"Education for propagation of Knowledge, Achievement and Cultural"

-Shikshan Maharshi Dr.Bapuji Salunkhe

Shri Swami Vivekanand Shikshan Sanstha's

Vivekanand College, Kolhapur (Empowered Autonomous)

A Project report on

"Role of Mathematics in Robotics"

SUBMITED BY

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Class: B.Sc.III

Under Guidance

Ms. P. P. Kulkarni

Submitted to

Vivekanand College

(Empowered Autonomous)

For the year 2023-24

CERTIFICATE

I undersigned hereby declare that the project report on "Role of Mathematics in Robotics" is developed by myself.

This project work is completed under the guidance of Ms. P.P. Kulkarni. This project work is based on the information collected by us.

I understand that any such copying is liable to be punished in a way the college authorities deem it.

Date: 23/03/2024

Place: Kolhapur

Teacher in Charge

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DECLARATION

I undersigned hereby declare that the Project report on "Role of Mathematics in Robotics" is completed under the guidance of Ms. P. P. Kulkarni. I declare that this is my original work which is submitted to Vivekanad College (Empowered Autonomous), Kolhapur in 2023-2024 academic year

We understand that any such copying is liable to be punished in a way the college authorities deem it.

Name: Vaishnavi S. Parit

Sign

ACKNOWLEDGEMENT

On the day of completion of this project, the numerous memories agreeing rushed in my mind with full of gratitude to this encouraged and helped me a lot at various stages of this work.

I offer sincere gratitude to all of them. I have great pleasure to express my deep sense of indebtedness and heart of full gratitude to my project guide Mr. S.P. Thorat sir for his expert and valuable guidance and continuous encouragement given to me during the course of project work. I am thankful to prin. Dr. R.R. Kumbhar sir (Principal, Vivekanand College) and Mr. S. P. Thorat sir (H.O.D Dept. of Mathematics) for allowing me to carry out our project work and extending me all the possible infra-structural facilities of department.

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Place: Kolhapur
Date: 23/03/2024
Name: Ms. Vaishnavi Parit

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INTRODUCTION

History

The world's first industrial robot called UNIMATE, it was designed in 1956 by American inventors George Devol and Joseph Engelberger.

350 B.C:
Greek mathematician Archytas succeeded in building in a mathematical bird which used steam of propel itself.

1920s:
In 1921, a Czech writer Karel Capek coined the term "Robot" in his play "R.U.R" (Rossum's Universal Robots)

1940s:
Issac Asimov gave us the three laws of robotics which can also be used to define what is robot.
William Grey Walter working in Burden Neurological Institute in Bristol, was able to create two autonomous robots named "Elmer" and "Elsie".

1970s:
Freddy and Freddy II were able to assemble wooden blocks and put rings on pegs using its video camera 3-DOF and 5-DOF mechanisms.

1980s:
Genghis was created by scientist at MIT in 1989. It was one of the first example cheap robots. Another great feature of it was its behavioral algorithm which makes the robot behave like a real insect.

2000s:
The new generation of robots like Robonaut 2 are the first humanoid robots in the history of robotics, that are used in space to help astronauts.

Mathematics Behind Robotics

Mathematics is an invaluable tool for robotic programming, especially it comes to manufacturing. Application of mathematics are everywhere- from understanding how robots interact with their environment to making sure components of a factory move precisely from one point to another.

Robotics does require some basic Mathematics skills. Our program teaches the mathematics used in formulas, imperial system, metrics systems, conversion and other relevant areas.

Mathematical concepts used to design Robotic model:

- **Calculus:** To optimize the trajectory of robot to achieve efficient and smooth motion.
- **Geometry:** To determine spatial relationships and designing robot concepts.
- **Linear Algebra:** Algebra has patterns and relationships which are used in the concepts of speed, time, Distance and force in robotics.

Degrees of Freedom

A 'Degrees of Freedom' (DoF) as it relates to robotic arms, is an independent joint that can provide freedom of movement for the manipulator, either in a rotational or translational (linear) sense. Every geometric axis that joint can rotate around or extend along is counted as a Single Degree of Freedom.

Dof = $m(N-1-J)+\sum_{i=1}^{n} f_i$

Where,

m = movable joints

N = number of links including ground

J = number of joints

f_i = number of freedom provided by joint i.

Ex. Consider a 3R robot, means it has three revolute joints.

Let, m= 3 N= 4 J= 3

By using Grubler's Formula,

DoF = $m(N-1-J) + \sum_{i=1}^{n} f_i$

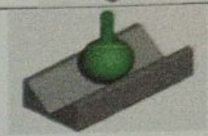
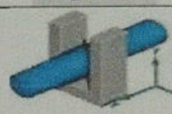
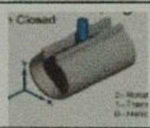
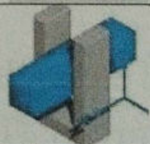
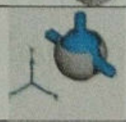
DoF = 3(4-1-3) + 3

DoF = 3 (0) + 3

DoF = 3

The robot has **3** degrees of freedom.

Degrees of Freedom according to Movement

Degree of Freedom	Free rotations	Free translations	Name	Kinematic Pair: Form Closure	Kinematic Pair: Force Closure
5	3	2	Sphere-plane		
4	3	1	Sphere-groove		
	2	2	Cylinder-plane		
3	3	0	Spheric		
	2	1	Sphere-slotted cylinder		
	1	2	Planar		
2	2	0	Slotted spheric		
	2	0	Toric		
	1	1	Cylinder		
	1	1	Slotted spheric		
1	1	0	Revolute		
	0	1	Prismatic		

Linear Algebra

When a robot rotates its camera to capture a panoramic view or translate to pick an object , linear algebra provides the tools to perform these actions correctly.

Ex.

Consider a robot's hand is supposed to peak up a part. A coordinate system, **P**, attached to the part is located relative to the "world" coordinate system, **W**, by the transformation matrix

$$^{W}T_{P}=\begin{bmatrix} 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & 2 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and the robot's base frame, **B**, is located relative to the world frame by

$$^{W}T_{B}=\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In order to put the hand on the part, we wish to align the hand frame, **H** and the part frame. What is the transformation $^{B}T_{H}$ (giving the hand frame relative to the robot base frame) that makes this happen?

Solution:

When the hand frame, **H**, and the part frame, **P**, are aligned then

$^{W}T_{H}$-$^{W}T_{P}$

This is shown in diagram to the right. Now we use the fact that the transformation from the world to the base and the transformation from the world to the base to the hand can be combined into a single transformation from the world to the hand, like this:

$^{W}T_{H}={}^{W}T_{B}\,{}^{B}T_{H}$

Substituting in the known matrices we have:

$$\begin{bmatrix} 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & 2 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix} {}^{B}T_{H}$$

Solving this matrices equation for ${}^{B}T_{H}$ gives:

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix} X \begin{bmatrix} 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & 2 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^{B}T_{H}$$

Or:

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & -9 \\ 0 & 0 & 0 & -1 \end{bmatrix} X \begin{bmatrix} 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & 2 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^{B}T_{H}$$

Or finally:

$$\begin{bmatrix} 0 & 1 & 0 & -2 \\ 0 & 0 & -1 & -3 \\ -1 & 0 & 0 & -9 \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^{B}T_{H}$$

As explained in the background, the three columns of the 3X3submatrix give the orientation of the hand coordinate system in terms of the robot's base coordinate system and the last column gives the origin of the hand coordinate system in terms of the origin of the base coordinate system.

Applications of Linear Algebra:

- To describe movement of robots
- To calculate speed
- To calculate acceleration
- To determine the position of robots on the manufacturing floor
- To capture a panoramic view
- To pick an object

Applications of Robotics

Security:

One of the top applications of robotics is in the field of security. Imagine a world where all security guards are robots. Even thieves would think twice before committing a crime! That's why robotics is being considered as a solution to enhance security measures. **Robots can serve as security agents**, protecting humans without being vulnerable to danger like human security guard**s.**

Entertainment:

Another top application of robotics is robots are also a big draw in the entertainment industry. While they cannot exactly become actors and actresses, they can be used behind the sets in movies and serials to manage the camera, provide special effects, etc. They can be used for boring repetitive tasks that are not suitable for a human as cinema is, after all, a creative industry. Robots can also be used to do stunt work that is very dangerous for humans but looks pretty cool in an action movie. Theme parks like Disney World are also using autonomous robots to enhance the magical experience of their customers.

Space Exploration:

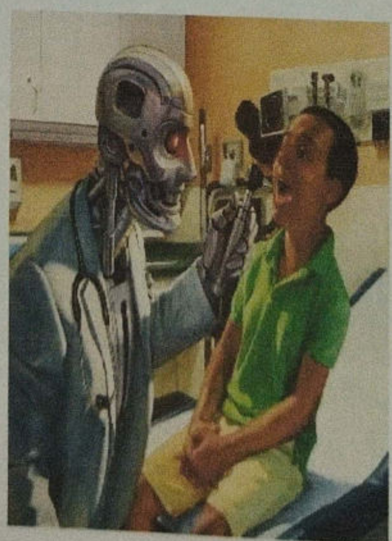
Many things in space are very dangerous for astronauts to do. Humans can't roam on Mars all day to collect soil samples or work on repairing a spaceship from the outside while it's in deep space! In these situations, robots are a great choice because there are no chances for the loss of human life. So, space institutions like NASA frequently use robots and autonomous vehicles to do things that humans can't. This can be one of the top applications of robotics.

For example, Mars Rover is an autonomous robot that travel on Mars and takes pictures of Martian rock formations that are interesting or important and then sends them back to Earth for NASA scientists to study.

Space Robot

Health Care:

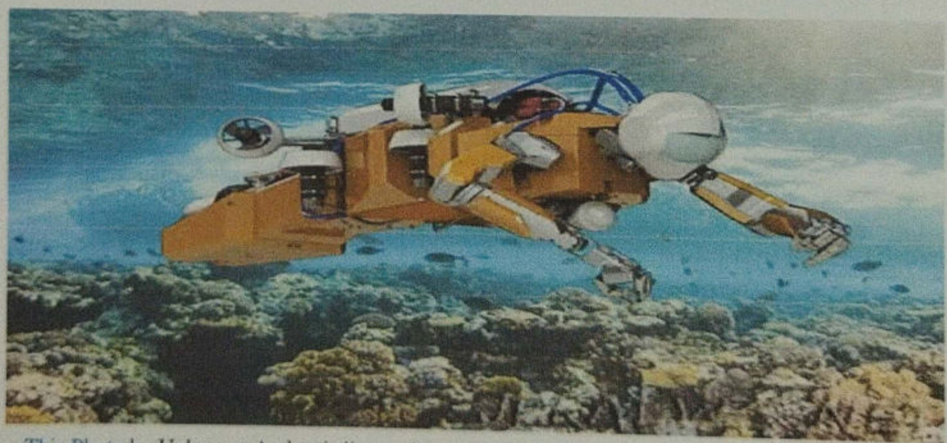
One of the top applications of robotics is robots have changed healthcare a lot. And all for the better! They can help doctors in performing operations more precisely, be used as prosthetic limbs, provide therapy to patients, etc. The possibilities are limitless. One example of this is the ***da Vinci robot*** that can help surgeons in performing complex surgeries relating to the heart, head, neck, and other sensitive areas. Other robotic devices are created like **exoskeletons** that can be used to provide additional support for people undergoing rehabilitation after spinal injuries, strokes, etc.



Underwater Exploration:

Robots are a great option for exploring places that humans cannot reach easily, like the depths of the ocean! There is a lot of water pressure deep in the ocean which means humans cannot go that down and machines such as submarines can only go to a certain depth as well.

A deep underwater is a mysterious place that can finally be explored using specially designed robots. These robots are remote-controlled, and they can go into the depths of the ocean to collect data and images about aquatic plant and animal life.



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CONCLUSION

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- Mathematics play a crucial role in developing algorithms for robot perception, motion, planning, and control, enabling robot to navigate complex environment.
- Robots can help us at that position on which human can not reach such as space exploration and underwater exploration
- Mathematics plays a critical role in the development and advancement of technology.
- Mathematics is not just a subject to classroom and textbook; it is fundamental component of our daily lifestyle.

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