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Shivaji University, Kolhapur

For the Degree of Bachelor of Science

In

Mathematics

By

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B.Sc. III (Mathematics)

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Under the Guidance of

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Vivekanand College (Empowered Autonomous),Kolhapur

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Vivekanand College, Kolhapur (Empowered Autonomous)

DEPARTMENT OF MATHEMATICS

Certificate

This is to certify that Mr/Ms. Mrunali Ajit Powar has successfully completed the project work on topic "Application of derivatives" towards the partial fulfilment for the course of Bachelor of Science (Mathematics) work of Vivekanand College, Kolhapur (Empowered Autonomous) affiliated to Shivaji University, Kolhapur during the academic year 2023-2024.

Place: Kolhapur

Date: 23/March/2024

Examiner

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Head Dept. of Mathematics

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Teacher in charge.

Examiner

DECLARATION

I undersigned hereby declare that project entitled "Application of derivatives" completed under the guidance of(Ms P.P.Kulkarni) based on the experiment results and cited data. I declare that this is my original work which is submitted to Vivekanand College, Kolhapur in 2023-2024 academic year.

0

Name: Mrunali Ajit Pawar.

Sign : M.APawar.

ACKNOWLEDGEMENT

On the day of completion of this project, the numerous memories agreeing rushed in my mind with full of gratitude to this encouraged and helped me a lot at various stages of this work.

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Place: Kolhapur.

Date: 23/March/2024

Name: Mrunali Ajit Pawar.

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Introduction

Newton and Leibniz quite independently of one another, were largely responsible for developing the ideas of integral calculus to the point where hitherto insurmountable problems could be solved by more or less routine methods. The successful accomplishments of these men were primarily due to the fact that they were able to fuse together the integral calculus with the second main branch of calculus, differential calculus.

The central idea of differential calculus is the notation of derivative. Like the integral, the derivative originated from a problem in geometry the problem finding the tangent line at a point of a curve. Unlike the integral. However, the derivative evolved very late in the history of mathematics. The concept was not formulated until early in the 17th century when the French mathematician Pierre de Fermat, attempted to determine the maxima and minima of certain special functions.

1. Definition of Derivative:

We begin with a function f defined at least on some open interval (a, b) on the x-axis. Then we choose a fixed-point x in this interval and introduce the difference quotient

$$\frac{f(x+h)-f(x)}{h}$$

Where the number h, which may be positive or negative (but not zero), is such that x + h also lies in (a, b). The numerator of this quotient measures the change in the function when x changes from x to x +h. The quotient itself is referred to as the average rate of change of f in the interval joining x to x + h.

Now we let h approach zero and see what happens to this quotient. If the quotient approaches some definite value as a limit (which implies that the limit is the same whether h approaches zero through positive values or through negative values), then this limit is called the derivative of f at x and is denoted by the symbol f'(x) (read as "f prime of x"). Thus, the formal definition of f'(x) may be stated as follows:

2. DEFINITION OF DERIVATIVE: The derivative f '(x) is defined by the equation

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

provided the limit exists. The number f '(x) is also called the rate of change of f at X.

Meaning of derivative:-

- The Derivative is the exact rate at which one quantity changes with respect to another.
- Geometrically, the derivative is the slope of curve at the point on the curve.
- The derivative is often called the "instantaneous" rate of change.
- The derivative of a function represents an infinitely small change the function with respect to one of its variables.
- The Process of finding the derivative is called "differentiation".

Application of Derivatives in Various Fields/Science such as in: -

- Biology
- Economics
- Chemistry
- Physics
- Mathematics
- Others (Psychology, sociology & geology)

3. Application of Derivative in Medical and Biology:

Sometimes we may question ourselves why students in biology or medical department still have to take mathematics and even physics. After reading this post, you will understand why.

3.1 Growth Rate of Tumor:

A tumor is an abnormal growth of cells that serves no purpose. There are certain level of a tumor regarding to its malignancy.

The first level is benign tumor. It does not invade nearby tissue or spread to other parts of the body the way cancer can. In most cases, the outlook with benign tumors is very good. But benign tumors can be serious if they press on vital structures such as blood vessels or nerves. Therefore, sometimes they require treatment and other times they do not.

The second level is premalignant or precancerous tumor which is not yet malignant, but is about to become so.

The last level is malignant tumors. These are cancerous tumors, they tend to become progressively worse, and can potentially result in death. Unlike benign tumors, malignant ones grow fast, they are ambitious, they seek out new territory, and they spread (metastasize).

The abnormal cells that form a malignant tumor multiply at a faster rate. Experts say that there is no clear dividing line between cancerous, precancerous and non-cancerous tumors-sometimes determining which is which may be arbitrary, especially if the tumor is in the middle of the spectrum. Some benign tumors eventually become premalignant, and then malignant.

The rate at which a tumor grows is directly proportional to its volume. Larger tumors grow faster and smaller tumors grow slower.

The rate at which a tumor grows is directly proportional to its volume. Larger tumors grow faster and smaller tumors grow slower.

The volume of a tumor is found by using the exponential growth model which is

$$V(t) = V_0 \cdot e^{kt}$$

$V_0 =$ initial volume

$e =$ exponential growth

$k =$ growth constant

$t =$ time

In order to find the rate of change in tumor growth, you must take the derivative of the volume equation (V(t))

$$V(t) = V_0 \cdot e^{kt}$$

$$V'(t) = V_0 \cdot e^{kt} \cdot k$$

Because e^{kt} is a complicated function, we use chain rule to derivate it.

$$y = e^{kt}$$

Let $u = kt$ $\qquad$ $y = e^{u}$

$$\frac{dy}{dt} = \frac{dy}{du}\frac{du}{dt} \qquad \frac{dy}{du} = e^{u}$$

$$\frac{du}{dt} = k \qquad \frac{dy}{dt} = k\,e^{u}$$

$$\frac{dy}{dt} = k\,e^{kt}$$

From the calculation above, we know that the derivative of e^{kt} is $k \cdot e^{kt}$

$$V'(t) = V_0 \cdot k \cdot e^{kt}$$

Because V(t) itself is equal to $V_0 \cdot e^{kt}$ we may conclude

$$V'(t) = k \cdot V$$

- There is the example to prove this theory:

3.2. Larger tumor:

Find the rate of change of a tumor when its initial volume is 10 cm^3 with a growth constant of 0.075 over a time period of 7 years

$$V(t) = V_0 \cdot e^{kt}$$

$$V(7) = 10 \times 2.178^{(0.075)7}$$

$$V(7) = 15.05 cm^3$$

$$V'(t) = k \cdot v$$

$$V'(t) = 0.075 \times 15.05$$

$$V'(t) = 1.13 cm^3 / \text{ year}$$

Then let's calculate the rate of change of smaller tumor with the same growth constant and time period.

3.3 Smaller tumor:

Find the rate of change of tumor when its initial volume is 2 cm^3 with a growth constant of 0.075 over a time period of 7 years

$$V'(t) = V_0 \cdot e^{kt}$$

$$V(7) = 2 \times 2.178^{(0.075)7}$$

$$V(7) = 3.01 cm^3$$

$$V'(t) = k \cdot v$$

$$V'(t) = 0.075 \times 3.01$$

$$V'(t) = 0.23 cm^3 / \text{ year}$$

With this calculation we know how important it is to detect a tumor as soon as possible. It is crucial to give a right treatment that will stop or slow down the growth of the tumor because bigger tumor intend to grow faster and, in some case, becoming a cancer that have a small chance to cured.

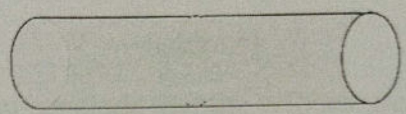
3.4 Blood Flow:

High blood pressure can affect the ability of the arteries to open and close. If your blood pressure is too high, the muscles in the artery wall will respond by pushing back harder. This will make them grow bigger, which makes your artery walls thicker. Thicker arteries mean that there is less space for the blood to flow through. This will raise your blood pressure even further.

Due to fat and cholesterol plaque that cling to the vessel, it becomes constricted. If an artery bursts or becomes blocked, the part of the body that gets its blood from that artery will be starved of the energy and oxygen it needs and the cells in the affected area will die.

If the burst artery supplies a part of the brain then the result is a stroke. If the burst artery supplies a part of the heart, then the area of heart muscle will die, causing a heart attack.

We can calculate the velocity of the blood flow and detect if there are something wrong with the blood pressure or the blood vessel wall. In this case, we portrait the blood vessel as a cylindrical tube with radius R and length L as illustrated below



Because of the friction at the walls of the vessel, the velocity of the blood is not the same in every point. The velocity of the blood in the center of the vessel is faster than the flow of the blood near the wall of the vessel. The velocity is decreases as the distance of radius from the axis (center of the vessel) increaseuntil v become O at the wall.

The relationship between velocity and radius is given by the low of laminar flow discovered by the France Physician Jean-Louis-Marie Poiseuille in 1840. This state that

$$V = \frac{P}{4\eta L}\left(R^2 - r^2\right)$$

V = initial volume

n = viscosity of the blood

P = Pressure difference between the ends of the blood vessel

L = length of the blood vessel

R = radius of the blood vessel

r = radius of the specific point inside the blood vessel that we want to know.

To calculate the velocity gradient or the rate of change of the specific point in the blood vessel we derivate the law of laminar flaw

$$V = \frac{P}{4\eta L}\left(R^2 - r^2\right)$$

$$V' = \frac{d}{dr}\left[\frac{P}{4\eta L}\left(R^2 - r^2\right)\right] = \frac{P}{4\eta L}\cdot\frac{d}{dt}\left(R^2 - r^2\right)$$

$$V' = \frac{P}{4\eta L}\left(0 - 2r\right)$$

$$V' = -\frac{2rP}{4\eta L}$$

Example:-

The left radial artery radius is approximately 2.2 mm and the viscosity of the blood is 0.0027 Ns/m^2. The length of this vessel is 20 mm and pressure differences are 0.05 N. What is the velocity gradient at r = 1 mm from center of the vessel?

$$V' = -\frac{2rp}{4\eta L}$$

$$V' = \frac{-2.1\times10^{-3}\times0.05}{4\times0.0027\times20\times10^{-3}}$$

$$V' = \frac{-10^{-4}}{2.16\times10^{-4}}$$

$$V' = -0.46 m/s$$

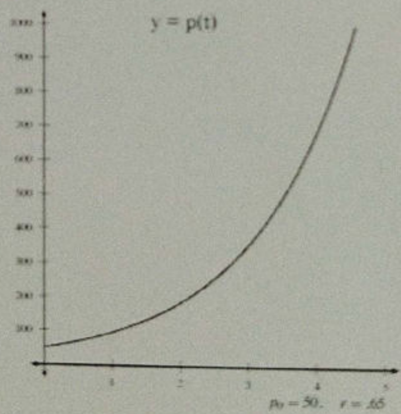
So, we can conclude that the velocity gradient is – 0.46 m/s. if the gradient of velocity is too high then the person may have a constriction in his/her blood vessel and needs further examination and treatment.

3.5 Population models

The population of a colony of plants, or animals, or bacteria, or humans, is often described by an equation involving a rate of change (this is called a "differential equation"). For instance, if there is plenty of food and there are no predators, the population will grow in proportion to how many are already there:

$$\frac{dp}{dt} = rp$$

Where r is constant. It's not hard to check that the function $p(t) = p_0e^{rt}$



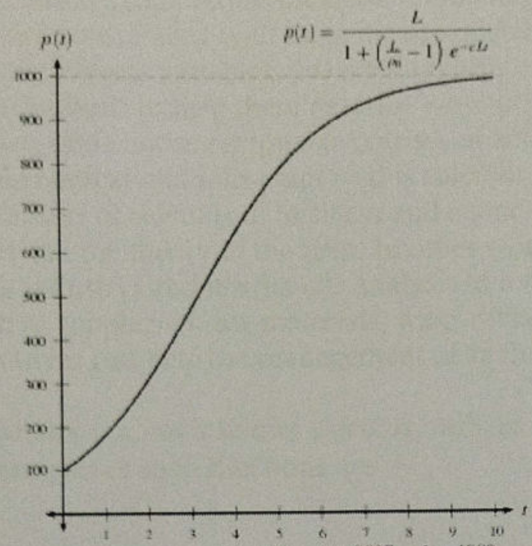
Satisfies this differential equation, where p_0 is the starting population. Colonies tend to grow exponential until they run out of space food or run into predators.

When there are limits on the food supply, the population is often governed by the logistic

EQUATION:-

$$\frac{dp}{dt} = cp(L-p)$$

Where c and L are constant. The population grows exponentially for a while, and then levels off at a horizontal asymptote of L



$p_0 = 100, \quad c = .0007, \quad L = 1000$

The logistic equation also governs the growth of epidemics, as well as for the example, the frequency of certain genes in a population.

4. Application of Derivative to Business and Economics:

In recent years, economic decision making has become more and more mathematically oriented. Faced with huge masses of statistical data, depending on hundreds or even thousands of different variables, business analysts and economists have increasingly turned to mathematical methods to help them describe what is happening, predict the effects of various policy alternatives, and choose reasonable courses of action from the myriad of possibilities. Among the mathematical methods employed is calculus. In this section we illustrate just a few of the many applications of calculus to business and economics. All our applications will center on what economists call the theory of the firm. In other words, we study the activity of a business (or possibly a whole industry) and restrict our analysis to a time period during which background conditions (such as supplies of raw materials, wage rates, and taxes) are fairly constant. We then show how derivatives can help the management of such a firm make vital production decisions.

Management, whether or not it knows calculus, utilizes many functions of the sort we have been considering. Examples of such functions are

C(x) cost of producing x units of the product.
R(x) revenue generated by selling x units of the product,
P(x) = R(x)-C(x) = the profit (or loss) generated by producing and (selling x units of the product.)

Note that the functions C(x), R(x), and P(x) are often defined only for non-negativ integers, that is, for x = 0,1,2,3. The reason is that it does not make sense to speak about the cost of producing -1 car or the revenue generated by selling 3.62 refrigerators. Thus, each function may give rise to a set of discrete points on a graph, as in Figure, in studying these functions, however, economists usually draw a smooth curve through the points and assume that C(x) is actually defined for all positive x. Of course, we must often interpret answers to problems in light of the fact that x is, in most cases, a nonnegative integer.

Cost Functions: If we assume that a cost function, C(x), has a smooth graph as in Figure, we can use the tools of calculus to study it. A typical cost function is analyzed in Example 1.

4.1 Marginal Cost Analysis:

Example:1:-

Suppose that the cost function for a manufacturer is given by

$$C(X)=(10^{-6})X^3-0.003X^2+5X+1000 \text{ dollars.}$$

a) Describe the behavior of the marginal cost.
b) Sketch the graph of C(x).

Solution:-

The first two derivatives of C(x) are given by

$$C'(X)=(3\times10^{-6})X^2-0.006X+5$$

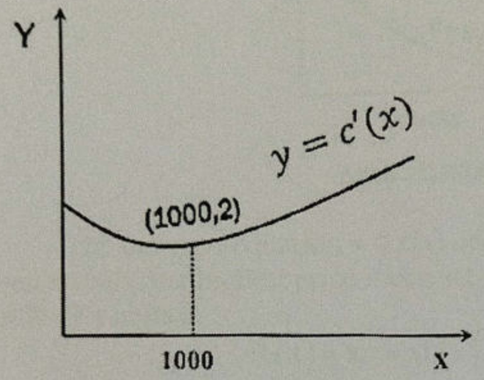
$$C''(X)=(6\times10^{-6})X-0.006$$

Let us sketch the marginal cost C'(x) first. From the behavior of C'(x), we will be able to graph C(x). The marginal cost function $y=(3\times10^{-6})X^2-0.006X+5$ has as its graph a parabola that opens upward. Since $y=C''(X)=0.000006(X-1000)$, we see that the parabola has a horizontal tangent at X = .1000 So, the minimum value of C'(x) occurs at X = 1000. The corresponding y-coordinate is

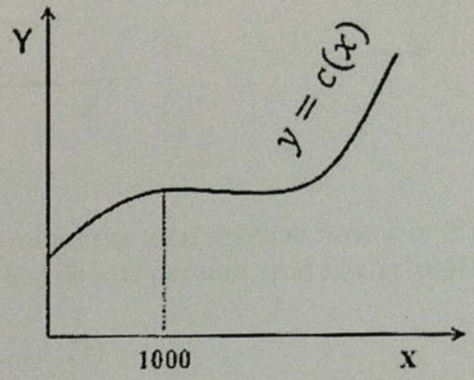
$$(3\times10^{-6})(1000)^2 - 0.006\times(1000)+5 = 3-6+5 = 2$$

The graph of y = C'(x) is shown in Figure. Consequently, at first, the marginal cost decreases. It reaches a minimum of 2 at production level 1000 and increases thereafter. This answers part (a). Let us now graph C(x). Since the graph shown in Figure is the graph of the derivative of C(x), we see that C'(x) is never zero, so there are no relative extreme points. Since C'(x) is always positive, C(x) is always increasing (as any cost curve should).

Moreover, since C'(x) decreases for x less than 1000 and increases for x greater than 1000, we see that C(x) is concave down for x less than 1000. is concave up for x greater than 1000, and has an inflection point at x = 1000. The graph of C(x) is drawn in Figure. Note that the inflection point of C(x) occurs at the value of x for which marginal cost is a minimum.



A marginal cost functions.



A cost functions.

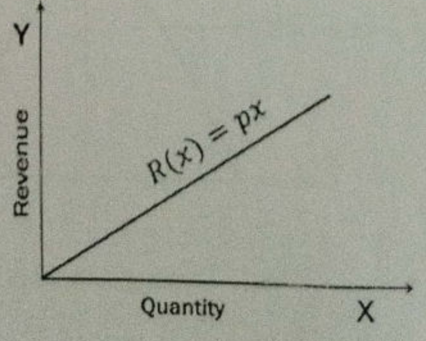
Actually, most marginal cost functions have the same general shape as the marginal cost curve of Example 1. For when x is small, production of additional units is subject to economies of production, which lowers unit costs. Thus, for x small, marginal cost decreases. However, increased production eventually leads to overtime, use of less efficient, older plants, and competition for scarce raw materials. As a result, the cost of additional units will increase for very large x. So, we see that C'(x) initially decreases and then increases.

Revenue Functions in general, a business is concerned not only with its costs, but also with its revenues. Recall that, if R(x) is the revenue received from the sale of x units of some commodity, then the derivative R'(x) is called the marginal revenue. Economists use this to measure the rate of increase in revenue per unit increase in sales.

If x units of a product are sold at a price p per unit, the total revenue R(x) is given by

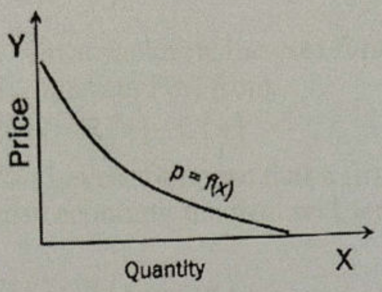
$$R(x) = x\cdot p$$

If a firm is small and is in competition with many other companies, its sales have little effect on the market price. Then, since the price is constant as far as the one firm is concerned, the marginal revenue R'(x) is the amount that the firm receives from the sale of one additional unit. In this case, the revenue function will have a graph Revenue as in figure.



A revenue curves.

An interesting problem arises when a single firm is the only supplier of a certain product or service, that is, when the firm has a monopoly. Consumers will buy large amounts of the commodity if the price per unit is low and less if the price is raised. For each quantity x, let f(x) be the highest price per unit that can be set to sell all x units to customers. Since selling greater quantities requires a lowering of the price, f(x) will be a decreasing function. Figure shows a typical demand curve that relates the quantity demanded, x, to the price, $p = f(x)$.



A demand curves.

The demand equation $p = f(x)$ determines the total revenue function. If the firm wants to sell x units, the highest price it can set is f(x) dollars per unit, and so the total revenue from the sale of x units is

$$R(x) = xp = xf(x) \text{...........} (1)$$

The concept of a demand curve applies to an entire industry (with many producers) as well as to a single monopolistic firm. In this case, many producers offer the same product for sale. If x denotes the total output of the industry, f(x) is the market price per unit of output and x f(x) is the total revenue earned from the sale of the x units.

4.2 Maximizing Revenue:

Example : 2 The demand equation for a certain product is $p = 6 - \frac{x}{2}$ dollars Find the level of production that results in maximum revenue.

Solution:

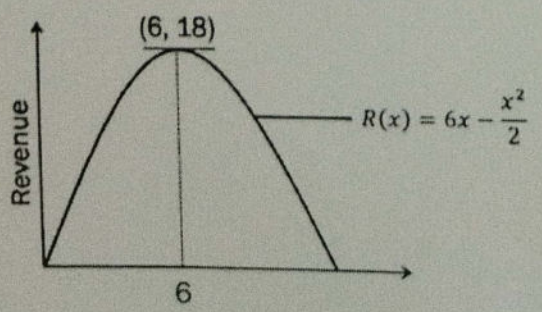
In this case, the revenue function R(x) is

$$R(x) = xp = x\left(6 - \frac{x}{2}\right)$$

$$= x \; 6x - \frac{x^2}{2} \text{ dollars.}$$

The marginal revenue is given by

$$R'(x) = 6 - x$$



Maximizing revenue.

The graph of R(x) is a parabola that opens downward. (see figure) It has a horizontal tangent precisely at those x for which R'(x) = 0 that is, for those x at which marginal revenue is O. The only such x is x = 6. The corresponding value of revenue is

$$R(x) = 6.6 - \frac{(6)^2}{2} = 18 \text{ dollars}$$

Thus, the rate of production resulting in maximum revenue is $x = 6$, which results in total revenue of 18 dollars.

Profit Functions: Once we know the cost function C(x) and the revenue function R(x), we can compute the profit function P(x) from

$$P(x) = R(x) - C(x)$$

Setting production Levels: Suppose that a firm has cost function C(X) and revenue function R(x). In a free-enterprise economy the firm will set production x in such a way as a maximize the profit function

$$P(x) = R(x) - c(x)$$

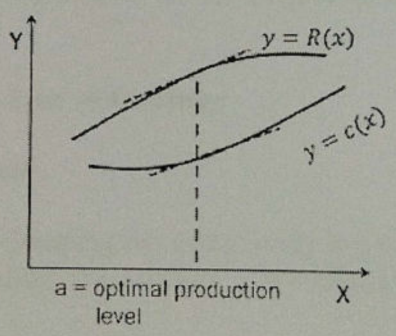
We have seen that if P(x) has a maximum at x = a, then P'(a) = 0In other words, since

$$P'(x) = R'(x) - C'(x)$$

$$R'(a) = C'(a) = 0$$

$$R'(a) = C'(a)$$

Thus, profit is maximized at a production level for which marginal revenue equals marginal cost. (See Figure)



6. Application of Derivative in Chemistry:

The change in temperature

- An object's temperature over time will approach the temperature of its surroundings (the medium).
- The greater the difference between the object's temperature and the medium's temperature, the greater the rate of change of the object's temperature.
- This change is a form of exponential decay.

6.1 Newton's Law of Cooling.

- It is a direct application for differential equations.
- Formulated by Sir Isaac Newton.
- Has many applications in our everyday life.
- Sir Isaac Newton found this equation behaves like what is called in Math (differential equations) so his used some techniques to find its general solution.

5. Derivation of Newton's Law of Cooling:

- Newton's observations:

  He observed that the temperature of the body is proportional to the difference between its own temperature and the temperature of the objects in contact with it.

- Formulating:

  First order separable DE

- Applying differential calculus:

$$\frac{dT}{dt} = -k(T - T_E)$$

Where k is the positive proportionality constant

- By separation of variables we get $\frac{dT}{(T - T_E)} = -kdt$
- By integrating both sides we get $\ln(T - T_E) + C = -kt$
- At time (t = O) the temperature is T_o

$$C = -\ln(T_o - T_E)$$

- By substituting $C = -\ln(T_o - T_E)$ we get

$$\ln\frac{(T-T_E)}{(T_o-T_E)}=-kt$$

$$T=T_E+(T_o-T_E)e^{-kt}$$

6.3 Applications on Newton's Law of Cooling:

Investigations.

>>It can be used to determine the time of death. .

Computer manufacturing.

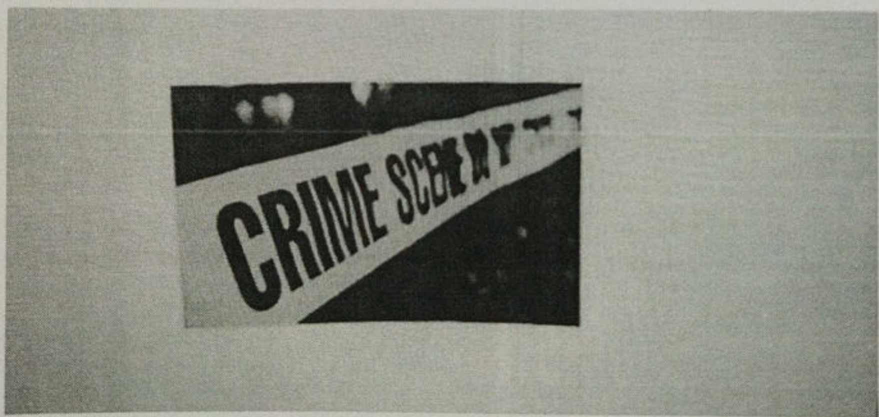
>>Processors.
>>Cooling systems

>>>Solar water Heater.

>>>Calculating the Surface area Of an object.

5. Applications of Newton's Law Of Cooling In Investigating A Crime scene:

The police came to a house at 10:23 am were urder had taken place. The detective measured the temperature of the victim's body and found that it was 26.7°C. Then he used a thermostat to measure the temperature of the room that was found to be 20°C through the last three days. After an hour he measured the temperature of the body again and found that the temperature was 25.8°C. Assuming that the body temperature was normal (37°C), what is the time of death?



Solution:

$$T=T_E+(T_o-T_E)e^{-kt}$$

Let the time at which the death took place be x hours before the arrival of the police men.

Substitute by the given values

$$T(x)=26.7=20+(37-20)e^{-kx}$$
$$T(x+1)=25.8=20+(37-20)e^{-k(x+1)}$$

Solve the 2 equations simultaneously

$$0.394 = e^{-kx}$$

$$0.341 = e^{-k(x+1)}$$

By taking the logarithmic function

$$\ln(0.394) = -kx \quad \text{..................}(1)$$

$$\ln(0.341) = -k(x+1) \quad \text{..............}(2)$$

By dividing (1) by (2)

$$\frac{\ln(0.394)}{\ln(0.341)} = \frac{-kx}{-k(x+1)}$$

$$0.8657 = \frac{x}{x+1}$$

Thus, $x \cong 7$ hours

Therefore, the murder took place 7 hours before the arrival of the detective which is at 3:23 pm

4.1 Applications of Newton's Law of Cooling in Processor Manufacturing:

A global company such as intel is willing to produce a new cooling system for their processor that can cool the processors from a temperature of 50°C to 27°C in just half an hour when the temperature outside is 20°C but they don't know what kind of materials they should use or what the surface area and the geometry of the shape are. So, what should they do? Simply they have to use the general formula of Newton's law of cooling

$$T = T_E + (T_o - T_E)e^{-kt}$$

And by substituting the numbers they get

$$27 = 20 + (50 - 20)e^{-0.5k}$$

Solving for k, we get

$$k = 2.9$$

So, they need a material with K = 2.9 (k is a constant that is related to the heat capacity, thermodynamics of the material and also the shape and the geometry of the material)

6. Application of Derivative in Physics:

Derivatives with respect to time:

In physics, we are often looking at how things change over time:

1. **Velocity** is the derivative of position with respect to time:

$$v(t) = \frac{d}{dt}(x(t))$$

2. **Acceleration** is the derivative of velocity with respect to time:

$$a(t) = \frac{d}{dt}(v(t)) = \frac{d^2}{dt^2}(x(t))$$

3. **Momentum** (usually denoted p) is mass times velocity, and **force (F)** is mass times acceleration, so the derivative of momentum is

$$\frac{dp}{dt} = \frac{d}{dt}(mv) = m\frac{dv}{dt} = ma = F$$

One of Newton's laws says that for every action there is an equal and opposite reaction, meaning that if particle 2 puts force F on particle 1, then particle 1 must put force – F on particle 2. But this means that the (momentum is constant), since

$$\frac{d}{dt}(p_1 + p_2) = \frac{dp_1}{dt} + \frac{dp_2}{dt} = F - F = 0$$

This is the **law of conservation of momentum**.

Derivatives with Respect to Position:

In physics, we also take derivatives with respect to x.

1. For so called "conservative" forces, there is a function V(x) such that the force depends only on position and is minus the derivative of V, namely $F(x) = -\frac{dv(x)}{dx}$ The function v(x) is called the potential energy.

   For instance, for a mass on a spring the potential energy is $\frac{1}{2}kx^2$, where k is a constant and the force is – kx.

2. The kinetic energy is $\frac{1}{2}mv^2$. Using the chain rule, we find that the total energy

$$\frac{d}{dt}\left(\frac{1}{2}mv^2 + V(x)\right) = mv\frac{dv}{dt} + V'(x)\frac{dx}{dt} = mva - Fv = (ma - F)v = 0$$

Since F = ma. This means that the total energy never changes.

These are just a few of the examples of how derivatives come up in physics. In fact, most of physics, and especially electromagnetism and quantum mechanics, is governed by differential equations in several variables.

5.1 Elasticity of Demand

The elasticity of demand E, is the percentage rate of decrease of demand per percentage increase in price. We obtain it from the demand equation according to the following formula:

$$E = \frac{dq}{dp} \cdot \frac{p}{q}$$

Where the demand equation expresses demand q, as a function of unit price p, we say that demand has unit elasticity if E = 1.

To find the unit price that maximizes revenue, we express E as a function of p, set E = 1, and then solve for p.

Example:

Suppose that the demand equation q = 20,000 – 2p.

Then $$E = -(-2)\frac{p}{20,000 - 2p} = \frac{p}{10,000 - p}$$

If p = 2000, then $E = \frac{1}{4}$, and demand is inelasticity at this price.

If p = 8000, then E = 4, and demand is elasticity at this price.

If p = 5000, then E = 1, and the demand has unit elasticity at this price.

8. Application of Derivative in Mathematics:

Applications of Maxima and Minima: Optimization Problems:

We solve **optimization problems** of the following form: Find the values of the unknowns x, y,... maximizing (or minimizing) the value of the **objective function f**, subject to certain constraints. The constraints are equations and inequalities relating or restricting the variables x, y.....

To solve such a problem, we use the constraint equations to write all of the variables in terms of one chosen variable, substitute these into the objective function f, and then find extrema as above. (We use any constraint inequalities to determine the domain of the resulting function of one variable.) Specifically:

1. Identify the unknown(s):

These are usually the quantities asked for in the problem.

2. Identify the objective function.

This is the quantity you are asked to maximize or minimize.

3. Identify the constraint(s).

These can be equations relating variables or inequalities expressing limitations on the values of variables.

4. **State the optimization problem.**
   This will have the form "Maximize (minimize] the objective function subject to the constraint(s)."

5. Eliminate extra variables.

If the objective function depends on several variables, solve the constraint equations to express all variables in terms of one particular variable. Substitute these expressions into the objective function to rewrite it as a function of a single variable. Substitute the expressions into any inequality constraints to help determine the domain of the objective function.

6. Find the absolute maximum (or minimum) of the objective function.

Example:

Here is a maximization problem: Maximize A = xy

Objective Function

subject to x +2y = 1000,
$x \geq 0$, and
$y \geq 0$ Constraints

Let us carry out the procedure for solving. Since we already have the problem stated as a optimization problem, we can start at step 5.

Eliminate extra variables.

We can do this by solving the constraint equation x + 2y = 100 for x (getting x = 100 – 2y) and substituting in the objective function and the inequality involving x:

Find the Absolut maximum (or minimum) of the objective function:-

Now, we have to find the maximum value of $A = 100y - 2y^2$.

Taking derivative of A with respect to y,

$$\frac{dA}{dy} = \frac{d}{dy}\left(100y - 2y^2\right) = 100 - 4y$$

For extreme points,

$$\frac{dA}{dy} = 0 \qquad 100 - 4y = 0$$

$$y = \frac{100}{4} \qquad y = 25$$

Put value of y in constant x, x + 2y = 100.

x = 100 – 2y
x = 100 – 2 (25)
x = 100 – 50
x = 50

Thus, extreme point is (50,25).

Maximum value of objective function,

$A = xy$

$A = (50)(25)$

$A = 1250$

Maximum, $A = 1250$

Analyzing Graphs :

We can use graphing technology to draw a graph, but we need to use differential calculus to understand what we are seeing. The most interesting features of a graph are the following.

Features of a Graph

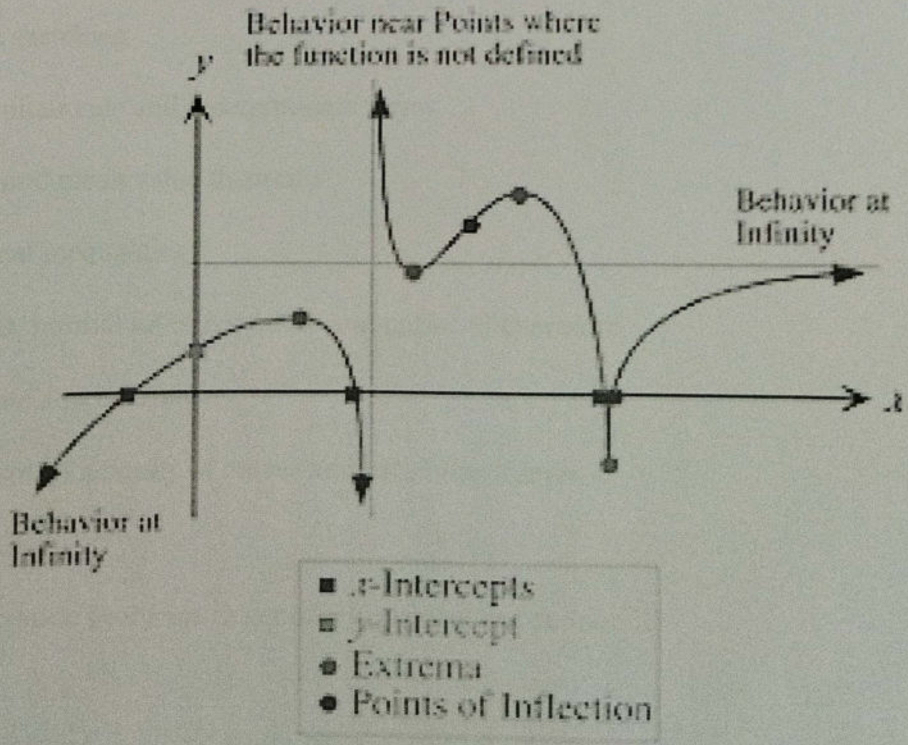
1. **The x – and y – intercepts:** If $y = f(x)$, find the x-intercept (s) by setting $y = 0$ and solving for x; find the y-intercept by setting $x = 0$.

2.Relative extrema: Use the processer to find relative extrema and locate the relative extrema.

3.Points of Inflection: Set $f''(x) = 0$ and solve for x to find candidate points of inflection.

4. Behavior near points where the function is not defined: If f(x) is not defined at x, consider $\lim_{x \to -} f(x)$ and $\lim_{x \to +} f(x)$ to see how the graph of f approaches this point.

5.Behavior at infinity: Consider $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to +\infty} f(x)$ if appropriate, to see haw the graph of f behaves far to the left and right.



To analyze this, we follow the procedure at left:

1. **The x and y-intercepts:**
   Setting y=0 and solving for x gives x= 0. This is the only x-intercept. Setting x=0 and solving for y gives y=0 : the y-intercept.

2. **Relative extrema:**
   The only extrema are stationary points found by setting f^{\prime}(x)=0 and solving for x, giving x=\underline{0} and x = -4. The corresponding points on the graph are the relative maximum (0, 0) and the relative minimum at (-4,8/9).

3. **Points of inflection:**
   Solving f^{\prime\prime}(x)=0 analytically is difficult, so we can solve it numerically (plot the second derivative and estimate where is crosses the x-axis) and find that the point of inflection lies at x\approx-6.1072

5. **Behavior near points where the function is not defined:**
   The function is not defined at x=-1 and x=2 The limits as x approaches these values from the left and right can be inferred from the graph:

Other Application of Derivatives in Mathematics:

- Approximation by differentials and newton's method
- Monotonic functions, relative and absolute extrema of functions
- Convex functions, inflection points and asymptotes
- Curve sketching
- L'Hospitals rule and indeterminate forms
- Roll's and mean value theorems
- Classical inequalities
- tangent, normal lines, curvature and radius of curvature
- Evaluate and involute
- Envelope of a family of curves and osculating curves
- related rates
- optimization problems in geometry, physics and economics

Application of Derivatives in Psychology:

The application of differential calculus to mental phenomena:

Dr. Montague's it was pointed out that we can get a very simple expression does for the 'specious present,' which was found to be $\frac{do}{ds}$, if we denote by o the objective and by s the subjective elements of a psychosis. The second derivative would determine the time flow. Without considering the important philosophical results of the theory we shall make the following observations about the method.

The author considers the ratio of the increments Δo and Δs, so which occur in the time Δt, and the fraction $\frac{\Delta o}{\Delta s}$ is supposed to approach or attain the limit $\frac{do}{ds}$ It will be of some interest to see what suppositions this statement involves. First of all, it is clear that we have to consider the limit $\frac{\frac{\Delta o}{\Delta t}}{\frac{\Delta s}{\Delta t}}$, because o is not an explicit function of s.

Though we know little or nothing about the sufficient conditions of differentiability, we can in this case readily indicate the following necessary conditions: (1) o and s must be continuous; (2) both must have a differential quotient with regard to t; (3) both differential quotients must be continuous; (4) $\frac{ds}{dt}$ must not be zero in the whole-time interval under consideration. it is hard to make those assumptions, nothing about the character of the functions dealt since s is apparently discontinuous in many points submitted to the well known tests.

It is evident that the author had in mind to measure a time period by its relation to a standard change and so to get rid duration, but he did not see that the conditions of the problem became so much more complicated by the implicit relation of o and All these tacit presuppositions would have become clear if the author had assumed that o is an explicit function of s, but such but such a relation, of which we can get no idea, would never have been granted. The establishing of the indirect relation between o and s by introducing them as functions of time hides the difficulty but does not remove it.

An example will show to what kind of conclusions we come, if we accept the author's view. $\frac{do}{ds}$ varies with time and we may pick out two moments for which this ratio has the same value, as it is always possible because $\frac{do}{ds}$ is continuous and $\frac{d^2o}{ds^2}$ changes sing. The conditions of Rolle's theorem are fulfilled, since continuity of $\frac{do}{ds}$ and existence of second derivative are supposed by the author, and therefore, the second derivative vanishes at least once. The vanishing of $\frac{d^2o}{ds^2}$ is characteristic for the state of ennui and the first conditions are approximately fulfilled if one sits in a quiet room and recalls something. It follows that one must be bored before one can recall anything. Psychological laws of this kind can be deduced easily by every mathematician.

There is not the least doubt that the whole theory of functions could be applied to a psychology of this kind, but the question remains, whether the conclusions logically deduced from our system admit of a verification by experiment. If we consider it an important feature of experimental psychology, that to every implication of our system corresponds an empirical fact and if possible, vice versa, we must renounce speculations about functions of which we know nothing.

Now supposing for a moment that there are no gaps and errors in the author's proof, could we deduce anything from his laws? Of course not. The function is totally unknown and we must measure empirically the value of $\frac{d^2o}{ds^2}$ It would be important to know the derivative if we could construct the function or if we could verify it in some other way, but as we cannot we must con elude that the use of symbols of which the end the meaning too general is of little help Finally it may be mentioned that the interesting attempt to measure a time period by the ratio of a change occurring in it to a standard change also occurring in it fails, because this ratio is a number which becomes a time only when multiplied by a time unit. For such a standard we choose a certain amount of change in o, to which we refer as a standard, for instance the movement of a pendulum. One of the principal features of a standard is constancy, and measurement is impossible without it. We have therefore either a measurement which varies with time or our whole speculations about the specious present break down, because the differential quotient of a constant vanishes everywhere.

Some Other Applications of Derivatives:

Derivatives are also use to calculate:

- Rate of heat flow in Geology.
- Rate of Improvement of performance in psychology
- Rate of the spread of a rumor in sociology.

Real Life Applications of Derivatives

10.1 Automobiles:

In an automobile there is always an odometer and a speedometer. These two gauges work in tandem and allow the driver to determine his speed and his distance that he has traveled. Electronic versions of these gauges simply use derivatives to transform the data sent to the electronic motherboard from the tires to miles per Hour (MPH) and distance (KM).

10.2 Radar Guns:

Keeping with the automobile theme from the previous slide, all police officers who use radar guns are actually taking advantage of the easy use of derivatives. When a radar gun is pointed and fired at your care on the highway. The gun is able to determine the time and distance at which the radar was able to hit a certain section of your vehicle. With the use of derivative, it is able to calculate the speed at which the car was going and also report the distance that the car was from the radar gun

.

10.3Business:

In the business world there are many applications for derivatives. One of the most important application is when the data has been charted on graph or data table such as excel. Once it has been input, the data can be graphed and with the applications of derivatives you can estimate the profit and loss point for certain ventures.

THANK YOU

THANK YOU