A PROJECT REPORT ON

Kaprekar number

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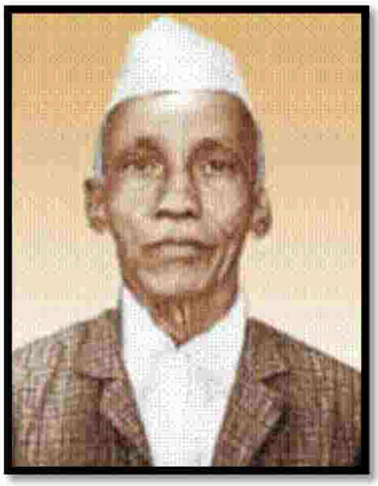
INTRODUCTION

In mathematics, a **Kaprekar number** for a given base is a non-negative integer, the representation of whose square in that base can be split into two parts—either or both of which may include leading zeroes—that add up to the original number. For instance, 45 is a Kaprekar number, because $45^2 = 2025$ and $20 + 25 = 45$. The number 1 is Kaprekar in every base, because $1^2 = 01$ in any base, and $0 + 1 = 1$. Kaprekar numbers are named after D. R. Kaprekar.

Dattatraya Ramchandra Kaprekar

Dattathreya Ramchandra Kaprekar (1905–1986) was an Indian recreational mathematician who described several classes of natural numbers including the Kaprekar, Harshad and Self numbers and discovered the Kaprekar constant, named after him. Despite having no formal postgraduate training and working as a schoolteacher, he published extensively and became well known in recreational mathematics circles.



Biography

Kaprekar received his secondary school education in Thane and studied at Fergusson College in Pune. In 1927 he won the Wrangler R. P. Paranjpe Mathematical Prize for an original piece of work in mathematics.

He attended the University of Mumbai, receiving his bachelor's degree in 1929. Having never received any formal postgraduate training, for his entire career (1930–1962) he was a schoolteacher at Nashik in Maharashtra, India. He published extensively, writing about such topics as recurring decimals, magic squares, and integers with special properties. He is also known as "Ganitanand"

Discoveries

Working largely alone, Kaprekar discovered a number of results in number theory and described various properties of numbers.[3] In addition to the Kaprekar constant and the Kaprekar numbers which were named after him, he also described self numbers or Devlali numbers, the Harshad numbers and Demlo numbers. He also constructed certain types of magic squares related to the Copernicus magic square.[4] Initially his ideas were not taken seriously by Indian mathematicians, and his results were published largely in low-level mathematics journals or privately published, but international fame arrived when Martin Gardner wrote about Kaprekar in his March 1975 column of Mathematical Games for Scientific American. Today his name is well-known and many other mathematicians have pursued the study of the properties he discovered.

Kaprekar constant

Main article: Kaprekar constant

In 1949, Kaprekar discovered an interesting property of the number 6174, which was subsequently named the Kaprekar constant.[5] He showed that 6174 is reached in the limit as one repeatedly subtracts the highest and lowest numbers that can be constructed from a set of four digits that are not all identical. Thus, starting with 1234, we have:

4321 − 1234 = 3087, then

8730 − 0378 = 8352, and

8532 − 2358 = 6174.

Repeating from this point onward leaves the same number (7641 − 1467 = 6174). In general, when the operation converges it does so in at most seven iterations.

A similar constant for 3 digits is 495. However, in base 10 a single such constant only exists for numbers of 3 or 4 digits; for other digit lengths or bases other than 10, the Kaprekar's routine algorithm described above may in general terminate in multiple different constants or repeated cycles, depending on the starting value.

Kaprekar number

Another class of numbers Kaprekar described are the Kaprekar numbers.[8] A Kaprekar number is a positive integer with the property that if it is squared, then its representation can be partitioned into two positive integer parts whose sum is equal to the original number (e.g. 45, since 45^2=2025, and 20+25=45, also 9, 55, 99 etc.) However, note the restriction that the two numbers are positive; for example, 100 is not a Kaprekar number even though 100^2=10000, and 100+00 = 100. This operation, of taking the rightmost digits of a square, and adding it to the integer formed by the leftmost digits, is known as the Kaprekar operation.

Some examples of Kaprekar numbers in base 10, besides the numbers 9, 99, 999, …, are (sequence A006886 in the OEIS):

Number	Square	Decomposition
703	$703^2 = 494209$	494+209 = 703
2728	$2728^2 = 7441984$	744+1984 = 2728
5292	$5292^2 = 28005264$	28+005264 = 5292
857143	$857143^2 = 734694122449$	734694+122449 = 857143

Devlali or Self number

In 1963, Kaprekar defined the property which has come to be known as self numbers,[9] which are integers that cannot be generated by taking some other number and adding its own digits to it. For example, 21 is not a self number, since it can be generated from 15: 15 + 1 + 5 = 21. But 20 is a self number, since it cannot be generated from any other integer. He also gave a test for verifying this property in any number. These are sometimes referred to as Devlali numbers (after the town where he lived); though this appears to have been his preferred designation, the term self number is more widespread. Sometimes these are also designated Colombian numbers after a later designation.

Harshad number

Kaprekar also described the Harshad numbers which he named harshad, meaning "giving joy" (Sanskrit harsha, joy +da taddhita pratyaya, causative); these are defined by the property that they are divisible by the sum of their digits. Thus 12, which is divisible by 1 + 2 = 3, is a Harshad number. These were later also called Niven numbers after a 1977 lecture on these by the Canadian mathematician Ivan M. Niven. Numbers which are Harshad in all bases (only 1, 2, 4, and 6) are called all-Harshad numbers. Much work has been done on Harshad numbers, and their distribution, frequency, etc. are a matter of considerable interest in number theory today.

Demlo number

Kaprekar also studied the Demlo numbers,[10] named after a train station 30 miles from Bombay on the then G. I. P. Railway where he had the idea of studying them. The best known of these are the Wonderful Demlo numbers 1, 121, 12321, …, which are the squares of the repunits 1, 11, 111, ….

2. Kaprekar number

Definition

Let X be a non-negative integer and n a positive integer. X is an n-Kaprekar number for base b if there exist non-negative integer A, and positive integer B satisfying:

$$X^2 = Ab^n + B, \text{ where } 0 < B < b^n$$

$$X = A + B$$

If d is any divisor of n, then X is also a d-Kaprekar number for base b^n. A Kaprekar number for base b is one which is an n-Kaprekar number for that base and some positive integer n.

More generally, we can define the set $K(N)$ for a given integer N as the set of integers X for which

$$X^2 = AN + B, \text{ where } 0 < B < N$$

$$X = A + B$$

An n-Kaprekar number for base b is then one which lies in the set $K(b^n)$, and a Kaprekar number for base b is one which lies in any one of the sets $K(b)$, $K(b^2)$, $K(b^3)$,….

Examples

297 is a Kaprekar number for base 10, because 297^2 = 88209, which can be split into 88 and 209, and 88 + 209 =

297. By convention, the second part may start with the digit 0, but must be nonzero. For example, 999 is a Kaprekar number for base 10, because $999^2 = 998001$, which can be split into 998 and 001, and 998 + 001 = 999. But 100 is not; although $100^2 = 10000$ and 100 + 00 = 100, the second part here is zero (i.e. not a positive integer).

The first few Kaprekar numbers in base 10 are:

1, 9, 45, 55, 99, 297, 703, 999, 2223, 2728, 4879, 4950, 5050, 5292, 7272, 7777, 9999, 17344, 22222, 38962, 77778, 82656, 95121, 99999, 142857, 148149, 181819, 187110, 208495, 318682, 329967, 351352, 356643, 390313, 461539, 466830, 499500, ... (sequence A006886 in the OEIS)

In particular, 9, 99, 999… are all Kaprekar numbers. More generally, for any base b, there exist infinitely many Kaprekar numbers, including all numbers of the form $b^n - 1$.

Other bases

In binary, the first 26 Kaprekar numbers are

1, 11, 110, 111, 1010, 1111, 11100, 11111, 100100, 110011, 111111, 1010101, 1011011, 1111000, 1111111, 10001000, 10010011, 10101011, 10111011, 11001101, 11111111, 101010110, 101011111, 101101101, 111110000, 111111111, ...

In ternary, the first 24 Kaprekar numbers are

1, 2, 22, 111, 112, 121, 222, 2102, 2222, 10220, 11111, 11112, 20021, 22222, 101010, 121220, 202202, 212010, 222222, 1101222, 1111111, 1111112, 2012021, 2222222, ...

In base 7, the first 45 Kaprekar numbers are

1, 3, 4, 6, 22, 25, 45, 66, 306, 333, 334, 361, 441, 642, 666, 1452, 2223, 4444, 5215, 6226, 6666, 11112, 15261, 22222, 33333, 33334, 44445, 55555, 66666, 120546, 125665, 136140, 152152, 224500, 303031, 321321, 345346, 363636, 442200, 514515, 530530, 546121, 651406, 652114, 666666, ...

In base 12, the first 41 Kaprekar numbers are

1, E, 56, 66, EE, 444, 778, EEE, 12XX, 1640, 2046, 2929, 3333, 4973, 5E60, 6060, 7249, 8889, 9293, 9E76, X580, X912, EEEE, 22223, 48730, 72392, 99999, EEEEE, 12E649, 16EX51, 1X1X1X, 222222, 22X54X, 26X952, 35186E, 39X39X, 404040, 4197X2, 450770, 5801E8, 5EE600, ...

In base 16, the first 72 Kaprekar numbers are

1, 6, A, F, 33, 55, 5B, 78, 88, AB, CD, FF, 15F, 334, 38E, 492, 4ED, 7E0, 820, B13, B6E, C72, CCC, EA1, FA5, FFF, 191A, 2A2B, 3C3C, 4444, 5556, 6667, 7F80, 8080, 9999, AAAA, BBBC, C3C4, D5D5, E6E6, FFFF, 1745E, 20EC2, 2ACAB, 2D02E, 30684, 3831F, 3E0F8, 42108, 47AE1, 55555, 62FCA, 689A3, 7278C, 76417, 7A427, 7FE00, 80200, 85BD9, 89AE5, 89BE9, 8D874, 9765D, 9D036, AAAAB, AF0B0, B851F, BDEF8, C1F08, C7CE1, CF97C, D5355, ...

Properties

- It was shown in 2000 that there is a bijection between the unitary divisors of $N - 1$ and the set $K(N)$ (defined above). Let $Inv(a,c)$ denote the multiplicative inverse of a modulo c, namely the least positive integer m such that $am \equiv 1 \pmod{c}$, and for each unitary divisor d of $N - 1$ let $\zeta(d) = d\,Inv(d, (N - 1)/d)$. Then the function ζ is a bijection from the set of unitary divisors of $N - 1$ onto the set $K(N)$. In particular, a number X is in the set $K(N)$ if and only if $X = d\,Inv(d, (N - 1)/d)$ for some unitary divisor d of $N - 1$.

- The numbers in $K(N)$ occur in complementary pairs, X and $N - X$. If d is a unitary divisor of $N - 1$ then so is $e = (N - 1)/d$, and if $d \,\mathrm{Inv}(d, e) = X$ then $e \,\mathrm{Inv}(e, d) = N - X$.
- In binary, all even perfect numbers are Kaprekar numbers. More generally, any numbers of the form $2^{n-1}(2^n - 1)$ or $2^{n-1}(2^n + 1)$, with n a positive integer, are binary Kaprekar numbers.
- For any base b congruent to 3 mod 4, all numbers of the form $(b^{2n+1} - 1)/2$ and $(b^{2n+1} + 1)/2$, with n a positive integer, are Kaprekar numbers for the base b. Numbers of the first form expressed in base b have $2n + 1$ digits, all equal to $(b - 1)/2$. Those of the second form are just one more than the corresponding one of the first form. In base 11, for instance, 555, 556, 55555, 55556, etc. are Kaprekar numbers, and in base 15, the numbers 777, 778, 77777, 77778 etc. are Kaprekar numbers.
- For any base b congruent to 5 mod 16, all numbers of the form $3(b^{4n-1} - 1)/4$, $(b^{4n-3} - 1)/4$, $(b^{4n-1} + 3)/4$, or $(3b^{4n-3} + 1)/4$, with n a positive integer, are Kaprekar numbers for the base b. The expansions of the first two of these forms in base b comprise strings of $4n$

-1 digits all equal to $3(b-1)/4$, and $4n-3$ digits all equal to $(b-1)/4$, respectively. Those of the third and fourth forms are just one more than the corresponding ones of the first two forms, respectively. In base 5, for instance, 1, 4, 112, 333, 11111, 33334 etc. are Kaprekar numbers.

- For any base b congruent to 13 mod 16, all numbers of the form $3(b^{4n-3}-1)/4$, $(b^{4n-1}-1)/4$, $(b^{4n-3}+3)/4$, or $(3b^{4n-1}+1)/4$, with n a positive integer, are Kaprekar numbers for the base b. The expansions of the first two of these forms in base b comprise strings of $4n-3$ digits all equal to $3(b-1)/4$, and $4n-1$ digits all equal to $(b-1)/4$, respectively. Those of the third and fourth forms are just one more than the corresponding ones of the first two forms, respectively. In base 13, for instance, 4, 9, 333, 99X, 33334, 99999 etc. are Kaprekar numbers.

3. 6174 (number)

← 6173	6174	6175 →

List of numbers — Integers

← 0 1k 2k 3k 4k 5k 6k 7k 8k 9k →

Cardinal	six thousand one hundred seventy-four
Ordinal	6174th (six thousand one hundred seventy-fourth)
Factorization	$2 \times 3^2 \times 7^3$
Greek numeral	,ϚΡΟΔ´
Roman numeral	$\overline{V}$MCLXXIV
Binary	1100000011110_2
Ternary	22110200_3
Quaternary	1200132_4
Quinary	144144_5
Senary	44330_6
Octal	14036_8
Duodecimal	$36A6_{12}$
Hexadecimal	$181E_{16}$
Vigesimal	$F8E_{20}$
Base 36	$4RI_{36}$

6174 is known as **Kaprekar's constant**[1][2][3] after the Indian mathematician D. R. Kaprekar. This number is notable for the following property:

1. Take any four-digit number, using at least two different digits. (Leading zeros are allowed.)

2. Arrange the digits in descending and then in ascending order to get two four-digit numbers, adding leading zeros if necessary.
3. Subtract the smaller number from the bigger number.
4. Go back to step 2 and repeat.

The above process, known as Kaprekar's routine, will always reach its fixed point, 6174, in at most 7 iterations. Once 6174 is reached, the process will continue yielding 7641 – 1467 = 6174. For example, choose 3524:

5432 – 2345 = 3087

8730 – 0378 = 8352

8532 – 2358 = 6174

7641 – 1467 = **6174**

The only four-digit numbers for which Kaprekar's routine does not reach 6174 are repdigits such as 1111, which give the result 0000 after a single iteration. All other four-digit numbers eventually reach 6174 if leading zeros are used to keep the number of digits at 4.

2111 – 1112 = 0999

9990 – 0999 = 8991 (rather than 999 – 999 = 0)

9981 – 1899 = 8082

8820 – 0288 = 8532

8532 – 2358 = **6174**

9831 reaches 6174 after 7 iterations:

9831 – 1389 = 8442

8442 – 2448 = 5994

9954 – 4599 = 5355

5553 – 3555 = 1998

9981 – 1899 = 8082

8820 – 0288 = 8532 (rather than 882 – 288 = 594)

8532 – 2358 = **6174**

4371 reaches 6174 after 7 iterations:

7431 - 1347 = 6084

8640 - 0468 = 8172 (rather than 864 - 468 = 396)

8721 - 1278 = 7443

7443 - 3447 = 3996

9963 - 3699 = 6264

6642 - 2466 = 4176

7641 - 1467 = **6174**

8774, 8477, 8747, 7748, 7487, 7847, 7784, 4877, 4787, and 4778 reach 6174 after 4 iterations:

8774 – 4778 = 3996

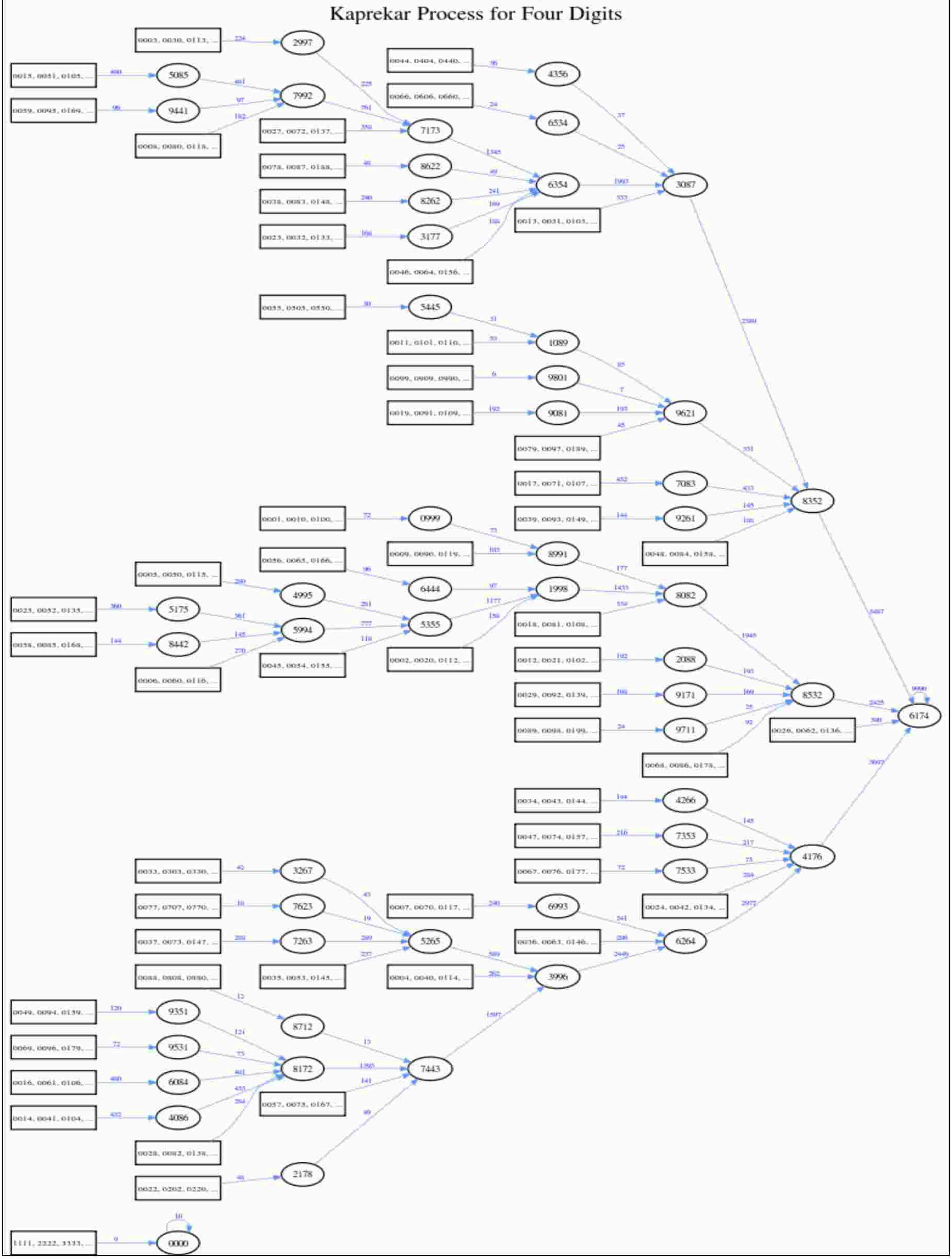
9963 – 3699 = 6264

6642 – 2466 = 4176

7641 – 1467 = **6174**

Note that in each iteration of Kaprekar's routine, the two numbers being subtracted one from the other have the same digit sum and hence the same remainder modulo 9. Therefore, the result of each iteration of Kaprekar's routine is a multiple of 9.

4.Kaprekar process for four digits



5.495 (number)

495 (**four hundred [and] ninety-five**) is the natural number following 494 and preceding 496. It is a pentatope number[1] (and so a binomial coefficient $\binom{12}{4}$).

← 494	495	496 →

List of numbers — Integers

← 0 100 200 300 400 500 600 700 800 900 →

Cardinal	four hundred ninety-five
Ordinal	495th (four hundred ninety-fifth)
Factorization	$3^2 \times 5 \times 11$
Greek numeral	ΥϘΕ´
Roman numeral	CDXCV
Binary	111101111_2
Ternary	200100_3
Quaternary	13233_4
Quinary	3440_5
Senary	2143_6
Octal	757_8
Duodecimal	353_{12}
Hexadecimal	$1EF_{16}$
Vigesimal	$14F_{20}$
Base 36	DR_{36}

Kaprekar transformation

The Kaprekar's routine algorithm is defined as follows for three-digit numbers:

1. Start with a three-digit number with at least two digits different.

2. Arrange the digits in ascending and then in descending order to get two three-digit numbers, adding leading zeros if necessary.
3. Subtract the smaller number from the bigger number.
4. Go back to step 2.

Repeating this process will always reach 495 in a few steps. Once 495 is reached, the process stops because 954 – 459 = 495.

Example

For example, choose 589:

985 − 589 = 396

963 − 369 = 594

954 − 459 = **495**

The only three-digit numbers for which this function does not work are repdigits such as 111, which give the answer 0 after a single iteration. All other three-digits numbers work if leading zeros are used to keep the number of digits at 3:

211 – 112 = 099

990 – 099 = 891 (rather than 99 - 99 = 0)

981 – 189 = 792

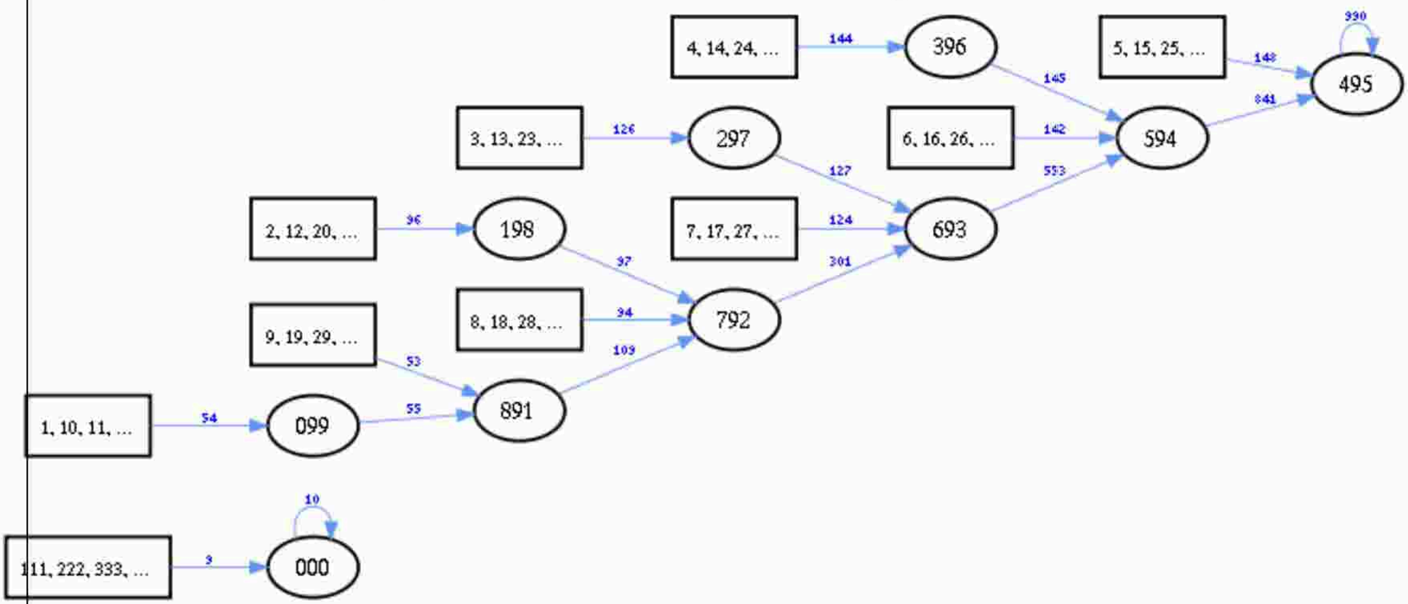
972 – 279 = 693

963 – 369 = 594

954 − 459 = **495**

The number 6174 has the same property for the four-digit numbers.

Kaprekar Process for Three Digits



Other "Kaprekar constants"

Note that there can be analogous fixed points for digit lengths other than four, for instance if we use 3-digit numbers then most sequences (i.e., other than repdigits such as 111) will terminate in the value 495 in at most 6 iterations. Sometimes these numbers (495, 6174, and their counterparts in other digit lengths or in bases other than 10) are called "Kaprekar constants".

Other properties

6174 is a Harshad number, since it is divided by the sum of its digits:

$6174 = (6 + 1 + 7 + 4) \times 343.$

6174 is a 7-smooth number, i. e. all its prime factors are not greater than 7.

6174 is a practical number, because an arbitrary number less than 6174 can be represented as a sum of various factors of the number 6174. This is not an uncommon property, and the nearest neighboring practical numbers are 6160, 6162, 6180, 6188.

6174 can be written as the sum of the first three degrees of the natural number 18:

$18^3 + 18^2 + 18 = 5832 + 324 + 18 = 6174.$

The sum of squares of the prime factors of 6174 is an exact square:

$2^2 + 3^2 + 3^2 + 7^2 + 7^2 + 7^2 = 4 + 9 + 9 + 49 + 49 + 49 = 169 = 13^2$.

THE MAIN RESULT

For each integer $N > 1$, let $K(N)$ denote the set of positive integers k for which there exists integers q and r such that

$$k^2 = qN + r \qquad (0 <= r < N) \qquad \textbf{(1)}$$

$$k = q + r \quad . \qquad \textbf{(2)}$$

As a matter of convention, we shall ignore the vacuous solution $k = N$ (for which $q = N$ and $r = 0$). **(1)** and **(2)** imply

$$k(k - 1) = q(N - 1) \qquad \textbf{(3)}$$

Since we disregard the vacuous solution, we have $1 <= k <= N - 1$ (for if $k >= N$ then **(3)** implies $q > k$, contradicting **(2)**).

The set $K(N)$ is nonempty, for always 1 is in $K(N)$. Suppose k were in $K(N)$. Since $(k, k - 1) = 1$, it follows from **(3)** that $d \mid k$ and $d' \mid k - 1$ for some positive d and d' such that $dd' = N - 1$ and $(d, d') = 1$. Let $k' = N - k$. Because $1 <= k <= N - 1$, we have $k' > 0$. Since $k' = (N -$

1) - (k - 1) , we have $d' \mid k'$. Thus $k = dm$ and $k' = d'm'$ for some positive m and m' , whence follows

$$dm + d'm' = N = dd' + 1 \qquad \textbf{(4)}$$

Definition: If (a , b) = 1 , we denote by **Inv**(a , b) the least positive integer m such that am = 1 (mod b). It follows that m = **Inv**(a , b) if and only if 1 <= m < b and am =1 (mod b).

It is not difficult to show the next result.

Lemma 1: Suppose (a , b) = 1 . Then m = **Inv**(a , b) and n = **Inv**(b , a) if and only if m and n are positive and am+ bn = ab + 1 .

Applying Lemma 1 to **(4)** gives

$$k = d\,\mathbf{Inv}(d , d') ; \qquad k' = d'\,\mathbf{Inv}(d ', d) . \qquad \textbf{(5)}$$

Conversely, let dd' = N - 1 , (d, d') = 1, and let m = **Inv**(d, d') and m' = **Inv**(d', d) . Then by Lemma 1 we have dm + d'm' = N . Therefore

$$d^2m^2 = (N - d'm')^2$$

$$= N^2 - N d'm' - (dm + d'm') d'm' +$$

$(d'm')^2$

$$= N^2 - N\,d'm' - mm'dd'$$

$$= (N - d'm' - mm')\ N + mm' .$$

Thus

$$(dm)^2 = (N - d'm' - mm')$$

$N + mm'$ $(mm' < N)$,

$$dm = (N - d'm' - mm') + mm' .$$

That is, dm satisfies **(1)** and **(2)** (with $q = N - d'm' - mm'$ and $r = mm'$), whence dm is in K(N). Note that $d'm'$ is in K(N) by symmetry. We have proved the following results:

Theorem 1: k is in K(N) if and only if k = d **Inv**(d, (N-1)/d) for some unitary divisor d of N - 1 .

Corollary A: The elements k of K(N) occur in complementary pairs. For each k in K(N), N - k is in K(N).

Let w(M) denote the number of distinct primes dividing M; then M has exactly $2^{w(M)}$ unitary divisors. The following result is immediate:

Corollary B: K(N) contains exactly $2^{w(N-1)}$ elements.

The convention that N not be an element of K(N) was taken to ensure the bijection between the elements of K(N) and the unitary divisors of N - 1 .

7.Kaprekar's Operation :

In 1949 the mathematician D. R. Kaprekar from Devlali, India, devised a process now known as Kaprekar's operation. First choose a four digit number where the digits are not all the same (that is not 1111, 2222,...). Then rearrange the digits to get the largest and smallest numbers these digits can make. Finally, subtract the smallest number from the largest to get a new number, and carry on repeating the operation for each new number.

It is a simple operation, but Kaprekar discovered it led to a surprising result. Let's try it out, starting with the number 2005, the digits of last year. The maximum number we can make with these digits is 5200, and the minimum is 0025 or 25 (if one or more of the digits is zero, embed these in the left hand side of the minimum number). The subtractions are:

5200 - 0025 = 5175

7551 - 1557 = 5994

9954 - 4599 = 5355

5553 - 3555 = 1998

9981 - 1899 = 8082

8820 - 0288 = 8532

8532 - 2358 = 6174

7641 - 1467 = 6174

When we reach 6174 the operation repeats itself, returning 6174 every time. We call the number 6174 a kernel of this operation. So 6174 is a kernel for Kaprekar's operation, but is this as special as 6174 gets? Well not only is 6174 the only kernel for the operation, it also has one more surprise up its sleeve. Let's try again starting with a different number, say 1789.

9871 - 1789 = 8082

8820 - 0288 = 8532

8532 - 2358 = 6174

We reached 6174 again!

A very mysterious number...

When we started with 2005 the process reached 6174 in seven steps, and for 1789 in three steps. In fact, you reach 6174 for all four digit numbers that don't have all the digits the same. It's marvellous, isn't it? Kaprekar's operation is so simple but uncovers such an interesting result. And this will become even more intriguing when we think about the reason why all four digit numbers reach this mysterious number 6174.

Only 6174?

The digits of any four digit number can be arranged into a maximum number by putting the digits in descending order, and a minimum number by putting them in ascending order. So for four digits a,b,c,d where

$9 \geq a \geq b \geq c \geq d \geq 0$

and a, b, c, d are not all the same digit, the maximum number is abcd and the minimum is dcba.

We can calculate the result of Kaprekar's operation using the standard method of subtraction applied to each column of this problem:

which gives the relations

$D = 10 + d - a$ (as $a > d$)

$C = 10 + c - 1 - b = 9 + c - b$ (as $b > c - 1$)

$B = b - 1 - c$ (as $b > c$)

$A = a - d$

for those numbers where $a>b>c>d$.

A number will be repeated under Kaprekar's operation if the resulting number ABCD can be written using the initial four digits a,b,c and d. So we can find the kernels of Kaprekar's operation by considering all the possible combinations of {a, b, c, d} and checking if they satisfy the relations above. Each of the 4! = 24 combinations gives a system of four simultaneous equations with four unknowns, so we should be able to solve this system for a, b, c and d.

It turns out that only one of these combinations has integer solutions that satisfy $9 \geq a \geq b \geq c \geq d \geq 0$. That combination is ABCD = bdac, and the solution to the simultaneous equations is a=7, b=6, c=4 and d=1. That is ABCD = 6174. There are no valid solutions to the simultaneous equations resulting from some of the digits in {a,b,c,d} being equal. Therefore the number 6174 is the only number unchanged by Kaprekar's operation — our mysterious number is unique.

For three digit numbers the same phenomenon occurs. For example applying Kaprekar's operation to the three digit number 753 gives the following:

753 - 357 = 396

963 - 369 = 594

954 - 459 = 495

954 - 459 = 495

The number 495 is the unique kernel for the operation on three digit numbers, and all three digit numbers reach 495 using the operation. Why don't you check it yourself?

How fast to 6174?

It was about 1975 when I first heard about the number 6174 from a friend, and I was very impressed at the time. I thought that it would be easy to prove why this phenomenon occurred but I could not actually find the reason why. I used a computer to check whether all four digit numbers reached the kernel 6174 in a limited number of steps. The program, which was about 50 statements in Visual Basic, checked all of 8991 four digit numbers from 1000 to 9999 where the digits were not all the same.

The table below shows the results: every four digit number where the digits aren't all equal reaches 6174 under Kaprekar's process, and in at most seven steps. If you do not reach 6174 after using Kaprekar's operation seven times, then you have made a mistake in your calculations and should try it again!

Iteratio	**Freque**

n	ncy
0	1
1	356
2	519
3	2124
4	1124
5	1379
6	1508
7	1980

Which way to 6174?

My computer program checked all 8991 numbers, but in his article Malcolm Lines explains that it is enough to check only 30 of all the possible four digit numbers when investigating Kaprekar's operation.

As before let's suppose that the four digit number is abcd, where

$$9 \geq a \geq b \geq c \geq d \geq 0.$$

Let us calculate the first subtraction in the process. The maximum number is 1000a+100b+10c+d and the minimum number is 1000d+100c+10b+a. So the subtraction is:

$$1000a + 100b + 10c + d - (1000d + 100c + 10b + a)$$

= 1000(a-d) + 100(b-c) + 10(c-b) + (d-a)

= 999(a-d) + 90(b-c)

The possible value of (a-d) is from 1 to 9, and (b-c) is from 0 to 9. By running through all the possibilities, we can see all the possible results from the first subtraction in the process. These are shown in Table 1.

		999X(a-d)								
		1	2	3	4	5	6	7	8	9
90X (b-c)	0	999	1998	2997	3996	4995	5994	6993	7992	8991
	1	1089	2088	3087	4086	5085	6084	7083	8082	9081
	2	1179	2178	3177	4176	5175	6174	7173	8172	9171
	3	1269	2268	3267	4266	5265	6264	7263	8262	9261
	4	1359	2358	3357	4356	5355	6354	7353	8352	9351
	5	1449	2448	3447	4446	5445	6444	7443	8442	9441
	6	1539	2538	3537	4536	5535	6534	7533	8532	9531
	7	1629	2628	3627	4626	5625	6624	7623	8622	9621
	8	1719	2718	3717	4716	5715	6714	7713	8712	9711
	9	1809	2808	3807	4806	5805	6804	7803	8802	9801

Table 1: Numbers after the first subtraction in Kaprekar's process

We are only interested in numbers where the digits are not all equal and

$a \geq b \geq c \geq d$,

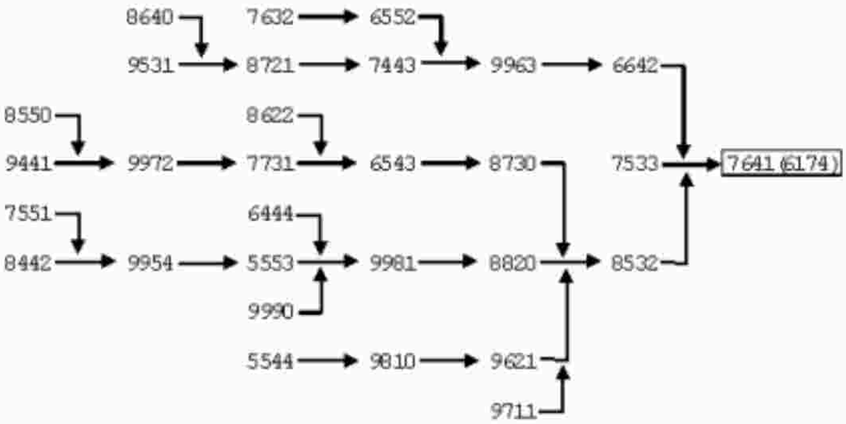
therefore we only need to consider those where (a-d) ≥ (b-c). So we can ignore the grey region in Table 1 which contains those numbers where (a-d) < (b-c).

Now we arrange the digits of the numbers in the table in descending order, to get the maximum number ready for the second subtraction:

		999X(a-d) 1	2	3	4	5	6	7	8	9
	0	9990	9981	9972	9963	9954	9954	9963	9972	9981
	1	9810	8820	8730	8640	8550	8640	8730	8820	9810
	2		8721	7731	7641	7551	7641	7731	8721	9711
	3			7632	6642	6552	6642	7632	8622	9621
90X	4				6543	5553	6543	7533	8532	9531
(b-c)	5					5544	6444	7443	8442	9441
	6						6543	7533	8532	9531
	7							7632	8622	9621
	8								8712	9711
	9									9801

Table 2: Maximum numbers, ready for the second subtraction

We can ignore the duplicates in Table 2 (the grey regions), and are left with just 30 numbers to follow through the rest of the process. The following figure shows the routes which these numbers take to reach 6174.



How these 30 numbers reach 6174

From this figure you can see how all the four digit numbers reach 6174 and reach it in at most seven steps. Even so I still think it is very mysterious. I guess Kaprekar, who discovered this number, was extremely clever or had a lot of time to think about it!

We have seen that four and three digit numbers reach a unique kernel, but how about other numbers? It turns out that the answers for those is not quite as impressive. Let try it out for a two digit number, say 28:

$$82 - 28 = 54$$
$$54 - 45 = 9$$
$$90 - 09 = 81$$
$$81 - 18 = 63$$
$$63 - 36 = 27$$
$$72 - 27 = 45$$
$$54 - 45 = 9$$

It doesn't take long to check that all two digit numbers will reach the loop 9→81→63→27→45→9. Unlike for three and four digit numbers, there is no unique kernel for two digit numbers.

But what about five digits? Is there a kernel for five digit numbers like 6174 and 495? To answer this we would need to use a similar process as before: check the 120 combinations of {a,b,c,d,e} for ABCDE such that

$$9 \geq a \geq b \geq c \geq d \geq e \geq 0$$

and

abcde - edcba = ABCDE.

Thankfully the calculations have already been done by a computer, and it is known that there is no kernel for Kaprekar's operation on five digit numbers. But all five digit numbers do reach one of the following three loops:

71973→83952→74943→62964→71973

75933→63954→61974→82962→75933

59994→53955→59994

As Malcolm Lines points out in his article, it will take a lot of time to check what happens for six or more digits, and this work becomes extremely dull! To save you from this fate, the following table shows the kernels for two digit to ten digit numbers (for more see Mathews Archive of Recreational Mathematics). It appears that Kaprekar's operation takes every number to a unique kernel only for three and four digit numbers.

Digits	**Kernel**
2	None
3	495
4	6174
5	None
6	549945, 631764
7	None

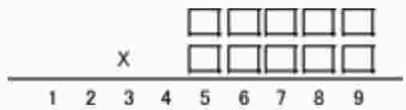
8	63317664, 97508421
9	554999445, 864197532
10	6333176664, 9753086421, 9975084201

Beautiful, but is it special?

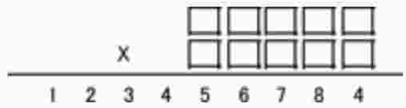
We have seen that all three digit numbers reach 495, and all four digit numbers reach 6174 under Kaprekar's operation. But I have not explained why all such numbers reach a unique kernel. Is this phenomenon incidental, or is there some deeper mathematical reason why this happens? Beautiful and mysterious as the result is, it might just be incidental.

Let's stop and consider a beautiful puzzle by Yukio Yamamoto in Japan.

If you multiply two five digit numbers you can get the answer 123456789. Can you guess the two five digit numbers?



This is a very beautiful puzzle and you might think that a big mathematical theory should be hidden behind it. But in fact it's beauty is only incidental, there are other very similar, but not so beautiful, examples. Such as:



(We can give you a hint to help you solve these puzzles, and here are the answers.)

If I showed you Yamamoto's puzzle you would be inspired to solve it because it is so beautiful, but if I showed you the second puzzle you might not be interested at all. I think Kaprekar's problem is like Yamamoto's number guessing puzzle. We are drawn to both because they are so beautiful. And because they are so beautiful we feel there must be something more to them when in fact their beauty may just be incidental. Such misunderstandings have led to developments in mathematics and science in the past.

Is it enough to know all four digit numbers reach 6174 by Kaprekar's operation, but not know the reason why? So far, nobody has been able to say that all numbers reaching a unique kernel for three and four digit numbers is an incidental phenomenon. This property seems so surprising it leads us to expect that a big theorem in number theory hides behind it. If we can answer this question we could find this is just a beautiful misunderstanding, but we hope not.

Note from the editors: many readers noticed that repeatedly adding up the digits of any of the kernels of

Kaprekar's operation always equals 9. Find out why in this follow-up to the article.

7. Python code for Kaprekar number:

A Kaprekar number is a number that is the result of a specific operation applied to a given number. The Kaprekar routine involves taking a non-negative integer, arranging its digits in ascending and then descending order to create two new numbers, subtracting the smaller number from the larger one, and repeating this process until the result equals the original number.

Here's a Python program that finds Kaprekar numbers within a given range:

```
def kaprekar_numbers(start, end):
    kaprekar_nums = []
    for num in range(start, end+1):
        square = num * num
        num_digits = len(str(num))
        square_str = str(square)

        for i in range(1, num_digits):
            part1 = int(square_str[:i])
            part2 = int(square_str[i:])
            if part1 != 0 and part2 != 0 and part1 + part2 == num:
                kaprekar_nums.append(num)
                break
    return kaprekar_nums

start_range = int(input("Enter the starting range: "))
end_range = int(input("Enter the ending range: "))

kaprekar_nums = kaprekar_numbers(start_range, end_range)
print("Kaprekar numbers in the range {} to {}: {}".format(start_range, end_range, kaprekar_nums))
```

You can input the range within which you want to find Kaprekar numbers, and the program will output the Kaprekar numbers within that range.

For example, if you input the range from 1 to 100, it will output:

Kaprekar numbers in the range 1 to 100: [1, 9, 45, 55, 99]

8.Applications of kaprekar numbers

While Kaprekar numbers may not have direct practical applications in fields like engineering or finance, they have significance in recreational mathematics and can offer insights into number theory and mathematical patterns. Here are some ways in which Kaprekar numbers can be applied:

1.Recreational Mathematics: Kaprekar numbers provide interesting puzzles and challenges for enthusiasts of recreational mathematics. They can be used as the basis for brain teasers, puzzles, and mathematical games, providing entertainment while also encouraging the exploration of number patterns.

2. Number Theory: Studying Kaprekar numbers can deepen understanding of number theory concepts. They offer insights into properties of numbers, such as digital roots, congruences, and modular arithmetic. Exploring Kaprekar numbers can lead to the discovery of new patterns and relationships among numbers.

3. Algorithm Development: Although not directly applicable in practical computing tasks, the study of Kaprekar numbers can inspire the development and analysis of algorithms. Researchers may explore efficient methods for generating or identifying Kaprekar numbers, which can contribute to algorithmic techniques and optimizations in computational mathematics.

4. Educational Tools: Kaprekar numbers can serve as educational tools for teaching various mathematical concepts, including number properties, operations, and problem-solving strategies. Teachers can use Kaprekar numbers to engage students in hands-on activities, fostering curiosity and critical thinking skills.

5. Inspiration for Research: The study of Kaprekar numbers can inspire further research in mathematics and related fields. Researchers may be motivated to investigate generalizations or variations of Kaprekar numbers, explore connections to other areas of mathematics, or seek applications in cryptography or theoretical computer science.

6. Cultural and Historical Significance: Kaprekar numbers are named after Indian mathematician D. R. Kaprekar, highlighting their cultural and historical significance. Exploring Kaprekar numbers can provide insights into the contributions of Indian mathematicians and the development of mathematical ideas across different cultures and time periods.

While Kaprekar numbers may not have direct practical applications in everyday life, their study enriches our understanding of mathematics and stimulates curiosity and creativity in exploring mathematical concepts and patterns.

9. Case Sudies:

Certainly! Here are a few case studies showcasing interesting aspects of Kaprekar numbers:

1. Kaprekar's Routine:

- One fascinating property of Kaprekar numbers is their behavior when iteratively applying Kaprekar's routine. Kaprekar devised a process where you take any arbitrary four-digit number (with at least two distinct digits), rearrange its digits to form the largest and smallest numbers possible, and then subtract the smaller number from the larger one. Repeating this process eventually converges to 6174, known as Kaprekar's constant.
- Case study: Start with the number 3524. Rearrange the digits to get the largest number 5432 and the smallest number 2345. The difference is 5432 - 2345 = 3087. Repeating the process, we get 8730 - 378 = 8352, then 8532 - 2358 = 6174. Further iterations will remain at 6174.

2. Generalizations and Variations:

- Kaprekar numbers aren't limited to base-10 arithmetic. They exist in other numerical bases as well. Exploring Kaprekar numbers in different bases can yield interesting patterns and variations.

- Case study: Investigate Kaprekar-like numbers in base-5 arithmetic. Analyze their properties and compare them to Kaprekar numbers in base-10.

3. Palindrome and Repunit Kaprekar Numbers:

- Palindromic numbers (numbers that read the same backward as forward) and repunit numbers (numbers consisting entirely of the digit 1) also exhibit Kaprekar properties in certain cases.

- Case study: Explore the properties of palindromic Kaprekar numbers and repunit Kaprekar numbers. Investigate whether there are infinitely many of them and identify any unique characteristics.

4. Kaprekar Numbers in Other Operations:

- While Kaprekar numbers are primarily studied in the context of squaring and subtraction, they can also be investigated in other mathematical operations.

- Case study: Investigate Kaprekar-like properties in multiplication, division, or exponentiation. Analyze whether similar patterns and properties emerge.

5. Applications in Cryptography:

- Kaprekar numbers and related concepts have been explored in cryptographic algorithms and protocols.
- Case study: Research how Kaprekar numbers can be utilized in encryption schemes or cryptographic protocols. Explore their potential advantages and limitations in the field of cryptography.

These case studies provide a glimpse into the diverse aspects and applications of Kaprekar numbers, showcasing their relevance and significance in various mathematical contexts. Further exploration and analysis can lead to deeper insights and discoveries in the realm of number theory and mathematics.

10. Conclusion

In conclusion, the study of Kaprekar numbers offers a fascinating journey into the realm of number theory, revealing intriguing patterns, properties, and applications. Through this project, we've delved into various aspects of Kaprekar numbers, from their historical origins to their computational methods and real-world implications. Here's a summary of the key points:

1. Historical Significance: Named after Indian mathematician D. R. Kaprekar, Kaprekar numbers have captivated mathematicians and enthusiasts alike with their unique properties.

2. Mathematical Properties: Kaprekar numbers possess distinct characteristics, such as yielding two numbers when squared whose sum equals the original number. Their behavior under different operations and numerical bases adds depth to their study.

3. Computational Methods: Algorithms for identifying and generating Kaprekar numbers within a given range have been explored, showcasing the intersection of mathematics and computer science.

4. Applications: While Kaprekar numbers may not have direct practical applications, their study inspires curiosity and creativity. They find applications in recreational mathematics, cryptography, and potentially other fields yet to be discovered.

5. Case Studies: Through case studies, we've examined various facets of Kaprekar numbers, including their convergence properties, generalizations to different bases, and potential applications in cryptography.

Finally, Kaprekar numbers stand as a testament to the beauty and richness of mathematical exploration. They serve as a reminder of the infinite possibilities inherent in numbers and the enduring allure of mathematical puzzles.

In essence, the study of Kaprekar numbers transcends mere arithmetic; it represents a journey of discovery, insight, and intellectual curiosity. As we conclude this project, let us continue to explore the wonders of mathematics, seeking inspiration in the enigmatic properties of numbers like Kaprekar numbers.

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