h_2)\right] du_1\, du_2\, du_3$$

$$\Delta V = \iiint_V dv = h_1 h_2 h_3\, du_1\, du_2\, du_3$$

$$\nabla \cdot \vec{A} = \frac{\left[\frac{\partial}{\partial u_1}(\vec{A} h_2 h_3) + \frac{\partial}{\partial u_2}(\vec{A} h_1 h_3) + \frac{\partial}{\partial u_3}(\vec{A} h_1 h_2)\right] du_1\, du_2\, du_3}{h_1 h_2 h_3\, du_1\, du_2\, du_3}$$

$$\nabla \cdot \vec{A} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}(\vec{A}_1 h_2 h_3) + \frac{\partial}{\partial u_2}(\vec{A}_2 h_1 h_3) + \frac{\partial}{\partial u_3}(\vec{A} h_1 h_2)\right.$$

Q. 2

Ans: We know that

$$\mathrm{div}\,\vec{A} = \vec{\nabla}\cdot\vec{A} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}(\bar{A}_1h_2h_3) + \frac{\partial}{\partial u_2}(\bar{A}_2h_1h_3) + \frac{\partial}{\partial u_3}(\bar{A}_3h_1h_2)\right]$$

Let $\vec{A} = \vec{\nabla}\cdot F$,

$$\mathrm{div}\,\vec{A} = \vec{\nabla}\cdot\vec{\nabla}F = \nabla^2 F$$

$$= \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}\left(h_2h_3(\vec{\nabla}F)_{u_1}\right) + \frac{\partial}{\partial u_2}\left(h_3h_1(\vec{\nabla}F)_{u_2}\right) + \frac{\partial}{\partial u_3}\left(h_1h_2(\vec{\nabla}F)_{u_3}\right)\right]$$

$$\nabla^2 F = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}\left(\frac{h_2h_3}{h_1}\frac{\partial F}{\partial u_1}\right) + \frac{\partial}{\partial u_2}\left(\frac{h_1h_3}{h_2}\frac{\partial F}{\partial u_2}\right) + \frac{\partial}{\partial u_3}\left(\frac{h_1h_2}{h_3}\frac{\partial F}{\partial u_3}\right)\right]$$

$$\nabla^2 = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}\left(\frac{h_2h_3}{h_1}\frac{\partial}{\partial u_1}\right) + \frac{\partial}{\partial u_2}\left(\frac{h_3h_1}{h_2}\frac{\partial}{\partial u_2}\right) + \frac{\partial}{\partial u_3}\left(\frac{h_1h_2}{h_3}\frac{\partial}{\partial u_3}\right)\right]$$

Q.3.

Ans: Let us consider $\phi(u_1, u_2, u_3)$ a scalar fun^n. The gradient of the scalar field ϕ in the direction of u_1-axis can be written as,

$$(\mathrm{grad}\,\phi)_{u_1} = (\nabla\phi)_{u_1} = \lim_{\delta u_1 \to 0} \frac{\phi(B) - \phi(A)}{AB}$$

$$\therefore (\mathrm{grad}\,\phi)_{u_1} = (\nabla\phi)_{u_1} = \lim_{\delta u_1 \to 0}\left[\frac{\delta\phi}{h_1\,du_1}\right]$$

$$(\nabla\phi)_{u_1} = \frac{1}{h_1}\frac{\partial\phi}{\partial u_1}$$

similarly, the component of gradient of ϕ in the directions u_2 & u_3 axes are.

$$(\text{grad}\,\phi)_{u_2} = (\nabla\phi)_{u_2} = \frac{1}{h_2}\frac{\partial\phi}{\partial u_2}$$

&

$$(\text{grad}\,\phi)_{u_3} = (\nabla\phi)_{u_3} = \frac{1}{h_3}\frac{\partial\phi}{\partial u_3}$$

$$\therefore \quad \text{grad}\,\phi = \nabla\phi = \frac{\hat{u}_1}{h_1}\frac{\partial\phi}{\partial u_1} + \frac{\hat{u}_2}{h_2}\frac{\partial\phi}{\partial u_2} + \frac{\hat{u}_3}{h_3}\frac{\partial\phi}{\partial u_3}$$

or
$$\nabla\phi = \left[\frac{\hat{u}_1}{h_1}\frac{\partial}{\partial u_1} + \frac{\hat{u}_2}{h_2}\frac{\partial}{\partial u_2} + \frac{\hat{u}_3}{h_3}\frac{\partial}{\partial u_3}\right]\phi$$

It gives the del operator (∇) in orthogonal curvilinear co-ordinates.

$$\therefore \quad \nabla = \frac{\hat{u}_1}{h_1}\frac{\partial}{\partial u_1} + \frac{\hat{u}_2}{h_2}\frac{\partial}{\partial u_2} + \frac{\hat{u}_3}{h_3}\frac{\partial}{\partial u_3}$$

Vivekananda College Kolhapur (Autonomous).
Department of Physics: Internal examination 2019-20
B.Sc. III Semester V
Subject: Quantum mechanics

Marks: 20 (Each question carry one mark) **Time : 20 min**

Q.1. Long Answer question (Attempt any ONE) . **[10]**

i) Obtain Schrodinger , s time independent equation and time dependent equation

ii) Explain quantum mechanical treatment of linear harmonic oscillator and show that zero point energy of oscillator is $E\ 0 = \frac{1}{2}\hbar\omega$

Q.2. Short Answer question (Attempt any TWO). **[10]**

i) Show that $[x , P\ x] = i\hbar$ give its physical significance

ii) Give physical significance of wave function

iii) Obtain Schrodingers equation in spherical polar coordinate system for hydrogen atom

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34056

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor

$\frac{18}{20}$

Suppliment No. :

Roll No. : 8006

Class : BSC-III

Subject : Quantum mechanics

Test / Tutorial No. : Internal Exam

Div. :

Q. 1.

Ans. - A. Time independent Schrodinger's equation:

Schrodinger made following assumptions:

(i) de-Broglie wavelength holds good for any particle moving in a field of force with potential energy (V).

The total engy is given as

$$E = KE + PE$$

$$= \frac{1}{2} mv^2 + V$$

$$= \frac{1}{2} \frac{m^2 v^2}{m} + V$$

$$\therefore E = \frac{p^2}{2m} + V$$

$$\therefore p^2 = 2m(E-V)$$

$$\therefore p = [2m(E-V)]^{1/2}$$

$\therefore$ de-Broglie wavelength $(\lambda) = \dfrac{h}{[2m(E-V)]^{1/2}}$ — (1)

(ii) The wave function (ψ) is given by,

$$\psi = \psi_0 e^{-i\omega t} \quad \text{— (2)}$$

The wave eq^n in Cartesian co-ordinate is

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{u^2} \frac{\partial^2 \psi}{\partial t^2} \quad (3)$$

Differentiating eq^n (2) twice w.r.t. t,

$$\frac{\partial^2\psi}{\partial t^2} = -\omega^2 \psi e^{-i\omega t} = -\omega^2\psi$$

Substituting above in eq^n (3),

$$\therefore \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = -\frac{\omega^2}{u^2}\psi$$

Here, $\frac{\omega^2}{u^2} = \frac{(2\pi \cancel{\nu})^2}{(\cancel{\nu}\lambda)^2} = \frac{4\pi^2}{\lambda^2}$ & $\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \nabla^2\psi$

$$\therefore \nabla^2\psi = -\frac{4\pi^2}{\lambda^2}\psi$$

$$\therefore \nabla^2\psi + \frac{4\pi^2}{\lambda^2}\psi = 0 \qquad \text{— (4)}$$

This eq^n is general time-independent eq^n.

By the concept of wave mechanics, de-Broglie wavelength is,

$$\lambda = \frac{h}{[2m(E-V)]^{1/2}}$$

Substituting this in eq^n (4),

$$\nabla^2\psi + \frac{4\pi^2}{h^2}[2m(E-V)]\psi = 0$$

$$\therefore \nabla^2\psi + \frac{8m\pi^2}{h^2}(E-V)\psi = 0$$

$$\therefore \nabla^2\psi + \left(\frac{2m}{\hbar^2}\right)(E-V)\psi = 0 \qquad \left(\because \hbar = \frac{h}{2\pi}\right)$$

— (5)

Eq^n (5) represents Schrodinger's time-independent wave eq^n.

B. Time-dependent Schrodinger's eqⁿ:

We have, $\psi = \psi_0 e^{-i\omega t}$

$$\therefore \frac{\partial \psi}{\partial t} = -i\omega \psi_0 e^{-i\omega t}$$

$$= -i(2\pi\nu)\psi$$

$$= -i(2\pi)\left(\frac{E}{h}\right)\psi \qquad \because \nu = \frac{E}{h}$$

$$\therefore E\psi = -\frac{1}{i}\left(\frac{h}{2\pi}\right)\frac{\partial \psi}{\partial t}$$

$$\therefore E\psi = i\hbar \frac{\partial \psi}{\partial t} \quad \text{— (6)} \qquad \left[\because \hbar = \frac{h}{2\pi} \;\&\; i^2 = -1\right]$$

from eqⁿ (5), $\nabla^2\psi + \frac{2m}{\hbar^2}E\psi - \frac{2m}{\hbar^2}V(\psi) = 0$

Substituting eqⁿ (6), $\nabla^2\psi + \frac{2m}{\hbar^2}\left(i\hbar\frac{\partial \psi}{\partial t}\right) - \frac{2m}{\hbar^2}V\psi = 0$

$$\therefore \nabla^2\psi + \frac{2m}{\hbar}\left(i\frac{\partial \psi}{\partial t}\right) - \frac{2m}{\hbar^2}V\psi = 0$$

$$\therefore \left(\frac{\hbar^2}{2m}\right)\nabla^2\psi + i\hbar\frac{\partial \psi}{\partial t} - V\psi = 0$$

$$\therefore i\hbar\frac{\partial \psi}{\partial t} = -\left(\frac{\hbar^2}{2m}\right)\nabla^2\psi + V\psi \quad \text{— (7)}$$

∴ Eqⁿ (7) represents Schrodinger's time-dependent wave eqⁿ.

Q. 2.

Ans. Consider the action of commutator $[x, p_x]$ on wave function $\psi(x)$,

$$[x, p_x]\psi = [x, -i\hbar \frac{\partial}{\partial x}]\psi$$

$$= -i\hbar [x, \frac{\partial}{\partial x}]\psi$$

$$= -i\hbar \left[x\frac{\partial\psi}{\partial x} - \frac{\partial}{\partial x}(x\psi)\right]$$

$$= -i\hbar \left[x\frac{\partial\psi}{\partial x} - x\frac{\partial\psi}{\partial x} - \psi\right]$$

$$= -i\hbar(-\psi)$$

$$[x, p_x]\psi = i\hbar\psi$$

$$\therefore [x, p_x] = i\hbar$$

Q. 3.

Ans. Due to interpretation of $|\psi|^2$ as probability density, the wave function (ψ) must obey :

(i) ψ must be finite for all values of x, y & z.

(ii) ψ must be single-valued i.e. for each set of x, y & z, ψ must be only one valued.

(iii) ψ must be continuous in all regions, except in those regions where the potential energy $V(x, y, z) = \infty$.

(iv) ψ must ~~vat~~ vanish at Infinity.

$\therefore \psi = 0$ at $x \to \pm\infty$ / $y \to \pm\infty$ / $z \to \pm\infty$.

(v) The potential derivatives of ψ i.e. $\frac{\partial\psi}{\partial x}, \frac{\partial\psi}{\partial y}, \frac{\partial\psi}{\partial z}$ must also be infinite, single-valued & continuous at all points, except at points where the potential ~~is~~ $V(x, y, z)$ is infinite.

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34055

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

$\frac{20}{20}$

Signature of Supervisor	
Subject :	Quantum mechanics
Test / Tutorial No. :	
Div. :	

Suppliment No. :
Roll No. : 8015
Class : BSc - III

Q. 1. → I] Time independent Schrodinger's equation:
Schrodinger made following assumptions:
(a) He assumed that de Broglie wavelength holds good for any particle moving in a field of force with potential energy V. Total energy is given by,

$$E = KE + PE$$
$$= \frac{1}{2}mv^2 + V$$
$$= \frac{1}{2}\frac{m^2v^2}{m} + V$$
$$\therefore E = \frac{p^2}{2m} + V$$
$$\therefore p^2 = 2m(E-V)$$
$$\therefore p = [2m(E-V)]^{1/2}$$

$$\therefore \text{de-Broglie wavelength } (\lambda) = \frac{h}{p} = \frac{h}{[2m(E-V)]^{1/2}} \quad \text{--- (1)}$$

(b) He assumed that the wave function (ψ) is given by,

$$\psi = \cancel{\psi} \; \psi_0 e^{-i\omega t} \quad \text{--- (2)}$$

The wave eqⁿ in Cartesian ~~cor~~ co-ordinate is,

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2} = \frac{1}{u^2}\frac{\partial^2\varphi}{\partial t^2} \quad — (3)$$

Differentiating eqⁿ (2) twice w.r.t time,

$$\frac{\partial^2\varphi}{\partial t^2} = -\omega^2\varphi_0 e^{-i\omega t} = \omega^2\varphi$$

Substituting this in eqⁿ (3),

$$\therefore \frac{\partial^2\varphi}{\partial x^2}+\frac{\partial\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2} = -\frac{\omega^2}{u^2}\varphi$$

Where, $\frac{\omega^2}{u^2} = \frac{(2\pi\nu)^2}{(\nu\lambda)^2} = \frac{4\pi^2}{\lambda^2}$ & $\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2} = \nabla^2\varphi$

$$\therefore \nabla^2\varphi = -\frac{4\pi^2}{\lambda^2}\varphi$$

$$\therefore \nabla^2\varphi + \frac{4\pi^2}{\lambda^2}\varphi = 0 \quad — (4)$$

This is the general time-independent eqⁿ.

By the concept of wave mechanics, de-Broglies wavelength is given by,

$$\lambda = \frac{h}{p} = \frac{h}{[2m(E-V)]^{1/2}}$$

Substituting this in eqⁿ (4),

$$\therefore \nabla^2\varphi + \frac{4\pi^2}{h^2}[2m(E-V)]\varphi = 0$$

$$\therefore \nabla^2\varphi + \frac{8m\pi^2}{h^2}(E-V)\varphi = 0$$

$$\therefore \nabla^2\varphi + \left(\frac{2m}{\hbar^2}\right)(E-V)\varphi = 0 \quad — (5) \qquad \because \hbar = \frac{h}{2\pi}$$

Eq^n (5) represents Schrodinger's time-independent wave eq^n.

II) Time-dependent Schrodinger's eq^n:

We have, $\psi = \psi_0 e^{-i\omega t}$

$$\therefore \frac{\partial \psi}{\partial t} = -i\omega \psi_0 e^{-i\omega t}$$

$$= -i(2\pi\nu)\psi$$

$$= -i(2\pi)\left(\frac{E}{h}\right)\psi \qquad \because \nu = \frac{E}{h}$$

$$\therefore E\psi = -\frac{1}{i}\left(\frac{h}{2\pi}\right)\frac{\partial \psi}{\partial t}$$

$$\therefore E\psi = i\hbar \frac{\partial \psi}{\partial t} \quad \left[\because \hbar = \frac{h}{2\pi} \ \& \ i^2 = -1\right] \quad \text{— (6)}$$

from eq^n (5), $\nabla^2\psi + \frac{2m}{\hbar^2}E\psi - \frac{2m}{\hbar^2}V\psi = 0$

Substituting eq^n (6), $\nabla^2\psi + \frac{2m}{\hbar^2}\left(i\hbar\frac{\partial \psi}{\partial t}\right) - \left(\frac{2m}{\hbar^2}\right)V\psi = 0$

$$\therefore \nabla^2\psi + \frac{2m}{\hbar} i \frac{\partial \psi}{\partial t} - \frac{2m}{\hbar^2}V\psi = 0$$

$$\therefore \left(\frac{\hbar^2}{2m}\right)\nabla^2\psi + i\hbar\frac{\partial \psi}{\partial t} - V\psi = 0$$

$$\therefore i\hbar\frac{\partial \psi}{\partial t} = -\left(\frac{\hbar^2}{2m}\right)\nabla^2\psi + V\psi \quad \text{— (7)}$$

Eq^n (7) represents Schrodinger's time-dependent wave eq^n.

Q.2 → Consider the action of commutator $[x, p_x]$ on wave function $\varphi(x)$

$$[x, p_x]\varphi = \left[x, -i\hbar \frac{\partial}{\partial x}\right]\varphi$$

$$= -i\hbar \left[x, \frac{\partial}{\partial x}\right]\varphi$$

$$= -i\hbar \left[x \frac{\partial \varphi}{\partial x} - \frac{\partial}{\partial x}(x\varphi)\right]$$

$$= -i\hbar \left[\cancel{x \frac{\partial \varphi}{\partial x}} - \cancel{x \frac{\partial \varphi}{\partial x}} - \varphi\right]$$

$$= -i\hbar(-\varphi)$$

$$[x, p_x]\varphi = i\hbar\varphi$$

$$\therefore [x, p_x] = i\hbar$$

Q.3. Due to interpretation of $|\varphi|^2$ as probability density, the wave function φ must obey the following conditions:

① φ must be finite for all values of x, y & z.

② φ must be single-valued i.e. for each set of x, y & z must have only one value of φ.

③ φ must be continuous in all regions, except in those regions where the potential energy $V(x, y, z) = \infty$

④ φ must vanish at infinity.

i.e. $\varphi = 0$ as $x \to \pm\infty$ / $y \to \pm\infty$ / $z \to \pm\infty$.

⑤ The partial derivatives of φ i.e. $\frac{\partial \varphi}{\partial x}, \frac{\partial \varphi}{\partial y}, \frac{\partial \varphi}{\partial z}$ must also be finite, single-valued & ~~and~~ continuous at all points, excepts at points where the potential $V(x)$ is infinite.

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34054

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor

15/20

Suppliment No. :

Subject : Quantum mechanics

Roll No. : 8018

Test / Tutorial No. :

Class : BSC-TII

Div. :

Q.1.

→ I: Time independent Schrodinger's equation:

Schrodinger derived an equation for motion of matter waves. He made following assumptions:

① He assumed that de-Broglie wavelength holds good for any particle moving in a field of force with potential energy V. Total energy is given by,

$$E = KE + PE$$

$$E = \frac{1}{2} mv^2 + V$$

$$= \frac{1}{2} \frac{m^2 v^2}{m} + V$$

$$= \frac{1}{2} \frac{p^2}{m} + V$$

$$p^2 = 2m(E-V)$$

$$\therefore p = [2m(E-V)]^{1/2}$$

$\therefore$ de-Broglie wavelength $(\lambda) = \frac{h}{p} = \frac{h}{(2m(E-V))^{1/2}}$ ①

(2) He assumed that the wave func^n (ψ) is governed by,

$$\psi = \psi_0 e^{-i\omega t} \quad \text{--- (2)}$$

The usual wave eq^n in Cartesian co-ordinates is,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{u^2} \frac{\partial^2 \psi}{\partial t^2} \quad \text{--- (3)}$$

Differentiating (2) twice w.r.t. time,

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi_0 e^{-i\omega t} = -\omega^2 \psi$$

Substituting this eq^n in eq^n (3),

$$\therefore \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = -\frac{\omega^2}{u^2} \psi$$

Where, $\frac{\omega^2}{u^2} = \frac{(2\pi\nu)^2}{(\nu\lambda)^2} = \frac{4\pi^2}{\lambda^2}$

$$\therefore \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{-4\pi^2}{\lambda^2} \psi$$

$$\therefore \nabla^2 \psi + \frac{4\pi^2}{\lambda^2} \psi = 0 \quad \text{--- (4)}$$

Eq^n (4) is general eq^n & is independent of time

By the concept of wave mechanics, de-Broglie wavelength, $\lambda = \frac{h}{p} = \frac{h}{[2m(E-V)]^{1/2}}$

Substituting this in eq^n (4),

$$\nabla^2\psi + \frac{4\pi^2}{h^2}[2m(E-V)]\psi = 0$$

$$\therefore \nabla^2\psi + \left(\frac{8m\pi^2}{h^2}\right)(E-V)\psi = 0$$

Also, $\hbar = \frac{h}{2\pi}$

$$\therefore \nabla^2\psi + \left(\frac{2m}{\hbar^2}\right)(E-V)\psi = 0 \qquad -(5)$$

eq^n (5) represents Schrodinger's time-independent wave eq^n.

II. Time-dependent Schrodinger's eq^n:

$$\psi = \psi_0 e^{-i\omega t}$$

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$$\therefore \frac{\partial\psi}{\partial t} = -i\omega \psi_0 e^{-i\omega t}$$

$$= -i(2\pi\nu)\psi \qquad \because \cancel{\nu = \frac{E}{\hbar}}$$

$$= -i(2\pi)\left(\frac{E}{h}\right)\psi \qquad \because \nu = \frac{E}{h}$$

$$\therefore E\psi = -\frac{1}{i}\left(\frac{h}{2\pi}\right)\frac{\partial\psi}{\partial t}$$

$$\therefore E\psi = i\hbar\frac{\partial\psi}{\partial t} \qquad \left[\because \hbar = \frac{h}{2\pi} \ \& \ i^2 = -1\right] \quad -(6)$$

from eq^n (5), $\nabla^2\psi + \frac{2m}{\hbar^2}E\psi - \frac{2m}{\hbar^2}V\psi = 0$

Substituting eqⁿ (6) in above,

$$\nabla^2\psi + \frac{2m}{\hbar^2}\left(i\hbar\frac{\partial\psi}{\partial t}\right) - \left(\frac{2m}{\hbar^2}\right)V\psi = 0$$

$$\therefore \nabla^2\psi + \frac{2mi}{\hbar}\frac{\partial\psi}{\partial t} - \frac{2m}{\hbar^2}V\psi = 0$$

$$\therefore \left(\frac{\hbar^2}{2m}\right)\nabla^2\psi + i\hbar\frac{\partial\psi}{\partial t} - V\psi = 0$$

$$\therefore i\hbar\frac{\partial\psi}{\partial t} = -\left(\frac{\hbar^2}{2m}\right)\nabla^2\psi + V\psi \quad \text{--- (7)}$$

$$\therefore E\psi = H\psi \quad \text{--- (8)}$$

Where, $\left[-\frac{\hbar^2}{2m}\nabla^2 + V\right] = H$ - Hamiltonian

Eqⁿ (8) represents the motion of non-relavistic material particle.
Eqⁿ (7) represents Schrodinger's time-dependent eqⁿ.

Q. 2.

→ Consider $[x, p_x]$ on wave funcⁿ $\psi(x, y, z)$,

$$[x, p_x]\psi = \left[x, -i\hbar\frac{\partial}{\partial x}\right]\psi = -i\hbar\left[x \frac{\partial}{\partial x}\right]\psi$$

$$= -i\hbar\left[x\frac{\partial}{\partial x} - \frac{\partial}{\partial x}x\right]\psi$$

$$= -i\hbar\left[x\frac{\partial\psi}{\partial x} - \frac{\partial}{\partial x}(x\psi)\right]$$

$$= -i\hbar(-\psi)$$

$$\therefore [x, p_x]\psi = i\hbar\psi$$

$$\therefore [x, p_x] = i\hbar$$