Vivekanand College Kolhapur (An Empowered Autonomous Institute)
Department of Mathematics
B.Sc.-I Sem -I Internal Examination 2025-26

Course Name: Basic Algebra (DSC03MAT11) Day & Date: Monday,13/10/2025
Time: 12:00PM to 1:30PM Marks: 10

Q.1. Select correct alternative from each of the following [02]

i) If $\frac{(\cos\theta + i\sin\theta)^9}{(\cos\theta + i\sin\theta)^{15}} =$ ______

a) $\cos 6\theta + i\sin 6\theta$ b) $\cos 5\theta + i\sin 5\theta$ c) $\cos 6\theta - i\sin 6\theta$ d) $\cos 5\theta - i\sin 5\theta$

ii) If $A = \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$ then AB=.........

a) $\begin{bmatrix} -4 & -4 \\ -7 & 7 \end{bmatrix}$ b) $\begin{bmatrix} -7 & 4 \\ -7 & 4 \end{bmatrix}$ c) $\begin{bmatrix} 7 & -4 \\ -4 & -7 \end{bmatrix}$ d) $\begin{bmatrix} -4 & 7 \\ -7 & -4 \end{bmatrix}$

Q.2. Attempt any two. [08]

i) Find the roots of $i^{\frac{1}{4}}$.

ii) Define Hermitian matrix and show that $\begin{bmatrix} 9 & 4-i & 3+2i \\ 4+i & 1 & 6+5i \\ 3-2i & 6-5i & 2 \end{bmatrix}$.

iii) Prove that $(\sqrt{3}+i)^n + (\sqrt{3}-i)^n = 2^{n+1}\cos\frac{n\pi}{6}$.

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Vivekanand College Kolhapur (An Empowered Autonomous Institute)
Department of Mathematics
B.Sc.-I Sem -I Internal Examination 2025-26

Course Name: Calculus (DSC03MAT12) Day & Date: Monday,13/10/2025
Time: 12:00PM to 1:30PM Marks: 10

Q.1. Select correct alternative from each of the following [02]

i) $\lim_{x\to 0}\frac{\sin x}{x} = $ ___

a) 0 b) 1 c) 2 d) 3

ii) If $z = x^2 + y^2$ then $\frac{\partial z}{\partial x} = $ _____

a) y^2 $b)$ x^2 $c)2x$ $d)$ $2x + 2y$

Q.2. Attempt any two. [08]

i) State and prove L-Hospital rule.

ii) Find n^{th} derivative of $\frac{x}{1+3x+2x^2}$.

iii) Solve. $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$.

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Name- Viraj Vijay Jadhav

॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥

– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

VIVEKANAND COLLEGE, KOLHAPUR

(An Empowered Autonomous Institute)

Assignment

Student's Sign :

Seat No./ Roll No. : 7202

Seat No./ Roll No. In words Seven Thousand two hundred and two

03596

02+08 = 10/10

प्र. क्र. Q. No.

Q.1 i) b) 1

ii) c) $2x$

02

Q.2

i) Statement-

If $f(x)$ and $g(x)$ are the two functions which can be expanded by using Taylor's series in the neighbourhood of $x=a$ and $f(a)=g(a)=0$ then,

$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$ provided that later limit exists.

proof:

We have $f(x)$ and $g(x)$ are the two real valued functions which can be expanded by the taylor's series in the neighbourhood of $x=a$.

We get,

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$$f(x) = f(a) + \frac{(x-a)}{1!}f'(x) + \frac{(x-a)^2}{2!}f''(x) + \frac{(x-a)^3}{3!}f'''(x) + \cdots \quad \text{——} (1)$$

and

$$g(x) = g(a) + \frac{(x-a)}{1!}g'(x) + \frac{(x-a)^2}{2!}g''(x) + \frac{(x-a)^3}{3!}g'''(x) + \cdots \quad \text{——} (2)$$

Also

$$f(a) = g(a) = 0 \quad \text{——} (3)$$

from (1) (2) and (3)

$$\frac{f(x)}{g(x)} = \frac{f(a) + \frac{(x-a)}{1!}f'(x) + \frac{(x-a)^2}{2!}f''(x) + \frac{(x-a)^3}{3!}f'''(x) + \cdots}{g(a) + \frac{(x-a)}{1!}g'(x) + \frac{(x-a)^2}{2!}g''(x) + \frac{(x-a)^3}{3!}g'''(x) + \cdots}$$

$$\frac{f(x)}{g(x)} = \frac{(x-a)f'(x) + \frac{(x-a)^2}{2!}f''(x) + \frac{(x-a)^3}{3!}f'''(x) + \cdots}{(x-a)g'(x) + \frac{(x-a)^2}{2!}g''(x) + \frac{(x-a)^3}{3!}g'''(x) + \cdots}$$

Taking limit as $x \to a$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{(x-a)f'(x) + \frac{(x-a)^2}{2!}f''(x) + \frac{(x-a)^3}{3!}f'''(x)}{(x-a)g'(x) + \frac{(x-a)^2}{2!}g''(x) + \frac{(x-a)^3}{3!}g'''(x)}$$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{(x-a)f'(x)}{(x-a)g'(x)}$$

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--- higher powers are vanishes as $x \to a$

Now

$x \to a \quad x \neq a \quad x - a \neq 0$

$$\therefore \boxed{\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}}$$

--- proved.

04

ii) Let,

$$y = \frac{x}{1+3x+2x^2}$$

$$y = \frac{x}{2x^2+3x+1}$$

Here, $2x^2+3x+1=0$

$2x^2+2x+1x+1=0$

$2x(x+1)+1(x+1)=0$

$(2x+1)(x+1)=0$

$$\therefore y = \frac{x}{(2x+1)(x+1)}$$

Let, By partial fraction.

$$y = \frac{x}{(2x+1)(x+1)} = \frac{A}{(2x+1)} + \frac{B}{(x+1)} \quad \text{--- (1)}$$

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Multiply and divide bothsides by $(2x+1)(x+1)$

$x = A(x+1) + B(2x+1)$ — (2)

put $x=-1$ in eqn (2) and put $x = -\frac{1}{2}$ in eqn (2)

$-1 = B(2(-1)+1)$ and $-\frac{1}{2} = A\left(-\frac{1}{2}+1\right)$

$-1 = -B$

$\boxed{B = 1}$

$-\frac{1}{2} = \frac{A}{2}$

$\boxed{A = -1}$

eqn (1) becomes.

$y = \frac{-1}{2x+1} + \frac{1}{x+1}$

$\therefore y_n = \frac{(-1)(-1)^n n! 2^n}{(2x+1)^{n+1}} + \frac{(-1)^n n! 1^n}{(x+1)^{n+1}}$

$$\boxed{y_n = (-1)^n n! \left[\frac{1}{(x+1)^{n+1}} - \frac{1}{(2x+1)^{n+1}}\right]}$$

04

Nome- Viraj Vijay Jadhav.

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VIVEKANAND COLLEGE, KOLHAPUR

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Assignment

Student's Sign :	Viraj.
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	03558

प्र. क्र. Q. No.

iii) Let,

$$L = \lim_{x \to 0} \left(\frac{\tan x}{x} \right)^{\frac{1}{x^2}}$$

~~log~~ Taking log on both sides.

$$\log L = \lim_{x \to 0} \log \left(\frac{\tan x}{x} \right)^{1/x^2}$$

$$\log L = \lim_{x \to 0} \frac{1}{x^2} \log \left(\frac{\tan x}{x} \right)$$

$$\log L = \lim_{x \to 0} \frac{\log(\tan x) - \log(x)}{x^2} \text{ --- } \left(\frac{0}{0}\right) \text{ Form}$$

By L-hospital rule

$$\log L = \lim_{x \to 0} \frac{\left[\frac{1}{\tan x} \cdot \sec^2 x\right] - \frac{1}{x}}{2x}$$

$$\log L = \lim_{x \to 0} \frac{\left[\frac{\cos x}{\sin x} \times \frac{1}{\cos^2 x}\right] - \frac{1}{x}}{2x}$$

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$$\log L = \lim_{x \to 0} \left[\frac{\frac{2}{2\sin x\cos x} - \frac{1}{x}}{2x} \right]$$

$$\log L = \lim_{x \to 0} \left[\frac{\frac{2}{\sin 2x} - \frac{1}{x}}{2x} \right]$$

$$\log L = \lim_{x \to 0} \left[\frac{2x - \sin 2x}{2x^2 \sin 2x} \right]$$

$$\log L = \lim_{x \to 0} \left[\frac{2x - \sin 2x}{2x^2 \cdot 2x} \right] \left[\frac{2x}{\sin 2x} \right]$$

$$\log L = \lim_{x \to 0} \left[\frac{2x - \sin 2x}{4x^3} \right]$$

$$\text{--- As } \lim_{x \to 0} \frac{2x}{\sin 2x} = 1$$

$$\log L = \lim_{x \to 0} \left[\frac{2x - \sin 2x}{4x^3} \right] \text{ --- } \left(\frac{0}{0}\right) \text{ Form}$$

By L-hospital Rule.

$$\log L = \lim_{x \to 0} \left[\frac{2 - 2\cos 2x}{12x^2} \right] \text{ --- } \left(\frac{0}{0}\right) \text{ Form}$$

By L-hospitalrule

$$\log L = \lim_{x \to 0} \left[\frac{4\sin 2x}{24x} \right] \cdots \left(\frac{0}{0}\right) \text{ Form}$$

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By L-hospital rule

$$\log L = \lim_{x \to 0} \left[\frac{8\cos 2x}{24} \right]$$

$$\log L = \frac{8\cos(0)}{24}$$

$$\log L = \frac{8}{24}$$

$$\log L = \frac{1}{3}$$

$$\boxed{L = e^{1/3}}$$

04

Vivekanand College, Kolhapur (An Empowered Autonomous Institute)
B.Sc. II Sem III Minor (Internal Examination) 2025-26
Course: MIN-V: 2MIN03MAT31: Computational Mathematics for Sciences- I

Day & Date: Saturday, 04/10/2025 Time: 02:00-03:00 PM Total Marks: 20

Instructions:

1.All the questions are compulsory.

2.Figure to the right indicates full marks.

Q.1. Select the correct alternative for each of the following. [4]

1) If $x = rcos(\theta), y = rsin(\theta)$ then $\frac{\partial(x,y)}{\partial(r,\theta)} =$ ________.

a) $\frac{1}{r}$ b) r^2 c) r d) $\frac{1}{r^2}$

2) If J is Jacobian of u, v with respect to x, y and J' is Jacobian of x, y with respect to u, v then ________.

a) $JJ' = 1$ b) $JJ' = 0$ c) $\frac{J}{J'} = 1$ d) $\frac{J}{J'} = 0$

3) If $z = f(x, y)$ is a homogeneous function of degree n, then ________.

a) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$ b) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = n(n-1)z$ c) $x\frac{\partial z}{\partial y} + y\frac{\partial z}{\partial x} = nz$ d) $x\frac{\partial z}{\partial y} + y\frac{\partial z}{\partial x} = n(n-1)z$

4) If $z = x^2y^2$ then $\frac{\partial z}{\partial x} =$ ______ and $\frac{\partial z}{\partial y} =$ ______.

a) $2xy^2, 2x^2y$ b) $2xy, 2xy$ c) $2x^2y^2, 2x^2y^2$ d) $2xy^2, 2x^2y^2$

Q.2. Attempt any ONE of the following. [8]

1) If p, q are functions of u, v and u, v are functions of x, y then prove that $\frac{\partial(p,q)}{\partial(u,v)} = \frac{\partial(p,q)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(u,v)}$.

2) If z is a homogeneous function of degree n in x and y and $f(u) = z$ then prove that, $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = g(u)(g'(u) - 1)$ where $g(u) = n\frac{f(u)}{f'(u)}$.

Q.3. Attempt any two of the following. [8]

1) Verify Euler's theorem for $z = ax^2 + 2hxy + by^2$

2) If $u^3 + v^3 = x + y, u^2 + v^2 = x^3 + y^3$, show that $\frac{\partial(u,v)}{\partial(x,y)} = \frac{\frac{1}{2}(y^2 - x^2)}{uv(u-v)}$.

3) If $x = e^v \sec(u)$, $y = e^v \tan(u)$, verify that $\frac{\partial(x,y)}{\partial(u,v)} = \frac{1}{\frac{\partial(u,v)}{\partial(x,y)}}$.

Vivekanand college, Kolhapur (An Empowered Autonomous Institute)
Department of Mathematics
B.Sc. Part II Sem-III
Internal Examination: October, 2025

Subject Code: Differential equation-II (DSC03MAT31) **Day and Date: Wednesday,01/10/2025**
Total Marks: 20 **Time: 2.00 PM- 03.00 PM**

Q.1. Choose the correct alternative. **[04]**

1. The linear differential equation is of the form….

A) $y = px + f(p)$ B) $y'' + Py' = Q$ C) $\frac{dy}{dx} + Py = Q$ D) $p = f(x,y)$

2. The Particular Integral of differential equation $(D^2 - 4D + 3)y = e^{2z}$ is…….

A) $-e^{2x}$ B) $-e^{-2x}$ C) e^{2x} D) $-e^{4x}$

3.The solution of differential equation $\sin(px - y) = p$ is…….

A) $y = px + \sin p$ B) $y = px - sin^{-1}p$ C) $y = px + sin^{-1}p$ D) $y = px - \sin p$

4. The equation $y = px + \frac{1}{p}$ is an example of…….

A) Linear equation B) Clairaut's equation C) Exact equation D) Homogeneous equation

Q.2. Attempt any one. **[08]**

1) Define Clairaut's differential equation and explain the method of solving and hence Solve $(y - px^2) = a^2(1 + p^2)$
2) Find the solution of second order linear differential equation $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$ where P, Q, R are functions of x only. The general solution of $f(D)y = R$ when one solution of $f(D)y = 0$ is known.

Q.3. Attempt any two. **[08]**

1) Solve $(p - xy)(p - x^2)(p - y^2) = 0$.
2) Solve $x = y + a\log p$.
3) Solve $x^2\frac{d^2y}{dx^2} - 2(x^2 + x)\frac{dy}{dx} + (x^2 + 2x + 2)y = 0$.

Vivekanand College, Kolhapur (An Empowered Autonomous Institute)

B.Sc. II Mathematics (Minor)- Internal Examination:2025

Course Name: Improper Integrals and Special functions

Day & Date: Friday & 03-10-2025 Time: 3.00 pm to 4.30 pm Total Marks: 20

Q.1. Select the correct alternative for each of the following. [8]

i) $\Gamma(n)$ =-------.

a) $\int_0^\infty x^{n+1}e^{-x}dx$ b) $\int_0^\infty x^{n-1}e^{x}dx$ c) $\int_0^1 x^{n-1}e^{-x}dx$ d) $\int_0^\infty x^{n-1}e^{-x}dx$

ii) For $n > 0, \Gamma(n+1) =$ ---------

a) $n\Gamma(n)$ b) $(n+1)\Gamma(n+1)$ c) $(n+1)\Gamma(n)$ d) $n\Gamma(n+1)$

iii) $\beta(m+1, n+1) =$ -----------

a) $\int_0^1 (x)^{m-1}(1-x)^{n-1}dx$ b) $\int_0^1 (x)^{m}(1-x)^{n}dx$ c) $\int_0^1 (x)^{m}(1-x)^{n-1}dx$ d) $\int_0^1 (x)^{m-1}(1-x)^{n}dx$

v) $\beta(m, n) =$ --------

a) $\frac{\Gamma(n)+\Gamma(m)}{\Gamma(n.m)}$ b) $\frac{\Gamma(n)\,\Gamma(m)}{\Gamma(n+m)}$ c) $\frac{\Gamma(n).\Gamma(m)}{\Gamma(n-m)}$ d) $\frac{\Gamma(n)+\Gamma(m)}{\Gamma(n+m)}$

Q.2. Attempt any one of the following. [8]

i) Evaluate $\int_0^\infty 2^{-4x^2}dx$.

ii) Prove that (a) $n\beta(m+1, n) = m\beta(m, n+1)$

(b) $\beta(m, n) = \beta(m+1, n) + \beta(m, n+1)$

iii) Prove that $\beta(m, n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx$.

Q.3. Attempt any two of the following. [8]

i) Evaluate $\int_0^\infty \frac{x^a}{a^x}dx$

ii) Evaluate $\int_0^1 x^m(\log x)^n dx$.

iii) Prove that $\int_0^1 x^{n-1}(1-x)^n dx = \frac{m!n!}{(m+n+1)!}$

iv) Prove that $\beta(m, n) = \beta(n, m)$.

Jiya Salim Mulla

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VIVEKANAND COLLEGE, KOLHAPUR

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Assignment

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	03260

$04 + 08 + 08 = \frac{20}{20}$

प्र. क्र. Q. No.

Q.1. 1. c] $\frac{dy}{dx} + Py = Q$

2. A] $-e^{2x}$

3. B] $y = px - \sin^{-1} p$

4. B] clairaut's equation

04

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Q. 2.

1] Clairaut's Differential Equation -

An Equation of the form $y = px + f(p)$, where $p = \frac{dy}{dx}$ & $f(p)$ is the function of p i[s]

called as Clairaut's Differential equation.

Method of Solving -

we have, $y = px + f(p)$ —— ①

differentiate w.r.t. x

$$\therefore \frac{dy}{dx} = p + x\frac{dp}{dx} + f'(p)\frac{dp}{dx}$$

$$\therefore p = p + [x + f'(p)]\frac{dp}{dx} \qquad [\because p = \frac{dy}{dx}]$$

$$\therefore p - p = [x + f'(p)]\frac{dp}{dx}$$

$$\therefore \frac{dp}{dx}[x + f'(p)] = 0 \quad \text{—— ②}$$

$$\therefore \frac{dp}{dx} = 0 \quad \& \quad x + f'(p) = 0$$

but, we reject the term $(x + f'(p) = 0)$, becau[se] it does not contained $\frac{dp}{dx}$.

$$\therefore \frac{dp}{dx} = 0$$

$$\therefore dp = 0\,dx$$

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Integrating both sides we get,

$$\int dP = \int 0 \, dx + c_1$$

$$\therefore \quad P = c_1 \qquad (3)$$

now, put $P = c$ in eq^n (1), we get

$$\therefore \quad y = cx + f(c)$$

which is required general solution.

now,

$$(y - px)^2 = a^2(1+p^2)$$

~~Squaring~~ Taking square root on both sides,

$$\therefore \quad y - px = \pm a\sqrt{1+p^2}$$

$$\therefore \quad y = px \pm a\sqrt{1+p^2}$$

above eq^n is in the form of Clairaut's eq^n

$$P = C$$

$$\therefore \quad y = cx \pm a\sqrt{1+c^2}$$

which is required general solution.

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Q. 3.

1] $(p-xy)(p-x^2)(p-y^2)=0$ ... given

$\therefore p-xy=0$ or $p-x^2=0$ or $p-y^2=0$

$\frac{dy}{dx}-xy=0$ or $\frac{dy}{dx}-x^2=0$ or $\frac{dy}{dx}-y^2=0$

(1) $\frac{dy}{dx}-xy=0$

$\therefore \frac{dy}{dx}=xy$

$\therefore \frac{1}{y}\,dy = x\,dx$

Integrating both sides, we get

$$\int \frac{1}{y}\,dy = \int x\,dx$$

$$\therefore \log y = \frac{x^2}{2}+c_1$$

$$\therefore \log y - \frac{x^2}{2} - c_1 = 0 \quad \text{——} \; (1)$$

(2) $\frac{dy}{dx}-x^2=0$

$\therefore \frac{dy}{dx}=x^2$

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कोल्हापूर
१९६४

VIVEKANAND COLLEGE, KOLHAPUR

(An Empowered Autonomous Institute)

Assignment

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	03259

प्र. क्र. Q. No.

now, Integrating both sides, we get

$$\int dy = \int x^2 \, dx$$

$$y = \frac{x^3}{3} + C_2$$

$$y - \frac{x^3}{3} - C_2 = 0 \quad \text{——} \quad (2)$$

(3) $\frac{dy}{dx} - y^2 = 0$

$\therefore \frac{dy}{dx} = y^2$

$\therefore \frac{1}{y^2} dy = dx$

Integrating both sides, we get

$$\int \frac{1}{y^2} dy = \int dx$$

$\therefore -\frac{1}{y} = x + C_3$ —— 3

$\therefore x + \frac{1}{y} + C_3 = 0$ —— (3)

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$$\left(\log y - \frac{x^2}{2} - c_1\right)\left(y - \frac{x^3}{3} - c_2\right)\left(x + \frac{1}{y} + c_3\right) = 0$$

which is required general solution.

Q.3.

2] $x = y + a \log p$ —— given ①

differentiate w.r.t. y

$$\frac{dx}{dy} = 1 + a\frac{1}{p}\frac{dp}{dy}$$

$$\therefore \frac{1}{p} = 1 + \frac{a}{p}\frac{dp}{dy} \quad \ldots \left[\because \frac{dy}{dx} = p\right]$$

$$\therefore \frac{1}{p} - 1 = \frac{a}{p}\frac{dp}{dy}$$

$$\therefore \frac{1-p}{p} = \frac{a}{p}\frac{dp}{dy}$$

$$\therefore \frac{p(1-p)}{p} = a\frac{dp}{dy}$$

$$\therefore 1-p = a\frac{dp}{dy}$$

$$\therefore dy = \frac{a}{1-p}\,dp$$

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प्र. क्र.
Q. No.

$$\int dy = \int \frac{a}{1-P}\, dP$$

$$\therefore \quad y = -a \log(1-P) + C \quad \text{——— (2)}$$

eliminiting P from eq^n (1) & (2) we get requi general solution.

Q. 3

3] $x^2 \frac{d^2y}{dx^2} - 2(x^2+x)\frac{dy}{dx} + (x^2+2x+2)y = 0$

divide x^2 on both sides,

$$\therefore \frac{d^2y}{dx^2} - 2\left(\frac{x^2+x}{x^2}\right)\frac{dy}{dx} + \left(\frac{x^2+2x+2}{x^2}\right)y = 0$$

$$\therefore \frac{d^2y}{dx^2} - 2\left(1+\frac{1}{x}\right)\frac{dy}{dx} + \left(1+\frac{2}{x}+\frac{2}{x^2}\right)y = 0 \quad \text{——— (1)}$$

Here,

$$P = -2\left(1+\frac{1}{x}\right), \quad Q = 1+\frac{2}{x}+\frac{2}{x^2} \quad \& \quad R = 0$$

let $y = u.v$ is general solution of given eq^n where u is not part of C.F.

Thus, The Transformed eq^n is,

$$\frac{d^2v}{dx^2} + v\left(Q - \frac{P^2}{4} - \frac{1}{2}\frac{dP}{dx}\right) = \frac{R}{u} \quad \text{——— (2)}$$

first we ~~calc~~ simplify the bracket term,

$$Q - \frac{P^2}{4} - \frac{1}{2}\frac{dP}{dx} = 1+\frac{2}{x}+\frac{2}{x^2} - 4\left(1+\frac{1}{x}\right)^2 - \frac{1}{2}\frac{dP}{dx}$$

04	Section	Q. No.											
		Marks											

प्र. क्र.
Q. No.

but $\frac{dP}{dx} = \frac{d}{dx}\left(-2\left(1+\frac{1}{x}\right)\right)$

$= \frac{d}{dx}\left(-2 - \frac{2}{x}\right) \quad = -\left(-\frac{2}{x^2}\right) = \frac{2}{x^2}$

$\therefore Q - \frac{P^2}{4} - \frac{1}{2}\frac{dP}{dx} = 1 + \frac{2}{x} + \frac{2}{x^2} - \left(1+\frac{1}{x}\right)^2 - \frac{1}{2}\left(\frac{2}{x^2}\right)$

$= 1 + \frac{2}{x} + \frac{2}{x^2} - 1 - \frac{2}{x} - \frac{1}{x^2} - \frac{1}{x^2}$

$= 0$

$\therefore \frac{d^2v}{dx^2} + 0v = 0 \quad$ [from eqn ②]

$\therefore \frac{d^2v}{dx^2} = 0$

Integrating both sides

$\therefore \frac{dv}{dx} = c_1$

again Integrating both sides,

$\therefore v = c_1 x + c_2$

now, $u = e^{-\frac{1}{2}\int P\,dx}$

$u = e^{-\frac{1}{2}\int -2(1+\frac{1}{x})dx}$

$u = e^{\int(1+\frac{1}{x})dx}$

$u = e^{x+\log x}$

$u = e^x e^{\log x}$

$u = x.e^x$

04

$\therefore$ Required general solution is $y = u.v$

Vivekanand College, Kolhapur (An Empowered Autonomous Institute)

B.Sc. III Internal Examination:2025

Course Name: Real Analysis | Course Code: DSC03MAT51 (NEP 2.0)

Day & Date: 29/09/2025 | Time: 12:30 PM to 01:30 PM | Total Marks: 20

Instructions:

1.All the questions are compulsory.

2.Figure to the right indicates full marks.

Q.1. Select the correct alternative for each of the following. **[04]**

i) If a sequence has an upper bound, then it is said to be -------.

a) monotone b) bounded above

c) Cauchy d) divergent

ii) limit of sequence $a_n = \frac{1}{n}$ is:

a) 0 b) 1 c) ∞ d) -1

iii) A series $\sum a_n$ absolutely convergent if --------

a) $a_n \to 0$ b) $\sum|a_n|$ converges

c) series diverges d) $a_n \to 0$ bounded

iv) An alternating series is one in which the terms are

a) all positive b) all negative

c) alternating in sign d) non-negative

Q.2. Attempt any one of the following. **[08]**

i) A non-increasing sequence which is bounded below then it is convergent.

ii) Prove that Every Cauchy sequence is convergent.

Q.3. Attempt any Two of the following. **[08]**

i) Find whether following series are convergent

a) $\sum_{n=1}^{\infty} \frac{n+2}{10^{10}(n+3)}$. b) $\sum_{n=1}^{\infty} \cos\left(\frac{\pi}{n}\right)$

ii) Find the limit of sequences $\sqrt{3}, \sqrt{3\sqrt{3}}, \sqrt{3\sqrt{3\sqrt{3}}}, \ldots\ldots.$

iii) If $\{S_n\}_{n=1}^{\infty}$is a sequence of real numbers then $\lim_{n\to\infty} \inf S_n \leq \lim_{n\to\infty} \sup S_n$.

iv) Find the limit superior and limit inferior of the sequence $S_n = \{(-1)^n + 1\}_{n=1}^{\infty}$.

***

॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥
– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

VIVEKANAND COLLEGE, KOLHAPUR

(An Empowered Autonomous Institute)

Assignment

Student's Sign :	Patil
Seat No./ Roll No. :	8235
Seat No./ Roll No. In words	Eight Two three Five.
	03142

Q.	1	2	3	total
	04	08	08	20

प्र. क्र.
Q. No.

Q.1.

i) b] bounded above

ii) a] 0.

iii) b] $\Sigma |a_n|$ converges.

iv) c] alternating in sign.

02	Section	Q. No.											
		Marks											

प्र. क्र.
Q. No.

Q. 2.

i] Statement:

A non-increasing sequence which is bounde below then it is convergent.

Proof:

Given that, the $\{S_n\}_{n=1}^{\infty}$ be non-increasin sequence & it is bounded below. then the set $A = \{S_1, S_2, \ldots\}$ be non-empty subset of $\mathbb{R}$ $[A \subset \mathbb{R}]$ which is bounded then set has greatest lower bound.

Let, $L = \text{glb}\{S_1, S_2, \ldots\} = \text{lub of } A$.

then $\exists$ $\epsilon > 0$ then $L+\epsilon$ is not an lower bound for A. we prove, $S_n \to L$, $n \to \infty$

hence, for some $N \in \mathbb{Z}$.

$$L + \epsilon \geq S_n \quad \forall n. \quad \text{—— I}$$

but, $\{S_n\}_{n=1}^{\infty}$ be non-increasing sequenc which implies that,

$$L - \epsilon \leq S_n \quad \forall n \quad \text{—— II}$$

$\therefore$ from I & II.

$$L - \epsilon \leq S_n \leq L + \epsilon \quad \forall n$$

05

$\therefore |S_n - L| < \epsilon \quad \forall n$

$\therefore \lim_{n \to \infty} S_n = L$

$\therefore \{S_n\}_{n=1}^{\infty}$ converges to L.

$\therefore$ It is convergent.

	Section	Q. No.													03
		Marks													

प्र. क्र.
Q. No.

Q. 3.

i) a] $a_n = \sum_{n=1}^{\infty} \frac{n+2}{10^{10}(n+3)}$

$\therefore \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n+2}{10^{10}(n+3)}$ (taking limit

$= \frac{1}{10^{10}} \lim_{n\to\infty} \frac{n+2}{n+3}$

$= \frac{1}{10^{10}} \cdot \lim_{n\to\infty} \frac{\cancel{n}}{\cancel{n}} \left(\frac{1+2/n}{1+3/n} \right)$

$= \frac{1}{10^{10}}$ $\neq 0$

$\therefore$ It is not convergent.

b]. $a_n = \sum_{n=1}^{\infty} \cos\left(\frac{\pi}{n}\right)$

take limit on both side,

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \cos\left(\frac{\pi}{n}\right)$

$= \cos . 0$

$= 1.$ $\neq 0$

04

$\therefore$ It is not convergent.

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		Marks											

प्र. क्र.
Q. No.

ii) Given, $\sqrt{3}, \sqrt{3\sqrt{3}}, \sqrt{3\sqrt{3\sqrt{3}}}, \ldots$

Now, $S_1 = \sqrt{3} = 3^{1/2}$

$S_2 = \sqrt{3\sqrt{3}} = \sqrt{3 \cdot 3^{1/2}} = 3^{1/2} \cdot 3^{1/4}$

$S_3 = \sqrt{3\sqrt{3\sqrt{3}}} = \sqrt{3 \cdot 3^{1/2} \cdot 3^{1/4}} = 3^{1/2} \cdot 3^{1/4} \cdot 3^{1/6}$

$\vdots$

$Sn = 3^{\frac{1}{2}}; 3^{\frac{1}{4}}, 3^{\frac{1}{6}}, \ldots 3^{\frac{1}{n}}$

$\therefore Sn = 3^{\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\ldots\frac{1}{2^n}}$

Now we consider expanssion

$\Rightarrow \left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\ldots+\frac{1}{2^n}\right)$

$\Rightarrow \frac{1}{2}\left(1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots+\frac{1}{2^{n-1}}\right)$

$\Rightarrow$

$\cancel{\frac{1}{2}} \frac{\left(1-\left(\frac{1}{2}\right)^n\right)}{\left(1-\cancel{\frac{1}{2}}\right)}$

$\Rightarrow \left[1-\left(\frac{1}{2}\right)^n\right]$

Now,

$Sn = 3^{\left[1-\left(\frac{1}{2}\right)^n\right]}$

$\lim_{n\to\infty} Sn = \lim_{n\to\infty} 3^{\left[1-\left(\frac{1}{2}\right)^n\right]}$

$= 3.$

04

$\boxed{\therefore \lim_{n\to\infty} Sn = 3.}$

॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥
– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

VIVEKANAND COLLEGE, KOLHAPUR
(An Empowered Autonomous Institute)
Assignment

Student's Sign : Patil
Seat No./ Roll No. : 8235
Seat No./ Roll No. In words
03045

प्र. क्र.
Q. No.

iii) Statement
If $\{s_n\}_{n=1}^{\infty}$ be seq^n of real no. then,
$\lim_{n\to\infty} \sup s_n \geq \lim_{n\to\infty} \inf s_n$

Proof: Given,
If $\{s_n\}_{n=1}^{\infty}$ be seq^n of real no.
we will prove that in two steps.

i) If $\{s_n\}_{n=1}^{\infty}$ be bound above as well as b below sequence

$\therefore$ lub & glb of $\{s_1, s_2, \ldots\}$ are exist.

$\therefore$ glb $\{s_1, s_2, \ldots\} \leq$ lub $\{s_1, s_2, \ldots\}$

$\therefore \lim_{n\to\infty} \inf s_n \leq \lim_{n\to\infty}$ ~~g~~ $\sup s_n$. —— I.

ii) If $\{s_n\}_{n=1}^{\infty}$ is sequence is not bound then,

02	Section	Q. No.											
		Marks											

प्र. क्र.
Q. No.

$m_n = glb\{s_1, s_2 \ldots\} = -\infty$, $M_n = lub\{s_1, s_2, \ldots\}$

$\therefore m_n \le M_n$

$\therefore \lim_{n\to\infty} \inf s_n \le \lim_{n\to\infty} \sup s_n$ —— II

$\therefore$ From I & II we get

$$\boxed{\lim_{n\to\infty} \inf s_n \le \lim_{n\to\infty} \sup s_n.}$$

03

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		Marks												

प्र. क्र.
Q. No.

iv) Given, $S_n = \{(-1)^n + 1\}_{n=1}^{\infty}$.

$\therefore S_n = \{0, 2, 0, 2, 0, 2, \ldots\}$

B: $M_1 = \text{lub}\{s_1, s_2, s_3, \ldots\}$
$= \text{lub}\{0, 2, 0, 2, 0, \ldots\} = 2$

$M_2 = \text{lub}\{s_2, s_3, s_4, \ldots\}$
$= \text{lub}\{2, 0, 2, 0, \ldots\} = 2$

$M_3 = \text{lub}\{s_3, s_4, \ldots\}$
$\text{lub}\{0, 2, 0, 2, \ldots\} = 2$

$\vdots$

$M_n = \text{lub}\{s_n, s_{n+1}, \ldots\}$
$= 2$.

$\therefore \lim_{n\to\infty} M_n = 2$. $\therefore$ $\boxed{\lim_{n\to\infty} \overline{\sup}\, s_n = 2}$

2]lly, $m_1 = \text{glb}\{s_1, s_2, s_3, \ldots\}$
$= \text{glb}\{0, 2, 0, 2, \ldots\} = 0$

$m_2 = \text{glb}\{2, 0, 2, 0, \ldots\} = 0$

$m_3 = \text{glb}\{0, 2, 0, 2, \ldots\} = 0$

$\vdots$

$m_n = \text{glb}\{s_n, s_{n+1}, \ldots\}$
$= 0$.

$\therefore \lim_{n\to\infty} m_n = 0$. $\boxed{\therefore \lim_{n\to\infty} \underline{\inf}\, s_n = 0}$

04

04	Section	Q. No.											
		Marks											

प्र. क्र.
Q. No.

Q. 2. Statement

ii) Prove Every Cauchy sequence is convergen

Proof :

Given, $\{S_n\}_{n=1}^{\infty}$ be cauchy sequence of
no. & we know that
every cauchy sequence is bounded.
then $\exists$,
$\lim_{n\to\infty} \inf S_n$ & $\lim_{n\to\infty} \sup S_n$ are finite real
we will prove that, $\lim_{n\to\infty} \sup S_n = \lim_{n\to\infty} \inf$
Now, we know,

$$\lim_{n\to\infty} \inf S_n \leq \lim_{n\to\infty} \sup S_n \quad \text{--- I}$$

by def^n of cauchy sequence $\exists\ \epsilon > 0$

$$|S_m - S_n| < \epsilon/2 \quad \forall\ m, n \geq N.$$
$$|S_N - S_n| < \epsilon/2 \quad \forall\ n \geq N.$$

$$\Rightarrow -\frac{\epsilon}{2} \leq |S_n - S_N| \leq \frac{\epsilon}{2}$$

$$\rightarrow S_N - \frac{\epsilon}{2} \leq S_n \leq S_N + \frac{\epsilon}{2}$$

$$\Rightarrow S_N - \frac{\epsilon}{2} \leq \text{glb}\{S_1, S_2, \ldots\}$$
$$\leq \text{lub}\{S_1, S_2, \ldots\} \leq S_N +$$

$$\Rightarrow \text{lub}\{S_1, S_2, \ldots\} - \text{glb}\{S_1, S_2, \ldots\} \leq \left(S_N + \frac{\epsilon}{2}\right)$$

II ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार II
– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

कोल्हापूर
१९६४

VIVEKANAND COLLEGE, KOLHAPUR

(An Empowered Autonomous Institute)

Assignment

Student's Sign :	Patil
Seat No./ Roll No. :	8235
Seat No./ Roll No. In words	
	03031

प्र. क्र.
Q. No.

$\Rightarrow$ lub $\{s_1, s_2, \ldots\} \leq$ glb $\{s_1, s_2, \ldots\} + \epsilon$

$\Rightarrow \lim\limits_{n\to\infty} \sup s_n \leq \lim\limits_{n\to\infty} \inf s_n$ —— II

$\therefore$ From I & II we get.

$$\boxed{\lim_{n\to\infty} \sup s_n = \lim_{n\to\infty} \inf s_n.}$$

$\therefore$ It is convergent.

08

VIVEKANAND COLLEGE, KOLHAPUR (AN EMPOWERED AUTONOMOUS INSTITUTE)

Internal Examination B.SC. Part- III (Mathematics) (Sem-V)

Course code : DSE03MAT52 Date:-06/10/2025

Subject Name:-Numerical Methods Total Marks:- [20 marks]

Q.1) Choose the correct alternative **[04 marks]**

i) $f(x_0, x_1, x_2) =$ -----

A) $\frac{f(x_1,x_2)-f(x_0,x_1)}{x_0-x_1}$ B) $\frac{f(x_1,x_2)-f(x_0,x_1)}{x_0-x_2}$ C) $\frac{f(x_1,x_2)-f(x_0,x_1)}{x_2-x_0}$ D) $\frac{f(x_1,x_2)-f(x_0,x_1)}{x_1-x_0}$

ii) The 2^{nd} degree polynomial passing through the points $(0,2), (1,7), (2,14), (3,23)$ is -----

A) $x^2 - 4x + 2$ B) $x^2 - 4x - 2$ C) $x^2 + 4x + 2$ D) $x^2 + 2$

iii) The value of $\Delta^2 y_1$ is given by....

A) $y_2 + 2y_1 + y_0$ B) $y_3 + 2y_2 + y_1$ C) $y_2 - 2y_1 + y_0$ D) $y_3 - 2y_2 + y_1$

iv) Newton's Forward and Backward interpolation is useful when data is....

A) Equally spaced B) Unequally spaced C) Both A and B D) None of These

Q.2) Attempt any One of the following **[08 marks]**

i) Explain Newton's Backward interpolation method and find second degree polynomial passing through the points (0, 3), (1, 6), (2, 11), (3,18).

ii) Using Newton's divided difference table find f(x) as polynomial in x and find $f'(2)$.

X	-1	0	3	6	7
Y	3	-6	39	822	1611

Q.3) Attempt any Two of the following **[08 marks]**

i) Using method of separation of symbols show that $e^x \left(u_0 + x\Delta u_0 + \frac{x^2}{2!}u_0 + \cdots .\right) = u_0 + u_1 x + \frac{x^2}{2!}u_2 + \cdots.$

ii) Find the interpolating polynomial in Lagrange's form for the given data And hence interpolate f(0).

X	-2	-1	1	3
f(x)	-15	-4	0	20

iii) Find f(1.5) using Newton's forward interpolation formula for the data

X	0	1	2	3	4
f(x)	0	1	8	27	64

iv) Derive Lagrange interpolation formula for n=3.

VIVEKANAND COLLEGE, KOLHAPUR (AN EMPOWERED AUTONOMOUS INSTITUTE)

Internal Examination B.SC. Part- III (Mathematics) (Sem-V)

Course code : MIN03MAT51 Date:-06/10/2025

Subject Name:-Computational Mathematics Total Marks:- [20 marks]

Q.1) Choose the correct alternative **[04 marks]**

i) $f(x_0, x_1, x_2) =$ -----

A) $\frac{f(x_1,x_2)-f(x_0,x_1)}{x_0-x_1}$ B) $\frac{f(x_1,x_2)-f(x_0,x_1)}{x_0-x_2}$ C) $\frac{f(x_1,x_2)-f(x_0,x_1)}{x_2-x_0}$ D) $\frac{f(x_1,x_2)-f(x_0,x_1)}{x_1-x_0}$

ii) The 2^{nd} degree polynomial passing through the points $(0,2), (1,7), (2,14), (3,23)$ is -----

A) $x^2 - 4x + 2$ B) $x^2 - 4x - 2$ C) $x^2 + 4x + 2$ D) $x^2 + 2$

iii) The value of $\Delta^2 y_1$ is given by....

A) $y_2 + 2y_1 + y_0$ B) $y_3 + 2y_2 + y_1$ C) $y_2 - 2y_1 + y_0$ D) $y_3 - 2y_2 + y_1$

iv) Newton's Forward and Backward interpolation is useful when data is....

A) Equally spaced B) Unequally spaced C) Both A and B D) None of These

Q.2) Attempt any One of the following **[08 marks]**

i) Explain Newton's Backward interpolation method and find second degree polynomial passing through the points (0, 3), (1, 6), (2, 11), (3,18).

ii) Using Newton's divided difference table find f(x) as polynomial in x and find $f'(2)$.

X	-1	0	3	6	7
Y	3	-6	39	822	1611

Q.3) Attempt any Two of the following **[08 marks]**

i) Using method of separation of symbols show that

$$e^x \left(u_0 + x\Delta u_0 + \frac{x^2}{2!}\Delta^2 u_0 + \cdots\right) = u_0 + u_1 x + \frac{x^2}{2!}u_2 + \cdots.$$

ii) Find the interpolating polynomial in Lagrange's form for the given data And hence interpolate f(0).

X	-2	-1	1	3
f(x)	-15	-4	0	20

iii) Find f(1.5) using Newton's forward interpolation formula for the data

X	0	1	2	3	4
f(x)	0	1	8	27	64

iv) Derive Lagrange interpolation formula for n=3.

VIVEKANAND COLLEGE, KOLHAPUR (AN EMPOWERED AUTONOMOUS INSTITUTE)

Internal Examination B.SC. Part- II (Mathematics) (Sem-III)

Course code : 2DSC03MAT32 Date:-06/10/2025

Subject Name:-Numerical Methods Total Marks:- [20 marks]

Q.1) Choose the correct alternative **[04 marks]**

i) A root of polynomial with multiplicity one is called……..

A) algebraic root B) Transcendental root C) simple root D) quadratic root

ii) In Simpson's (3/8)th rule we divide [a, b] into…. equal subintervals.

A) n B) 2n C) 3n D) 4n

iii) Equation $f(x) = x^5 + x^4 - 3x^2 + 2x - 7 = 0$ has at most …. Positive roots.

A) 3 B) 2 C) 1 D) 0

iv) The value of $\Delta E^{-1} = \cdots$..

A) $\nabla\Delta$ B) ∇ C) E D) Δ

Q. 2] Attempt any one of the following. **[8]**

i) Evaluate $\int_0^3 \frac{x^2}{1+x^3}\, dx$ using Simpson's (3/8)th rule with 6 strips. Hence find $\ln(28)^{1/3}$.

ii) Perform four iteration of Regular falsi method to find positive real root of $xe^x = 2$.

Q. 3] Attempt any two of the following. **[8]**

i) Using Lagrange's interpolation formula interpolate f(0) from given data

X	-2	-1	1	3
F(x)	-15	-4	0	20

ii) Using Bisection method find the real root of equation $x - cosx = 0$ perform four iterations.

iii) Find second degree polynomial by Newton's forward interpolation passing through the points (0, 3), (1, 6), (2, 11), (3,18).

VIVEKANAND COLLEGE, KOLHAPUR (AN EMPOWERED AUTONOMOUS INSTITUTE)

Internal Examination B.SC. Part- II (Mathematics) (Sem-III)

Course code : 2DSC03MAT32 Date:-06/10/2025

Subject Name:-Numerical Methods Total Marks:- [20 marks]

Q.1) Choose the correct alternative **[04 marks]**

i) A root of polynomial with multiplicity one is called……..

A) algebraic root B) Transcendental root C) simple root D) quadratic root

ii) In Simpson's (3/8)th rule we divide [a, b] into…. equal subintervals.

A) n B) 2n C) 3n D) 4n

iii) Equation $f(x) = x^5 + x^4 - 3x^2 + 2x - 7 = 0$ has at most …. Positive roots.

A) 3 B) 2 C) 1 D) 0

iv) The value of $\Delta E^{-1} = \cdots$..

A) $\nabla\Delta$ B) ∇ C) E D) Δ

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i) Using Lagrange's interpolation formula interpolate f(0) from given data

X	-2	-1	1	3
F(x)	-15	-4	0	20

ii) Using Bisection method find the real root of equation $x - cosx = 0$ perform four iterations.

iii) Find second degree polynomial by Newton's forward interpolation passing through the points (0, 3), (1, 6), (2, 11), (3,18).

Vivekanand College, Kolhapur (An Empowered Autonomous Institute)

B.Sc. III Internal Examination:2025

Course Name: Real Analysis | Course Code: DSC03MAT51 (NEP 2.0)

Day & Date: 29/09/2025 | Time: 12:30 PM to 01:30 PM | Total Marks: 20

Instructions:

1.All the questions are compulsory.

2.Figure to the right indicates full marks.

Q.1. Select the correct alternative for each of the following. **[04]**

i) If a sequence has an upper bound, then it is said to be -------.

a) monotone b) bounded above

c) Cauchy d) divergent

ii) limit of sequence $a_n = \frac{1}{n}$ is:

a) 0 b) 1 c) ∞ d) -1

iii) A series $\sum a_n$ absolutely convergent if --------

a) $a_n \to 0$ b) $\sum |a_n|$ converges

c) series diverges d) $a_n \to 0$ bounded

iv) An alternating series is one in which the terms are

a) all positive b) all negative

c) alternating in sign d) non-negative

Q.2. Attempt any one of the following. **[08]**

i) A non-increasing sequence which is bounded below then it is convergent.

ii) Prove that Every Cauchy sequence is convergent.

Q.3. Attempt any Two of the following. **[08]**

i) Find whether following series are convergent

a) $\sum_{n=1}^{\infty} \frac{n+2}{10^{10}(n+3)}$. b) $\sum_{n=1}^{\infty} \cos\left(\frac{\pi}{n}\right)$

ii) Find the limit of sequences $\sqrt{3}, \sqrt{3\sqrt{3}}, \sqrt{3\sqrt{3\sqrt{3}}}, \ldots\ldots$

iii) If $\{S_n\}_{n=1}^{\infty}$ is a sequence of real numbers then $\lim_{n\to\infty} \inf S_n \leq \lim_{n\to\infty} \sup S_n$.

iv) Find the limit superior and limit inferior of the sequence $S_n = \{(-1)^n + 1\}_{n=1}^{\infty}$.

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॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥

– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

VIVEKANAND COLLEGE, KOLHAPUR

(An Empowered Autonomous Institute)

Assignment

Student's Sign :	Patil
Seat No./ Roll No. :	8235
Seat No./ Roll No. In words	Eight Two three Five.
	03142

Q.	1	2	3	total
	04	08	08	20

प्र. क्र.
Q. No.

Q.1.

i) b] bounded above

ii) a] 0.

iii) b] $\Sigma |a_n|$ converges.

iv) c] alternating in sign.

02	Section	Q. No.											
		Marks											

प्र. क्र.
Q. No.

Q. 2.

i] Statement:

A non-increasing sequence which is bounde below then it is convergent.

Proof:

Given that, the $\{S_n\}_{n=1}^{\infty}$ be non-increasin sequence & it is bounded below.
then the set $A = \{S_1, S_2, \ldots\}$ be non-empty subset of R [$A \subset R$] which is bounded then set has greatest lower bound.
Let, $L = glb \{S_1, S_2, \ldots\}$ = lub of A.
then $\exists\ \epsilon > 0$ then $L+\epsilon$ is not an lower bound for A. we prove, $S_n \to L$, $n \to \infty$
hence, for some $N \in \mathbb{Z}$.

$$L + \epsilon \geq S_n \quad \forall n. \quad \text{--- I}$$

but, $\{S_n\}_{n=1}^{\infty}$ be non-increasing sequence which implies that,

$$L - \epsilon \leq S_n \quad \forall n \quad \text{--- II}$$

$\therefore$ from I & II.

$$L - \epsilon \leq S_n \leq L + \epsilon \quad \forall n$$

05

$\therefore |S_n - L| < \epsilon \quad \forall n$

$\therefore \lim_{n\to\infty} S_n = L$

$\therefore \{S_n\}_{n=1}^{\infty}$ converges to L.

$\therefore$ It is convergent.

	Section	Q. No.												03
		Marks												

प्र. क्र.
Q. No.

Q. 3.

i) a] $a_n = \sum_{n=1}^{\infty} \frac{n+2}{10^{10}(n+3)}$

$\therefore \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n+2}{10^{10}(n+3)}$ (taking limit

$= \frac{1}{10^{10}} \lim_{n\to\infty} \frac{n+2}{n+3}$

$= \frac{1}{10^{10}} \cdot \lim_{n\to\infty} \frac{\cancel{n}}{\cancel{n}}\left(\frac{1+2/n}{1+3/n}\right)$

$= \frac{1}{10^{10}}$ $\neq 0$

$\therefore$ It is not convergent.

b]. $a_n = \sum_{n=1}^{\infty} \cos\left(\frac{\pi}{n}\right)$

take limit on both side,

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \cos\left(\frac{\pi}{n}\right)$

$= \cos . 0$

$= 1.$ $\neq 0$

04

$\therefore$ It is not convergent.

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		Marks											

प्र. क.
Q. No.

ii) Given, $\sqrt{3}, \sqrt{3\sqrt{3}}, \sqrt{3\sqrt{3\sqrt{3}}}, \ldots$

Now, $S_1 = \sqrt{3} = 3^{1/2}$

$S_2 = \sqrt{3\sqrt{3}} = \sqrt{3 \cdot 3^{1/2}} = 3^{1/2} \cdot 3^{1/4}$

$S_3 = \sqrt{3\sqrt{3\sqrt{3}}} = \sqrt{3 \cdot 3^{1/2} \cdot 3^{1/4}} = 3^{1/2} \cdot 3^{1/4} \cdot 3^{1/6}$

$\vdots$

$S_n = 3^{\frac{1}{2}}; 3^{\frac{1}{4}}, 3^{\frac{1}{6}}, \ldots 3^{\frac{1}{n}}$

$\therefore S_n = 3^{\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\ldots\frac{1}{2^n}}$

Now we consider expanssion

$\Rightarrow \left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\ldots+\frac{1}{2^n}\right)$

$\Rightarrow \frac{1}{2}\left(1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots+\frac{1}{2^{n-1}}\right)$

$\Rightarrow$

$\cancel{\frac{1}{2}} \frac{\left(1-\left(\frac{1}{2}\right)^n\right)}{\left(1-\frac{1}{\cancel{2}}\right)}$

$\Rightarrow \left[1-\left(\frac{1}{2}\right)^n\right]$

Now,

$S_n = 3^{\left[1-\left(\frac{1}{2}\right)^n\right]}$

$\lim_{n\to\infty} S_n = \lim_{n\to\infty} 3^{\left[1-\left(\frac{1}{2}\right)^n\right]}$

$= 3.$

04

$\boxed{\therefore \lim_{n\to\infty} S_n = 3.}$

॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥
– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

VIVEKANAND COLLEGE, KOLHAPUR

(An Empowered Autonomous Institute)

Assignment

Student's Sign :	Patil
Seat No./ Roll No. :	8235
Seat No./ Roll No. In words	
	03045

प्र. क्र. Q. No.

iii) Statement

If $\{s_n\}_{n=1}^{\infty}$ be seqn of real no. then,
$\lim_{n\to\infty} \sup s_n \geq \lim_{n\to\infty} \inf s_n$

Proof: Given,
If $\{s_n\}_{n=1}^{\infty}$ be seqn of real no.
we will prove that in two steps.

i) If $\{s_n\}_{n=1}^{\infty}$ be bound above as well as b below sequence

$\therefore$ lub & glb of $\{s_1, s_2, \ldots\}$ are exist.

$\therefore$ glb $\{s_1, s_2, \ldots\} \leq$ lub $\{s_1, s_2, \ldots\}$

$\therefore \lim_{n\to\infty} \inf s_n \leq \lim_{n\to\infty} \sup s_n$. ——— I.

ii) If $\{s_n\}_{n=1}^{\infty}$ is sequence is not bound then,

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		Marks											

प्र. क्र.
Q. No.

$m_n = \text{glb}\{s_1, s_2 \ldots\} = -\infty$, $M_n = \text{lub}\{s_1, s_2, \ldots$

$\therefore \quad m_n \le M_n$

$\therefore \lim\limits_{n\to\infty} \inf S_n \le \lim\limits_{n\to\infty} \sup S_n$ —— II

$\therefore$ From I & II we get

$$\boxed{\lim_{n\to\infty} \inf S_n \le \lim_{n\to\infty} \sup S_n .}$$

03

	Section	Q. No.												03
		Marks												

प्र. क्र. Q. No.

iv) Given, $s_n = \{(-1)^n + 1\}_{n=1}^{\infty}$.

$\therefore S_n = \{0, 2, 0, 2, 0, 2, \ldots\}$

$\therefore M_1 = lub\{s_1, s_2, s_3, \ldots\}$
$= lub\{0, 2, 0, 2, 0, \ldots\} = 2$
$M_2 = lub\{s_2, s_3, s_4 \ldots\}$
$= lub\{2, 0, 2, 0, \ldots\} = 2$
$M_3 = lub\{s_3, s_4, \ldots\}$
$lub\{0, 2, 0, 2, \ldots\} = 2$
$\vdots$
$M_n = lub\{s_n, s_{n+1}, \ldots\}$
$= 2$.

$\therefore \lim_{n\to\infty} M_n = 2$. $\therefore$ $\boxed{\lim_{n\to\infty} \sup s_n = 2}$

2] lly, $m_1 = glb\{s_1, s_2, s_3, \ldots\}$
$= glb\{0, 2, 0, 2, \ldots\} = 0$
$m_2 = glb\{2, 0, 2, 0 \ldots\} = 0$
$m_3 = glb\{0, 2, 0, 2, \ldots\} = 0$
$\vdots$
$m_n = glb\{s_n, s_{n+1}, \ldots\}$
$= 0$.

$\therefore \lim_{n\to\infty} m_n = 0$. $\boxed{\therefore \lim_{n\to\infty} \inf s_n = 0}$

04

04	Section	Q. No.											
		Marks											

प्र. क्र.
Q. No.

Q. 2. Statement

ii) Prove Every Cauchy sequence is convergen

Proof:

Given, $\{S_n\}_{n=1}^{\infty}$ be cauchy sequence of r
no. & we know that
every cauchy sequence is bounded.
then $\exists$,
$\lim_{n\to\infty} \inf S_n$ & $\lim_{n\to\infty} \sup S_n$ are finite real n
we will prove that, $\lim_{n\to\infty} \sup S_n = \lim_{n\to\infty} \inf$
Now, we know,

$$\lim_{n\to\infty} \inf S_n \leq \lim_{n\to\infty} \sup S_n \quad \text{—— I}$$

by defn of cauchy sequence $\exists\ \epsilon > 0$

$$|S_m - S_n| < \epsilon/2 \quad \forall\ m, n \geq N.$$
$$|S_N - S_n| < \epsilon/2 \quad \forall\ n \geq N.$$

$$\Rightarrow -\frac{\epsilon}{2} \leq |S_n - S_N| \leq \frac{\epsilon}{2}$$

$$\Rightarrow S_N - \frac{\epsilon}{2} \leq S_n \leq S_N + \frac{\epsilon}{2}$$

$$\Rightarrow S_N - \frac{\epsilon}{2} \leq g.l.b \{S_1, S_2, \ldots\}$$
$$\leq l.u.b \{S_1, S_2, \ldots\} \leq S_N +$$

$$\Rightarrow \cancel{S_n}\ l.u.b \{S_1, S_2, \ldots\} - g.l.b \{S_1, S_2, \ldots\} \leq \left(S_N + \frac{\epsilon}{2}\right)$$

॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥

– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

VIVEKANAND COLLEGE, KOLHAPUR

(An Empowered Autonomous Institute)

Assignment

Student's Sign :	[signature]
Seat No./ Roll No. :	8235
Seat No./ Roll No. In words	
	03031

प्र. क्र.
Q. No.

$\Rightarrow$ lub $\{s_1, s_2, ---\} \leq$ glb $\{s_1, s_2, ----\} + \epsilon$

$\Rightarrow \lim_{n\to\infty} \sup s_n \leq \lim_{n\to\infty} \inf s_n$ —— II

$\therefore$ From I & II we get.

$$\boxed{\lim_{n\to\infty} \sup s_n = \lim_{n\to\infty} \inf s_n.}$$

$\therefore$ It is convergent.

08

VIVEKANAND COLLEGE, KOLHAPUR (AN EMPOWERED AUTONOMOUS INSTITUTE)

Internal Examination B.SC. Part- III (Mathematics) (Sem-V)

Course code : DSE03MAT52 Date:-06/10/2025

Subject Name:-Numerical Methods Total Marks:- [20 marks]

Q.1) Choose the correct alternative **[04 marks]**

i) $f(x_0, x_1, x_2) =$ -----

A) $\frac{f(x_1,x_2)-f(x_0,x_1)}{x_0-x_1}$ B) $\frac{f(x_1,x_2)-f(x_0,x_1)}{x_0-x_2}$ C) $\frac{f(x_1,x_2)-f(x_0,x_1)}{x_2-x_0}$ D) $\frac{f(x_1,x_2)-f(x_0,x_1)}{x_1-x_0}$

ii) The 2^{nd} degree polynomial passing through the points $(0,2), (1,7), (2,14), (3,23)$ is -----

A) $x^2 - 4x + 2$ B) $x^2 - 4x - 2$ C) $x^2 + 4x + 2$ D) $x^2 + 2$

iii) The value of $\Delta^2 y_1$ is given by....

A) $y_2 + 2y_1 + y_0$ B) $y_3 + 2y_2 + y_1$ C) $y_2 - 2y_1 + y_0$ D) $y_3 - 2y_2 + y_1$

iv) Newton's Forward and Backward interpolation is useful when data is....

A) Equally spaced B) Unequally spaced C) Both A and B D) None of These

Q.2) Attempt any One of the following **[08 marks]**

i) Explain Newton's Backward interpolation method and find second degree polynomial passing through the points (0, 3), (1, 6), (2, 11), (3,18).

ii) Using Newton's divided difference table find f(x) as polynomial in x and find $f'(2)$.

X	-1	0	3	6	7
Y	3	-6	39	822	1611

Q.3) Attempt any Two of the following **[08 marks]**

i) Using method of separation of symbols show that $e^x \left(u_0 + x\Delta u_0 + \frac{x^2}{2!}u_0 + \cdots.\right) = u_0 + u_1 x + \frac{x^2}{2!}u_2 + \cdots.$

ii) Find the interpolating polynomial in Lagrange's form for the given data And hence interpolate f(0).

X	-2	-1	1	3
f(x)	-15	-4	0	20

iii) Find f(1.5) using Newton's forward interpolation formula for the data

X	0	1	2	3	4
f(x)	0	1	8	27	64

iv) Derive Lagrange interpolation formula for n=3.

VIVEKANAND COLLEGE, KOLHAPUR (AN EMPOWERED AUTONOMOUS INSTITUTE)

Internal Examination B.SC. Part- III (Mathematics) (Sem-V)

Course code : MIN03MAT51 Date:-06/10/2025

Subject Name:-Computational Mathematics Total Marks:- [20 marks]

Q.1) Choose the correct alternative **[04 marks]**

i) $f(x_0, x_1, x_2) =$ -----

A) $\frac{f(x_1,x_2)-f(x_0,x_1)}{x_0-x_1}$ B) $\frac{f(x_1,x_2)-f(x_0,x_1)}{x_0-x_2}$ C) $\frac{f(x_1,x_2)-f(x_0,x_1)}{x_2-x_0}$ D) $\frac{f(x_1,x_2)-f(x_0,x_1)}{x_1-x_0}$

ii) The 2^{nd} degree polynomial passing through the points $(0,2), (1,7), (2,14), (3,23)$ is -----

A) $x^2 - 4x + 2$ B) $x^2 - 4x - 2$ C) $x^2 + 4x + 2$ D) $x^2 + 2$

iii) The value of $\Delta^2 y_1$ is given by....

A) $y_2 + 2y_1 + y_0$ B) $y_3 + 2y_2 + y_1$ C) $y_2 - 2y_1 + y_0$ D) $y_3 - 2y_2 + y_1$

iv) Newton's Forward and Backward interpolation is useful when data is....

A) Equally spaced B) Unequally spaced C) Both A and B D) None of These

Q.2) Attempt any One of the following **[08 marks]**

i) Explain Newton's Backward interpolation method and find second degree polynomial passing through the points (0, 3), (1, 6), (2, 11), (3,18).

ii) Using Newton's divided difference table find f(x) as polynomial in x and find $f'(2)$.

X	-1	0	3	6	7
Y	3	-6	39	822	1611

Q.3) Attempt any Two of the following **[08 marks]**

i) Using method of separation of symbols show that

$$e^x \left(u_0 + x\Delta u_0 + \frac{x^2}{2!}\Delta^2 u_0 + \cdots \right) = u_0 + u_1 x + \frac{x^2}{2!} u_2 + \cdots.$$

ii) Find the interpolating polynomial in Lagrange's form for the given data And hence interpolate f(0).

X	-2	-1	1	3
f(x)	-15	-4	0	20

iii) Find f(1.5) using Newton's forward interpolation formula for the data

X	0	1	2	3	4
f(x)	0	1	8	27	64

iv) Derive Lagrange interpolation formula for n=3.

VIVEKANAND COLLEGE, KOLHAPUR (AN EMPOWERED AUTONOMOUS INSTITUTE)

Internal Examination B.SC. Part- II (Mathematics) (Sem-III)

Course code : 2DSC03MAT32 Date:-06/10/2025

Subject Name:-Numerical Methods Total Marks:- [20 marks]

Q.1) Choose the correct alternative **[04 marks]**

i) A root of polynomial with multiplicity one is called……..

A) algebraic root B) Transcendental root C) simple root D) quadratic root

ii) In Simpson's (3/8)th rule we divide [a, b] into…. equal subintervals.

A) n B) 2n C) 3n D) 4n

iii) Equation $f(x) = x^5 + x^4 - 3x^2 + 2x - 7 = 0$ has at most …. Positive roots.

A) 3 B) 2 C) 1 D) 0

iv) The value of $\Delta E^{-1} = \cdots$..

A) $\nabla\Delta$ B) ∇ C) E D) Δ

Q. 2] Attempt any one of the following. **[8]**

i) Evaluate $\int_0^3 \frac{x^2}{1+x^3}\, dx$ using Simpson's (3/8)th rule with 6 strips. Hence find $\ln(28)^{1/3}$.

ii) Perform four iteration of Regular falsi method to find positive real root of $xe^x = 2$.

Q. 3] Attempt any two of the following. **[8]**

i) Using Lagrange's interpolation formula interpolate f(0) from given data

X	-2	-1	1	3
F(x)	-15	-4	0	20

ii) Using Bisection method find the real root of equation $x - cosx = 0$ perform four iterations.

iii) Find second degree polynomial by Newton's forward interpolation passing through the points (0, 3), (1, 6), (2, 11), (3,18).

VIVEKANAND COLLEGE, KOLHAPUR (AN EMPOWERED AUTONOMOUS INSTITUTE)

Internal Examination B.SC. Part- II (Mathematics) (Sem-III)

Course code : 2DSC03MAT32 Date:-06/10/2025

Subject Name:-Numerical Methods Total Marks:- [20 marks]

Q.1) Choose the correct alternative **[04 marks]**

i) A root of polynomial with multiplicity one is called……..

A) algebraic root B) Transcendental root C) simple root D) quadratic root

ii) In Simpson's (3/8)th rule we divide [a, b] into…. equal subintervals.

A) n B) 2n C) 3n D) 4n

iii) Equation $f(x) = x^5 + x^4 - 3x^2 + 2x - 7 = 0$ has at most …. Positive roots.

A) 3 B) 2 C) 1 D) 0

iv) The value of $\Delta E^{-1} = \cdots$..

A) $\nabla\Delta$ B) ∇ C) E D) Δ

Q. 2] Attempt any one of the following. **[8]**

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i) Using Lagrange's interpolation formula interpolate f(0) from given data

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iii) Find second degree polynomial by Newton's forward interpolation passing through the points (0, 3), (1, 6), (2, 11), (3,18).

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Vivekanand College, Kolhapur

(Empowered Autonomous)

KOLHAPUR

DEPARTMENT OF MATHEMATICS

Internal Examination. Date: 24/02/2026

Notice B.Sc.-I Sem-II

All student of B.Sc. I are hereby informed that their Internal examination of Mathematics will be conducted on **Wednesday, 11th March 2026.** The examination will be conducted only one time; student are directed to attend the examination without fail. Syllabus and Schedule for examination as mentioned in following table.

Sr.No.	Date & Time	Name of Paper	Units
1.	Date : 11/03/2026 Time: 12.00 PM-1.30 PM	Differential Equation-I (DSC03MAT21)	1.Exact Differential Equation 2.Linear Differential Equation
2.		Discrete Mathematics (DSC03MAT22)	1.Predicates and Quantifiers 2. Isomorphic Graph 3. Handshaking Lemma

Note: All student must bring their assignment notebook on Internal examination.

G. B. Kolhe

Dr. S. P. Thorat

for HEAD

DEPARTMENT OF MATHEMATICS
VIVEKANAND COLLEGE, KOLHAPUR
(EMPOWERED AUTONOMOUS)

VIVEKANAND COLLEGE KOLHAPUR
ESTD JUNE 1964
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Vivekanand College, Kolhapur
(Empowered Autonomous)
KOLHAPUR

DEPARTMENT OF MATHEMATICS

Date: 17/03/2026

Notice

B.Com. I Sem II(2025-26)

Internal Examination

All the students of B.Com. I(2025-26) are hereby informed that their internal examination of mathematics will be conducted on **18th March 2026**. Internal Examination of Paper III and paper IV will be run same day in **online mode** (through Google form). Time table and Syllabus for examination will be mentioned in following table. Link will be provided 5 **minute before** examination on **WhatsApp group.**

Sr. No.	Name of paper	Topics	Date & Time
1	Business Mathematics-III	Module-I Differentiability Module II- Integration	Wednesday, 18th March 2026, 10:30 AM to 11:00 AM
2	Business Mathematics-IV	Module- I Permutation and Combination Module-II Transportation and Assignment Problems	Wednesday, 18th March 2026, 11:10 AM to 11:40 AM

Nature of Question Paper:

10 MCQ's of 1 marks each.

VIVEKANAND COLLEGE KOLHAPUR * (Empowered Autonomous) * ESTD JUNE 1964

G.B.Kolhe

(Mr. G.B.Kolhe)
HEAD
DEPARTMENT OF MATHEMATICS
VIVEKANAND COLLEGE, KOLHAPUR
(EMPOWERED AUTONOMOUS)

Vivekanand College, Kolhapur (An Empowered Autonomous Institute)

B.Sc. I (Semester-II), Examination:

Course Name: Differential Equation-I   Course Code: 2DSC03MAT21 (NEP 2.0)

Day & Date: 11/03/2026   Time: 12 pm- 1.30 pm   Total Marks: 10

Q.1. Select the correct alternative for each of the following. **[02]**

1) For equation $Mdx + Ndy = 0$, the equation is exact if…

a) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ b) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ c) $\frac{\partial M}{\partial x} \neq \frac{\partial N}{\partial y}$ d) $\frac{\partial M}{\partial x} \neq \frac{\partial N}{\partial y}$

2) The Integration factor of $\frac{dy}{dx} + Py = Q$ is…

a) $e^{-\int p dx}$ b) $e^{\int p dy}$ c) $e^{-\int p dy}$ d) $e^{\int p dx}$

Q.2) Attempt any one of the following. **[08]**

i) Solve $(x^2 + y^2 + 1)dx + 2xy\, dy = 0$.

ii) Solve $\frac{dy}{dx} - y \tan x = e^x$.

iii) Solve $x^2 y\, dx - (x^3 + y^3)dy = 0$.

*****

Vivekanand College, Kolhapur (An Empowered Autonomous Institute)
B.Sc. I (Semester-II), Internal Examination:

Course Name: Discrete Mathematics **Course Code: 2DSC03MAT22**

Day & Date: 11/03/2026 **Time:** 12.00pm - 1.30pm **Total Marks: 10**

Q.1. Select the correct alternative for each of the following. [2]

i) If a graph has total degree 8 then number of edges in graph are ____.

a) 16 b) 4 c) 8 d) 10

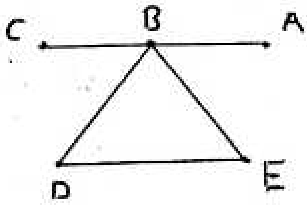
ii) Which of the following is a Proposition?

a)" Who won the match?" b) "7 is prime number "

c) "Open the door" d) "Is it raining?"

Q.2. Attempt any two of the following. [8]

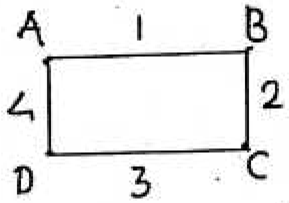
i) State and prove Hand shaking lemma and verify for following graph G_1



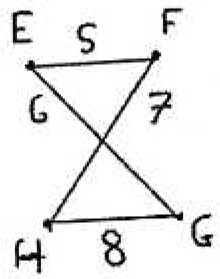
G_1

ii) verify the equivalence of implication: $p \to q \equiv {\sim}p \vee q$

iii) Check whether following pair of graphs are isomorphic or not



G_6



G_7

iv) Define Existential quantifier and write logical equivalence of following statement in distributing quantifiers "There exists a person who is either a teacher or a student."

****

Name: Viraj Vijay Jadhav

॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥

- शिक्षणमहर्षी डॉ. बापूजी साळुंखे

Sub- Differential Equation -I

VIVEKANAND COLLEGE, KOLHAPUR

(An Empowered Autonomous Institute)

Assignment

Student's Sign :	Pul.
Seat No./ Roll No. :	7202
Seat No./ Roll No. In words	Seven two zero two
	11820

02+08 = 10/10

प्र. क्र. Q. No.

Q. 1)

1) b) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

2) d) $e^{\int p dx}$

2

Q. 2)

i) Given D.E. is

$$(x^2+y^2+1)\,dx + 2xy\,dy = 0$$

comparing with $Mdx + Ndy = 0$

$\therefore M = x^2+y^2+1$ and $N = 2xy$

$\therefore \frac{\partial M}{\partial y} = 2y$ and $\frac{\partial N}{\partial x} = 2y$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

02	Section	Q. No.											
		Marks											

प्र. क्र. Q. No.

$\therefore$ Given D.E. is an exact D.E.

$\therefore$ Soln of the Diff. eqn is

$$\int_{y=\text{constant}} M\,dx + \int \left(\begin{array}{c}\text{terms not containing}\\ x \text{ in } N\end{array}\right) dy = c$$

$$\int (x^2+y^2+1)\,dx + \int (0)\,dy = C$$

$$\int x^2 dx + y^2\int dx + \int 1\,dx = c$$

$$\frac{x^3}{3} + xy^2 + x = c$$

ii) Given D.E. is

$$\frac{dy}{dx} - y\tan x = e^x$$

comparing with

$$\frac{dy}{dx} + Py = Q.$$

$\therefore P = -\tan x \qquad Q = e^x$

$$\therefore \text{I.F.} = e^{\int p\,dx}$$

$$= e^{-\int \tan x}$$

$$= e^{-\log(\sec x)}$$

$$= e^{\log(\sec x)^{-1}}$$

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Q. No.

$= (\sec x)^{-1}$

$I.F. = \frac{1}{\sec x}$

$\therefore$ Soln of D.E. is

$y(I.F.) = \int Q(I.F.)\,dx + c$

$y\left(\frac{1}{\sec x}\right) = \int \left(e^x \cdot \frac{1}{\sec x}\right) dx + c$

$y \cos x = \int e^x \cos x \, dx + c$ ——(1)

$= \cos x \int e^x - \left[\int e^x dx \cdot \frac{d}{dx}(\cos x) \cancel{dx}\right] dx$

$= e^x \cos x + \int (e^x \sin x)\,dx$

$= e^x \cos x + \left[e^x \sin x - \int e^x \cos x \, dx\right]$

$y \cancel{\cos x} = e^x \cos x + e^x \sin x - y\cancel{\cos x} - c$ ---From(1)

$\therefore \cancel{e^x(\sin x + \cos x) + c}$

$2y\cos x = e^x(\sin x + \cos x) - c$

4

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Q. No.

iii) Given, D.E. is.

$$x^2 y\,dx - (x^3 + y^3)\,dy = 0$$

comparing with $M dx + N dy = 0$

$M = x^2 y \qquad N = -x^3 - y^3$

$\frac{\partial M}{\partial y} = x^2 \qquad \frac{\partial N}{\partial x} = -3x^2$

$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

$\therefore$ This is not exact D.E.

consider $Mx + Ny = \cancel{x^3 y} - \cancel{x^3 y} - y^4$

$= -y^4 \neq 0.$

4

$\therefore$ I.F. $= -\frac{1}{y^4}$

Multiply the D.E. by I.F.

$$-\left(\frac{x^2 y}{y^4}\right)dx + \left(\frac{x^3 + y^3}{y^4}\right) = 0$$

This is exact D.E.

$\therefore$ Sol^n of D.E. is given by

$$\int\limits_{y=const} M dx + \int \left(\text{terms not containing } x \text{ in } N\right) dy = c$$

$$\int \left[-\frac{x^2}{y^3}\right] dx + \int \frac{1}{y}\, dy = c$$

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- शिक्षणमहर्षी डॉ. बापूजी साळुंखे

कोल्हापूर
१९६४

Student's Sign : Rul.

Seat No./ Roll No. : 7202

Seat No./ Roll No. seven two
In words zero two.

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VIVEKANAND COLLEGE, KOLHAPUR

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Assignment

क्र.
No.

$$-\frac{1}{y^3}\int x^2\,dx + \int \frac{1}{y}\,dy = c$$

$$-\frac{x^3}{3y^3} + \log y = c$$

Name : Viraj Vijay Jadhav

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Sub- Discrete Mathematics.

VIVEKANAND COLLEGE, KOLHAPUR

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Assignment

Student's Sign :	
Seat No./ Roll No. :	7202
Seat No./ Roll No. In words	seven two zero two
	11821

Q	1	2	3	total
	2	08		10/10

प्र. क्र. Q. No.

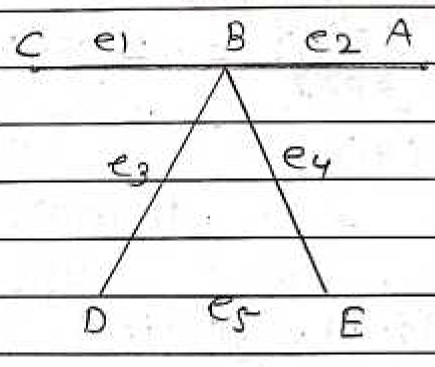
Q. 1)

i) b) 4

ii) b) 7 is prime number

Q. 2)

i) Given graph is.



(G_1)

In graph $G_1 = (V, E)$

$V = \{A, B, C, D, E\}$

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$E = \{e_1, e_2, e_3, e_4, e_5\}$

Now Degree of each vertex

$d(A) = 1$ $\quad d(D) = 2$

$d(B) = 4$ $\quad d(E) = 2$

$d(C) = 1$

Total Degree of G_1,

$$d(G_1) = \sum_{V \in G_1} d(V)$$

$$= d(A) + d(B) + d(C) + d(D) + d(E)$$

$$= 1 + 4 + 1 + 2 + 2$$

$$d(G_1) = 10 \quad \text{--- (1)}$$

By hand shaking lemma,

Total $d(G_1) = 2 \times$ No. of edges in G_1 — (2

LHS = Total $d(G_1)$

LHS = 10 $\quad$ --- from (1)

RHS = $2 \times$ No. of edges in G_1

$= 2 \times 5$

RHS = 10

03

$\therefore$ LHS = RHS

$\therefore$ Hand shaking lemma is verified.

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ii) Given statement,

$$p \to q \equiv \sim p \vee q$$

1	2	3	4	5
p	q	$\sim p$	$p \to q$	$\sim p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$\therefore$ All values in column(4) and coulmn(5) are identical

$\therefore \quad p \to q \equiv \sim p \vee q$

04

iii)

Both the graphs G_6 and G_7 have 4 vertices and 4 edges $\therefore$ ~~V=E~~

Bijective mapping between ~~edge~~ vertex set is

$A \longleftrightarrow E$

$B \longleftrightarrow F$

$C \longleftrightarrow H$

$D \longleftrightarrow G$.

Bijective mapping between edgeset

$e_1 \longleftrightarrow e_5$

$e_2 \longleftrightarrow e_7$

$e_3 \longleftrightarrow e_8$

$e_4 \longleftrightarrow e_6$

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$\therefore \quad G_6 \cong G_7$

$\therefore$ Graph G_6 and G_7 are Isomorphic with eachother.

04

iv)

Symbolic Form Fo statement, There exist a person who is either a teacher or a student is

$$\exists x \left[T(x) \vee S(x) \right]$$

where $T(x)$ = person x is teacher
$S(x)$ = person x is student.

Distributed Form:

$$\exists x \left[T(x) \vee S(x) \right] = \exists x\, T(x) \vee \exists x\, S(x)$$

Interpretation:

i) $\exists x\, T(x)$ = There exist a person x who is teacher.

ii) $\exists x\, S(x)$ = There exist a person x who is student

$\therefore \exists x\, T(x) \vee \exists x\, S(x)$ = There exis a person who is teacher or there exist a ~~stud~~ person who is student.

04

"Education for Knowledge, Science, and Culture"
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Shri Swami Vivekanand Shikshan Sanstha's

Vivekanand College, Kolhapur

(Empowered Autonomous)

KOLHAPUR

DEPARTMENT OF MATHEMATICS

Internal Examination

Date: 24/02/2026

Notice B.Sc.-II Sem-IV

All student of B.Sc. II are hereby informed that their Internal examination of Mathematics will be conducted on **Saturday, 14th March 2026.** The examination will be conducted only one time; student are directed to attend the examination without fail. Syllabus and Schedule for examination as mentioned in following table.

Sr. No.	Date & Time	Name of Paper	Units
1.	**Date : 14/03/2026 Time: 2.30 PM-4.00 PM**	Computational Mathematics for Mathematics (2MIN03MAT41)	Unit-I
2.		Laplace Transform (2MIN03MAT42)	Unit -I

G. B. Kolhe

Dr. S. P. Thorat

for HEAD

DEPARTMENT OF MATHEMATICS

VIVEKANAND COLLEGE, KOLHAPUR

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VIVEKANAND COLLEGE, KOLHAPUR (An Empowered Autonomous Institute)

B.Sc. Part-II (Mathematics) (Sem-IV) Internal Examination

SUBJECT: Computational Mathematics for Sciences- II COURSE CODE: 2MIN03MAT41 MARKS: 20

Q.1 Select the correct alternative. [04]

i) The central difference operator δ is defined by ______________.

a) $y_n - y_{n-1} = \delta y_n$ b) $y_n - y_{n-1} = \delta y_{n/2}$

c) $y_n - y_{n-1} = \delta y_{(n-1)/2}$ d) $y_n - y_{n-1} = \delta y_{n-1/2}$

ii) $\Delta^3 y_0 =$ ______________.

a) $y_3 - 3y_2 + 3y_1 - y_0$ b) $y_3 + 3y_2 + 3y_1 + y_0$

c) $y_3 - 3y_2 - 3y_1 - y_0$ d) $y_3 + 3y_2 - 3y_1 + y_0$

iii) $\sqrt{1 + \frac{1}{4}\delta^2} =$ ______________.

a) E b) μ

c) Δ d) ∇

iv) $E^{1/2} - E^{-1/2} =$ ______________.

a) Δ b) ∇

c) δ d) μ

Q.2 Attempt any ONE of the following. [08]

i) Determine the value of sin 38° using following table.

x	15	20	25	30	35	40
$\sin x$	0.2588190	0.3420201	0.4226183	0.5	0.5735764	0.6427876

ii) Show that $\Delta^n u_{x-n} = u_x - n u_{x-1} + \frac{n(n-1)}{2} u_{x-2} + \cdots + (-1)^n u_{x-n}$.

Q.3 Attempt any TWO of the following. [08]

i) Show that $\Delta = \nabla E = \delta E^{1/2}$.

ii) Find the cubic polynomial having $y(1) = 24, y(3) = 120, y(5) = 336, y(7) = 720$.

iii) Define interpolation, polynomial interpolation and interpolating polynomial.

Vivekanand College, Kolhapur (An Empowered Autonomous Institute)
B.Sc. II (Mathematics) (Semester-IV), Internal Examination:

Course Name: Integral Calculus   Course Code: 2DSC03MAT42(NEP 2.0)

Day & Date: 13/03/2026   Time: 2.30 pm – 4.00 pm   Total Marks: 20

Q.1. Select the correct alternative for each of the following. [04]

i) $\Gamma(n) =$ ———.

a) $\int_0^{\infty} x^{n+1} e^{-x} dx$ b) $\int_0^{\infty} x^{n-1} e^{x} dx$

c) $\int_0^{1} x^{n-1} e^{-x} dx$ d) $\int_0^{\infty} x^{n-1} e^{-x} dx$

ii) $\int_0^{\infty} x^7 e^{-2x^2} dx =$ ———

a) 1   b) 0   c) $\frac{3}{16}$ d) none of these

iii) $\beta(m,n) =$ ———

a) $\int_0^{\infty} \frac{(x)^{m-1}}{(1+x)^{n+1}} dx$ b) $\int_0^{1} (x)^{m-1}(1-x)^{n-1} dx$

c) $\int_0^{\pi/2} (sinx)^{2m-1}(cosx)^{2n-1} dx$ d) All of these

iv) $\int_0^{1} (x)^5 (1-x^3)^{10} dx =$ ———

a) $\frac{1}{3}\beta(2,11)$ b) $2\beta\left(\frac{1}{3}, 11\right)$ c) $11\,\beta\left(\frac{1}{3}, 2\right)$ d) $11\,\beta\left(\frac{1}{2}, 3\right)$

Q.2. Attempt any ONE of the following. [08]

i) Prove that $\beta(m,n) = \frac{\Gamma m\,\Gamma n}{\Gamma(m+n)}$ $m, n > 0$

ii) Prove that $\int_0^{\pi/2} (sinx)^p (cosx)^q dx = \frac{1}{2}\beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$

Q.3. Attempt any TWO of the following. [08]

i) Prove that $\frac{2^n \Gamma\left(n+\frac{1}{2}\right)}{\sqrt{\pi}} = 1.3.5 \ldots .(2n-1), n > 0$

ii) Prove that $\beta(m,n)\beta(m+n,p)\beta(m+n+p,q) = \frac{\Gamma m\,\Gamma n\,\Gamma p\,\Gamma q}{\Gamma m+n+p+q}$

iii) Evaluate the $\int_0^{\infty} \frac{x^4}{4^x} dx = \frac{4!}{(log4)^5}$

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Vivekanand College, Kolhapur (An Empowered Autonomous Institute)
B.Sc. II (Mathematics) (Semester-IV), Internal Examination:

Course Name: Differential Calculus Course Code: 2DSC03MAT41(NEP 2.0)

Day & Date: 13/03/2026 Time: 2.30 pm – 4.00 pm Total Marks: 20

Q.1. Select the correct alternative for each of the following. [04]

i) A function $f(x)$ is continuous at $x = a$ if…

a) $f(a)$ exist b) $\lim_{x\to a} f(x)$ exists
c) $\lim_{x\to a} f(x) = f(a)$ d) All of the above

ii) A removable discontinuity occurs when…

a) limit exists but function value is different or undefined
b) limit does not exist
c) function becomes infinite
d) function oscillates

iii) The function $f(x) = |x|$ is…

a) Differentiable everywhere
b) but not differentiable at $x = 0$
c) Discontinuous at $x = 0$
d) Differentiable only at $x = 0$

iv) $u = x + y$, $v = x - y$ then $\frac{\partial(x,y)}{\partial(u,v)} = \ldots$

a) $\frac{1}{2}$ b) -2
c) 2 d) 1

Q.2. Attempt any ONE of the following. [08]

i) If J is jacobian of u, v with respect to x, y and J' is jacobian of x, y with respect to u, v then $JJ' = 1$.

ii) Prove that

a) The composition of two continuous function is continuous function.

b) A Constant function is everywhere continuous.

Q.3. Attempt any TWO of the following. [08]

i) Find the value of 'a' if that function $f(x)$ defined by

$$f(x) = \begin{cases} 2x - 1 \ ; x < 2 \\ a \ ; x = 2 \\ x + 1 ; x > 2 \end{cases} \quad \text{is continuous at } x = 2.$$

ii) If $x^2 + y^2 + u^2 - v^2 = 0$ and $uv + xy = 0$ then show that $\frac{\partial(u.v)}{\partial(x,y)} = \frac{x^2-y^2}{u^2+v^2}$.

iii) If $u = x + y$, $v = \frac{x}{x+y}$ then find $\frac{\partial(u.v)}{\partial(x,y)}$.

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VIVEKANAND COLLEGE, KOLHAPUR (An Empowered Autonomous Institute)

B.Sc. Part- II (Mathematics) (Sem-IV) Internal Examination

Subject: Laplace Transform **Course Code: 2MIN03MAT42** **Marks:20**

Q. 1] Select the correct alternative for each of the following: **[4]**

i) If $L\{ F(t) \} = f(s)$ then $L\{ \int_0^t F(u)du\} = \ldots\ldots$

A) $f(s)$ B) $\frac{1}{s}f(s)$ C) $sf(s)$ D) $f'(s)$

ii) L{ $sin3t.cos3t\}$ = …….

A) $\frac{2}{s^2+4}$ B) $\frac{1}{s^2-4}$ C) $\frac{3}{s^2+36}$ D) $\frac{1}{s^2+36}$

iii) The value of $\int_0^\infty e^{-2t}\, t^2 \ dt =$ ______

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) $\frac{5}{6}$ D) $\frac{2}{3}$

iv) $L\{ y'(t) \} =$ _____

A) $s^2L\{y\} - sy(0) - y'(0)$ B) $sL\{y\} + y(0)$ C) $sL\{y\} - y(0)$ D)$sL\{y\} + sy(0) + y'(0)$

Q.2] Attempt any one of the following **[8]**

i) If $L\{F(t)\} = f(s)$, prove that $L\{F(at)\} = \frac{1}{a}f(\frac{s}{a})$.Also if $L\{F(t)\} = \frac{e^{-\frac{1}{s}}}{s}$ then find $L\{F(7t)\}$.

ii) Find the value of $L\{\frac{\cos 2t - \cos 3t}{t}\}$.

Q.3] Attempt any two of the following **[8]**

i) Find Laplace transform of the function, $F(t) = \begin{cases} e^t, & 0 < t \le 5 \\ 3, & t > 5 \end{cases}$

ii) Evaluate $L\{t^2 \cos 3t\}$ using Laplace transform.

iii) Evaluate $L\{te^{-t} \sinh t\}$ using Laplace transform.

Name: Shreya Bajirao Kumbhar

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Student's Sign : SK

Seat No./ Roll No. : 7794

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VIVEKANAND COLLEGE, KOLHAPUR

(An Empowered Autonomous Institute)

Assignment

Subject :- Computational Mathematics for science-I

Internal Examination (SEM IV) (Minor) BSC –

प्र. क्र. Q. No.

20/20

1)

i) d) $y_n - y_{n-1} = \delta y_{n-1/2}$

ii) ~~b)~~ $\cancel{y_3 + 3y_2 + 3y_1 + y_0}$

a) $y_3 - 3y_2 + 3y_1 - y_0$

iii) b) μ

iv) c) δ

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		Marks						

प्र. क्र. Q. No.

Q.2] i] Determine the value of $\sin 38^\circ$ using forward difference table

x	$\sin x = f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
15	0.2588190	0.0832011	-0.0026029	-0.0006186
20	0.3420201	0.0805982	-0.0032165	-0.0005888
25	0.4226183	0.0773817	-0.0038053	-0.0005599
30	0.5	0.0735764	-0.0043652	
35	0.5735764	0.0692112		
40	0.6427876			

$\Delta^4 f(x)$	$\Delta^5 f(x)$
0.0000248	0.0000041
0.0000289	

using formula,

$$\sin(38^\circ) = f_0(x) + P\Delta f_0 + \frac{P(P-1)}{2!}\Delta^2 f_0 + \frac{P(P-1)(P-2)}{3!}\Delta^3 f_0$$

$$+ \frac{P(P-1)(P-2)(P-3)}{4!}\Delta^4 f_0 + \frac{P(P-1)(P-2)(P-3)(P-4)}{5!} \times \Delta^5 f_0$$

* Given, $x_0 = 15$, $f_0 = 0.2588190$,
$\Delta f_0 = 0.0832011$, $\Delta^2 f_0 = -0.0026029$
$\Delta^3 f_0 = -0.0006136$, $\Delta^4 f_0 = 0.0000248$,

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	Marks												03

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Q. No.

$\Delta^5 f_0 = 0.0000041$ & $h = 5$ $x = 38$

$p = \frac{x - x_0}{h}$ $\frac{38 - 15}{5}$

$p = 4.6$

putting Values

$$\sin(38^\circ) = 0.2588190 + (4.6)(0.0832011) + \frac{(4.6)(4.6-1)}{2}(-0.0026029)$$

$$+ \frac{(4.6)(4.6-1)(4.6-2)}{6}(-0.0006136)$$

8

$$+ \frac{(4.6)(4.6-1)(4.6-2)(4.6-3)}{4\times3\times2\times1}(0.0000248)$$

$$+ \frac{(4.6)(4.6-1)(4.6-2)(4.6-3)(4.6-4)}{5\times4\times3\times2\times1}(0.0000041)$$

$$= 0.2588190 + 0.38272506 + (-0.0215520) + (-0.0044031936) + (0.000071185) + 0.0000014122368$$

$$= \underline{\underline{0.6156}}$$

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Q. 3)

i) Show that $\Delta = \nabla E = \delta E^{1/2}$

① Δ: L.H.S $= \Delta$

$\Delta y_i = y_{i+1} - y_i$

$\Delta y_i = y_{i+1} - y_i$

$\Delta y_i = E y_i - y_i$

$\Delta y_i = y_i (E - 1)$

$\Delta = E - 1$ —— ①

② ∇E

$(\nabla E) y_i = \nabla (E y_i)$

$= \nabla (y_{i+1})$

$= \nabla y_i$

$= y_{i+1} - y_i$ --- (by definition of d
$\nabla = (y_i - y_{i-1})$

$= E y_i - y_i$

$(\nabla E) y_i = y_i (E - 1)$

$\nabla E = E - 1$ —— ②

$\delta E^{1/2}$

$(\delta E^{1/2}) y_i = \delta (E^{1/2} y_i)$

$= \delta (y_{i+1/2})$

$(\delta E^{1/2}) y_i = \delta \left(y_{\frac{2i+1}{2}} \right)$

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VIVEKANAND COLLEGE, KOLHAPUR

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Assignment

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Seat No./ Roll No. In words	12263

प्र. क्र.
Q. No.

$$(\delta E^{1/2}) y_i = \delta\left(y_{\frac{2i+1}{2}}\right)$$

$$= y_{\frac{2(i+1/2)+1}{2}} - y_{\frac{2[(i+1/2)-1]}{2}}$$

$$= y_{(i+1/2)+1} - y_{(i+1/2)-1}$$

$$= y_{i+3/2} - y_{i-1/2}$$

$$(\delta E^{1/2}) y_i = E^{3/2} y_i - E^{-1/2} y_i$$

$$(\delta E^{1/2}) y_i = \cancel{E y_i} \; \cancel{y_i (E} \; (E-1)$$

$$\delta E^{1/2} y_i = y_i (E-1)$$

$$\delta E^{1/2} y_i = E-1 \quad \text{— (3)}$$

from (1) (2) & (3)

$$\Delta = \nabla E = \delta E^{1/2}$$

--- Hence proved.

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प्र. क्र.
Q. No.

-3)

ii] Given, $y(1) = 24$, $y(3) = 120$, $y(5) = 336$, $y(7) = 720$

x	$y_0(x)$	$\Delta y_0(x)$	$\Delta^2 y_0(x)$	$\Delta^3 y_0(x)$
1	24	96	120	48
3	120	216	168	
5	336	384		
7	720			

Given,

$x_0 = 1$, $y_0 = 24$, $\Delta y_0 = 96$, $\Delta^2 y_0 = 120$
$\Delta^3 y_0 = 48$, $h = 2$

$$p = \frac{x - x_0}{h}$$

$$\therefore p = \frac{x-1}{2}$$

by using Newton's forward difference formula

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$y(x) = 24 + \left(\frac{x-1}{2}\right)(96) + \frac{\left(\frac{x-1}{2}\right)\left(\frac{x-1}{2} - 1\right)}{2!} \times 120$$

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$$+ \frac{\left(\frac{x-1}{2}\right)\left(\frac{x-1}{2}-1\right)\left(\frac{x-1}{2}-2\right)}{6} \times 48$$

$$= 24 + (x-1)(48) + \frac{\left(\frac{x-1}{2}\right)\left(\frac{x-3}{2}\right)}{2} \times \overset{60}{\cancel{120}}$$

$$+ \frac{\left(\frac{x-1}{2}\right)\left(\frac{x-3}{2}\right)\left(\frac{x-5}{2}\right)}{\cancel{6}} \times \overset{8}{\cancel{48}}$$

$$\frac{\left(\frac{x-1}{2}\right)\left(\frac{x-3}{2}\right)}{} \quad \frac{(x-1)(x-3)}{4} \times 15$$

$$= 24 + 48x - 48 + (x-1)(x-3)(15) + (x-1)(x-3)(x-5)$$

$$= 24 + 48x - 48 + (x^2 - 3x - x + 3)15 + (x^2 - 3x - x + 3)(x-5)$$

$$= 48x - 24 + 15x^2 - 45x - 15x + 45 + x^3 - 3x^2 - x^2 + 3x - 5x^2 + 15x + 5x - 15$$

$$= \underline{\underline{x^3 + 6x^2 + 11x - 6}}$$

--- This is a polynomial equation

Jiya Salim Mulla

॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥
– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

VIVEKANAND COLLEGE, KOLHAPUR

(An Empowered Autonomous Institute)

Assignment

Integral calculus

Student's Sign :	Jsmulla.
Seat No./ Roll No. :	7736
Seat No./ Roll No. In words	seven seven three six
	11989

प्र. क्र. Q. No.				
Question	1	2	3	total
Marks	04	08	08	20/20

Q. 1

i] d] $\int_0^\infty e^{-x} x^{n-1} \, dx$

ii] c] $\frac{3}{16}$

iii] d] All of these

iv] a] $\frac{1}{3} \beta(2, 11)$

02	Section	Q. No.											
		Marks											

प्र. क.
Q. No.

Q. 2.
i] Statement :

$$\beta(m,n) = \frac{\Gamma m \, \Gamma n}{\Gamma m+n} \quad ; m,n > 0$$

Proof :

we know that,

$$\Gamma m = \int_0^\infty e^{-x} x^{m-1} \, dx$$

& 2^{nd} form of gamma function is,

$$\Gamma m = 2\int_0^\infty e^{-x^2} x^{2m-1} \, dx$$

&

$$\Gamma n = 2\int_0^\infty e^{-y^2} y^{2n-1} \, dy$$

now, consider,

$$\Gamma m \, \Gamma n = 2\int_0^\infty e^{-x^2} x^{2m-1} dx \times 2\int_0^\infty e^{-y^2} y^{2n-1} dy$$

$$= 4\int_0^\infty\int_0^\infty e^{-(x^2+y^2)} x^{2m-1} y^{2n-1} \, dx \, dy$$

now, by change of co-ordinate system we have transformations,

$x = r\cos\theta$ & $y = r\sin\theta$

Section	Q. No.													03
	Marks													

$r = 0$ or $\cos\theta = 0$ $\quad$ $r = 0$ or $\sin\theta = 0$

$\therefore \theta = \frac{\pi}{2}$ $\quad$ $\theta = 0^\circ$

if $x = \infty$ $\quad$ if $y = \infty$

$r\cos\theta = \infty$ $\quad$ $r\sin\theta = \infty$

$\therefore r = \infty$ $\quad$ $\therefore r = \infty$

$\therefore$ r lies between 0 to ∞

& θ lies between 0 to $\frac{\pi}{2}$

now,

$$J\left(\frac{x,y}{r,\theta}\right) = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix}$$

$$= \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}$$

$$= r\cos^2\theta + r\sin^2\theta$$

$$= r(\cos^2\theta + \sin^2\theta)$$

$$= r \qquad [\because \cos^2\theta + \sin^2\theta = 1]$$

now,

$$\Gamma m \, \Gamma n = 4\int_0^{\pi/2}\int_0^{\infty} e^{-r^2} r^{2m-1} \cos^{2m-1}\theta \, . \, r^{2n-1} \sin^{2n-1}\theta \; r \, dr \, d\theta$$

$$= 4\int_0^{\pi/2}\int_0^{\infty} e^{-r^2} r^{2m-1+2n-1+1} \cos^{2m-1}\theta \, . \, \sin^{2n-1}\theta \, dr \, d\theta$$

$$\Gamma m \, \Gamma n = 2\int_0^{\infty} e^{-r^2} r^{2(m+n)-1} \, dr \times 2\int_0^{\pi/2} \cos^{2m-1}\theta \, . \, \sin^{2n-1}\theta \, . \, d\theta \quad (1)$$

04	Section	Q. No.												
		Marks												

प्र. क्र. Q. No.

by 2^{nd} form of ~~b~~ beta fun^n we have,

$$\beta(m,n) = 2\int_0^{\pi/2} \cos^{2m-1}\theta \cdot \sin^{2n-1}\theta \cdot d\theta \qquad (2)$$

from eq^n (1) & (2)

$$\Gamma m \, \Gamma n = 2\int_0^{\infty} e^{-r^2} r^{2(m+n)-1}\, dr \times \beta(m,n) \qquad (3)$$

but,

$$\Gamma(m+n) = 2\int_0^{\infty} e^{-r^2} r^{2(m+n)-1}\, dr \qquad (4)$$

from eq^n (3) & (4) we get

$$\Gamma m \, \Gamma n = \Gamma(m+n) \times \beta(m,n)$$

$$\therefore \boxed{\beta(m,n) = \frac{\Gamma m \, \Gamma n}{\Gamma(m+n)}} \qquad m, n > 0$$

hence proved

08

VIVEKANAND COLLEGE, KOLHAPUR
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Assignment

Integral Calculus

Student's Sign :	[signature]
Seat No./ Roll No. :	7736
Seat No./ Roll No. In words	Seven Seven three six
	11834

we have,

$$\Gamma(n+1) = n! \quad \text{— } 2^{nd} \text{ property of gamma fun}^n$$

$$\therefore \Gamma(n+1/2) = (n+1/2-1)!$$

$$\therefore \Gamma(n+1/2) = (n-1/2)(n-1/2-1)!$$

$$\therefore \Gamma(n+1/2) = (n-1/2)(n-3/2)(n-3/2-1)!$$

$$\therefore \Gamma(n+1/2) = (n-1/2)(n-3/2)(n-5/2)(n-7/2)!$$

$$\vdots$$

$$\therefore \Gamma(n+1/2) = (n-1/2)(n-3/2)(n-5/2)\ldots\ldots 5/2 \cdot 3/2 \cdot 1/2\, \Gamma(1/2)$$

$$\therefore \Gamma(n+1/2) = \left(\frac{2n-1}{2}\right)\left(\frac{2n-3}{2}\right)\left(\frac{2n-5}{2}\right)\ldots\ldots \frac{5}{2} \cdot \frac{3}{2} \cdot \frac{1}{2}\sqrt{\pi}$$

$$\therefore \Gamma(n+1/2) = \frac{\sqrt{\pi}}{2^n}(2n-1)(2n-3)(2n-5)\ldots 5.3.1$$

$$\therefore \frac{2^n\,\Gamma(n+1/2)}{\sqrt{\pi}} = 1.3.5.\ldots\ldots(2n-5).(2n-3)(2n-1) \quad , n \geq 0$$

02	Section	Q. No.										
		Marks										

प्र. क्र.
Q. No.

thus,

$$\frac{2^n \Gamma(n+1/2)}{\sqrt{\pi}} = 1.3.5 \ldots\ldots (2n-1) \quad ; n > 0$$

— hence proved

04

Q. 3.

ii] $L.H.S. = \beta(m,n)\,\beta(m+n,p)\,\beta(m+n+p,q)$

$$= \frac{\Gamma m\,\Gamma n}{\Gamma m+n} \cdot \frac{\Gamma m+n\,\Gamma p}{\Gamma m+n+p} \cdot \frac{\Gamma m+n+p\,\Gamma q}{\Gamma m+n+p+q}$$

$$= \frac{\Gamma m\,\Gamma n\,\Gamma p\,\Gamma q}{\Gamma m+n+p+q}$$

$= R.H.S.$

thus,

$$\beta(m,n)\,\beta(m+n,p)\,\beta(m+n+p,q) = \frac{\Gamma m\,\Gamma n\,\Gamma p\,\Gamma q}{\Gamma m+n+p+q}$$

04

— hence proved

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– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

NAME – HARSH V. PAWAR

BSC-SY (SEM IV)

DIFFERENTIAL CALCULUS

VIVEKANAND COLLEGE, KOLHAPUR

(An Empowered Autonomous Institute)

Assignment

Student's Sign :	
Seat No./ Roll No. :	7725
Seat No./ Roll No. In words	
	11974

04 + 08 + 08 = 20/20

प्र. क्र. Q. No.

Q.1.

i) A Function $f(x)$ is continuous at $x=a$ if —

Ans. (d) All of these

ii) A removable discontinuity occurs when —

Ans. (a) Limits exist

iii) The functn $f(x)=|x|$ is —

Ans. (b) ~~Discontinuous~~ ~~at x=0~~ but not differentiable at x

iv) $u = x+y$, $v = x-y$ then $\frac{\partial(x,y)}{\partial(u,v)} =$

Ans (a) $-\frac{1}{2}$

04

Q.2.

i) Let u and v be the function of x and y

$u = f_1(x,y)$ and $v = f_2(x,y)$ —— ①

partial derivative w.r.t x and y of 1st order of eqn ①.

$$1 = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial u}$$

$$0 = \frac{\partial v}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial v}{\partial y}\cdot\frac{\partial y}{\partial u}$$

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– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

NAME – HARSH V. PAWAR
BSC-SY (SEM IV)
DIFFERENTIAL CALCULUS

VIVEKANAND COLLEGE, KOLHAPUR
(An Empowered Autonomous Institute)

Assignment

Student's Sign :	
Seat No./ Roll No. :	7725
Seat No./ Roll No. In words	
	11974

04+08+08 = 20/20

प्र. क्र. Q. No.

Q.1.

i) A function $f(x)$ is continuous at $x=a$ if —
Ans. (d) All of these

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iv) $u = x+y$, $v = x-y$ then $\frac{\partial(x,y)}{\partial(u,v)} =$
Ans (a) $-\frac{1}{2}$

04

Q.2.

i) Let u and v be the function of x and y
$u = f_1(x,y)$ and $v = f_2(x,y)$ —— ①

partial derivative w.r.t x and y of 1st order of eqn ①.

$$1 = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial u}$$

$$0 = \frac{\partial v}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial v}{\partial y}\cdot\frac{\partial y}{\partial u}$$

02	Section	Q. No.												
		Marks												

प्र. क्र.
Q. No.

$$0 = \frac{\partial v}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial v}{\partial y} \frac{\partial y}{\partial u}$$

$$1 = \frac{\partial v}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial v}{\partial y} \frac{\partial y}{\partial v}$$

Let J be the Jacobian of ~~x~~ u, v w.r.t x, y

i.e. $J = \dfrac{\partial(u,v)}{\partial(x,y)}$

And J' be the Jacobian of x, y w.r.t u, v.

i.e. $J' = \dfrac{\partial(x,y)}{\partial(u,v)}$

Now,

$$J \cdot J' = \frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(u,v)}$$

$$= \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix} \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix}$$

$$= \begin{vmatrix} \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial u} & \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial v} \quad \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial v} \\ \frac{\partial v}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial v}{\partial y} \cdot \frac{\partial y}{\partial u} & \frac{\partial v}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial v}{\partial y} \cdot \frac{\partial y}{\partial v} \end{vmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$J \cdot J' = 1$

Hence proved.

Section	Q. No.													03
	Marks													

प्र. क्र.
Q. No.

Q.3.

i) $$F(x) = \begin{cases} 2x-1 & ; x<2 \\ a & ; x=2 \\ x+1 & ; x>2 \end{cases}$$

The above Function is continuous at $x=2$.

$$LHL = \lim_{x\to 2^-} f(x) = \lim_{h\to 0} (2-h)$$

$$= \lim_{h\to 0} 2(2-h)-1$$

$$= \lim_{h\to 0} 4-1 = 3$$

$$RHL = \lim_{x\to 2^+} F(x) = \lim_{h\to 0} (2+h)$$

$$= \lim_{h\to 0} (2+h)+1$$

$$= \lim_{h\to 0} 2+1 = 3$$

As it is a continuous Function, So

$$LHL = RHL = F(2)$$

$$F(2) = 3$$

$$\boxed{a=3}$$

4

04	Section	Q. No.												
		Marks												

प्र. क्र.
Q. No.

(11) $f_1 = x^2+y^2+u^2-v^2=0$

$f_2 = uv+xy=0$

$$\frac{\partial(u,v)}{\partial(x,y)} = (-1)^2 \left[\frac{\dfrac{\partial(f_1,f_2)}{\partial(x,y)}}{\dfrac{\partial(f_1,f_2)}{\partial(u,v)}}\right]$$

So,

$$\frac{\partial(f_1,f_2)}{\partial(x,y)} = \begin{vmatrix} \dfrac{\partial f_1}{\partial x} & \dfrac{\partial f_1}{\partial y} \\ \dfrac{\partial f_2}{\partial x} & \dfrac{\partial f_2}{\partial y} \end{vmatrix} = \begin{vmatrix} 2x & 2y \\ y & x \end{vmatrix}$$

$$= 2x^2-2y^2$$

$$= 2(x^2-y^2)$$

Now,

$$\frac{\partial(f_1,f_2)}{\partial(u,v)} = \begin{vmatrix} \dfrac{\partial f_1}{\partial u} & \dfrac{\partial f_1}{\partial v} \\ \dfrac{\partial f_2}{\partial u} & \dfrac{\partial f_2}{\partial v} \end{vmatrix} = \begin{vmatrix} 2u & -2v \\ v & u \end{vmatrix}$$

$$= 2u^2+2v^2$$

$$= 2(u^2+v^2)$$

$$\frac{\partial(u,v)}{\partial(x,y)} = \frac{\cancel{2}(x^2-y^2)}{\cancel{2}(u^2+v^2)}$$

4

$$= \frac{x^2-y^2}{u^2+v^2}$$

Hence proved.

Harsha Balaso Havaldar

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VIVEKANAND COLLEGE, KOLHAPUR

(An Empowered Autonomous Institute)

Assignment

Student's Sign :

Seat No./ Roll No. : 7977

Seat No./ Roll No. In words seven nine seven seven

11958

Laplace Transform

प्र. क्र. Q. No.

04 + 08 + 08 = 20

Q.1

i] → B] $\frac{1}{s} f(s)$

01

ii] → c] $\frac{3}{s^2+36}$

01

iii] → A] $\frac{1}{4}$

01

iv] → ~~A] $s^2 L\{y\} - sy(0) - y'(0)$~~

c] $sL\{y\} - y(0)$

01

02 Section | Q. No. | Marks

प्र. क. Q. No.

$$F(t) = \begin{cases} e^t, & 0 < t \leq 5 \\ 3, & t > 5 \end{cases}$$

Q.3] i] Soln

We have,

$$L\{F(t)\} = \int_0^\infty e^{-st} \cdot F(t)\, dt$$

$$= \int_0^5 e^{-st} F(t)\, dt + \int_0^\infty e^{-st} F(t)\, dt$$

$$= \int_0^5 e^{-st} e^t\, dt + \int_5^\infty e^{-st}\, 3\, dt$$

$$= \int_0^5 e^{(1-s)t}\, dt + 3\int_5^\infty e^{-st}\, dt$$

$$= \left[\frac{e^{t(1-s)}}{(1-s)}\right]_0^5 + 3\left[\frac{e^{-st}}{-s}\right]_5^\infty$$

$$= \frac{1}{1-s}\left[e^{5(1-s)} - 1\right] + 3\left[\frac{0 + e^{-5s}}{s}\right]$$

ok

$$\boxed{= \frac{1}{1-s}\left[e^{5(1-s)} - 1\right] - \frac{3e^{-5s}}{s}}$$

Section	Q. No.												
	Marks												

Here, $L\{F(t)\} = \frac{3}{s^2+3^2} = f(s)$

Where $F(t) = \cos 3t$

$\therefore L\{t^2 F(t)\} = (-1)\frac{d^2}{ds^2} f(s)$

$L\{t^2 F(t)\} = \frac{d^2}{ds^2} \frac{s}{s^2+3^2}$

$= \frac{d}{ds}\left[\frac{(s^2+3^2)(1) - s(2s)}{(s^2+3^2)^2}\right]$

$= \frac{d}{ds}\left[\frac{3^2 - s^2}{(s^2+3^2)^2}\right]$

$= \frac{(s^2+3^2)^2 - 2s - (3^2 - s^2) - 2(s^2+3^2) \cdot 2s}{(s^2+3^2)^4}$

$= \frac{(s^2+3^2)}{(s^2+3^2)}\left[\frac{-2s(s^2+3^2) - 4s(s^2-3^2)}{(s^2+a^2)3}\right]$

$= -2s\left[\frac{(s^2+03^2 + 2(3^2-s^2)}{(s^2+3^2)3}\right]$

$= \{t^2 F(t)\} = -2s\left[\frac{3 \cdot 3^2 - s^2}{(s^2+3^2)^3}\right]$

4

04	Section	Q. No.											
		Marks											

प्र. क्र. Q. No.

$$L\{t^2 F(t)\} = -2s \frac{[27 - s^2]}{(s^2+3^2)^3}$$

Q.2] ii]

$$L\left\{\frac{\cos 2t - \cos 3t}{t}\right\}$$

$$F(t) = \cos 2t - \cos 3t$$

$$L\{F(t)\} = \left[\frac{9}{s^2+4} - \frac{s}{s^2+9}\right] = f(s)$$

$$L\left\{\frac{F(t)}{t}\right\} = \int_s^\infty f(s)\, ds$$

$$L\left\{\frac{\cos 2t - \cos 3t}{t}\right\} = \int_s^\infty \left(\frac{s}{s^2+4} - \frac{s}{s^2+9}\right) ds$$

$$= \frac{1}{2}\left[\int_9^\infty \left(\frac{2s}{s^2+4} - \frac{2s}{s^2+9}\right)\right] ds$$

$$= \frac{1}{2}\left[\log(s^2+4) - \log(s^2+9)\right]_9^\infty$$

$$= \frac{1}{2}\left[\log\left(\frac{s^2+4}{s^2+9}\right)\right]_9^\infty$$

$$= \frac{1}{2}\left[\log \frac{s^2[1+4/s^2]}{s^2[1+9/s^2]}\right]_9^\infty$$

Harsha Balaso Havaldar 7977

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– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

कोल्हापूर
१९६४

VIVEKANAND COLLEGE, KOLHAPUR

(An Empowered Autonomous Institute)

Assignment

Student's Sign :	
Seat No./ Roll No. :	
Seat No./ Roll No. In words	
	12478

प्र. क्र.
Q. No.

$$= \frac{1}{2}\left[\log 1 - \log\left(\frac{s^2+4}{s^2+9}\right)\right]$$

$$\boxed{= \frac{-1}{2} \log\left(\frac{s^2+4}{s^2+9}\right)}$$

08

"ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार" - शिक्षणमहर्षी डॉ. बापूजी साळुंखे

Shri Swami Vivekanand Shikshan Sanstha, Kolhapur.

VIVEKANAND COLLEGE, KOLHAPUR

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Date: 12/02/2026

Notice

Internal Exam 2025-26

B.Sc. III (Major & Minor) Sem VI

All students of B.Sc. III (Major & Minor) are hereby informed that the internal examination for Semester VI will be held from **24/02/2026 to 28/02/2026**. The examination will be conducted only once. Attendance is mandatory, and no exceptions will be permitted. The syllabus, timetable, and pattern of question paper for the Internal will be mentioned as follows:

Sr. No.	Name of the Paper	Syllabus	Day & Date	Time
Major Subjects				
1	Metric Space	Unit 1 & 2	Tuesday, 24/02/2026	12:00 PM to 01:00 PM
2	Linear Algebra	Unit 1 & 2	Wednesday, 25/02/2026	12:00 PM to 01:00 PM
3	Complex Analysis	Unit 1 & 2	Thursday, 26/02/2026	12:00 PM to 01:00 PM
4	Optimization Techniques	Unit 1 & 2	Friday, 27/02/2026	12:00 PM to 01:00 PM
Minor Subject				
1	Basics of Operation Research	Unit 1 & 2	Saturday, 28/02/2026	12:00 PM to 01:00 PM

Question Paper Pattern **20 Marks**

Q.1) Select the correct alternative. 4 MCQs **[04]**

Q.2) Attempt any one out of two Questions. **[08]**

Q.3) Attempt any two out of three Questions. **[08]**

VIVEKANAND COLLEGE KOLHAPUR (Empowered Autonomous) ESTD JUNE 1964

G.B.Kolhe

Dr.S.P.Thorat

HEAD
DEPARTMENT OF MATHEMATICS
VIVEKANAND COLLEGE, KOLHAPUR
(EMPOWERED AUTONOMOUS)

Vivekanand College, Kolhapur (An Empowered Autonomous Institute)

B.Sc. III (Mathematics) (Semester-VI), Examination:

Course Name: Metric Space **Course Code: DSC03MAT61 (NEP 1.0)**

Day & Date: **Time:** **Total Marks: 20**

Instructions:

1.All the questions are compulsory.

2.Figure to the right indicates full marks.

Q.1. Select the correct alternative for each of the following. **[04]**

1. For metric space <M ρ> the subset A is dense in M if______.

   A) $\bar{A} = M$ B) $\bar{A} = \emptyset$ C) $\bar{A} = A$ D) $\bar{A} \cap A = \emptyset$

2. If ℓ^∞ is set of all bounded sequences of real number and $x = \{x_n\}_{n=1}^{\infty}$, $y = \{y_n\}_{n=1}^{\infty}$ are in ℓ^∞ then show that <ℓ^∞, ρ> forms metric space where $\rho(x, y) =$ ____

   A)lub $|x - y|$ B) $\underset{1\le n\le p}{\text{lub}} |x_n + y_n|$ C) $\underset{1\le n\le p}{\text{lub}} |x|$ D) $\underset{1\le n\le p}{\text{lub}} |x_n - y_n|$

3. A metric space <M ρ> is connected if and only if it has_____.

   A) separation B) no separation C) subsets D) connected subsets.

4. In an metric space <M ρ> , A = $\emptyset$ is open set in space then $A' = M - A$ is ____set.

   A) closed B) neither open nor closed C) open D) both open and closed.

Q.2. Attempt any one of the following. **[08]**

1) Show that <$\mathbb{R}^2$, ρ> forms a metric space where $\rho(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$ for $x = (x_1, x_2)$, $y = (y_1, y_2)$ in $\mathbb{R}^2$.

2) Show that every convergent sequence in metric space is Cauchy sequence.

Q.3. Attempt any two of the following. **[08]**

1) Let (X, d) be a metric space and x, y, z, be any 3 points of X then $d(x, y) \ge |d(x, z) - d(z, y)|$.

2) Show that in any metric space $<M, \rho> M$ and φ are both open and closed sets.

3) Let A and B be a subset of X then show that

   i) $A \subset B \Rightarrow D(A) \subseteq D(B)$. ii) $D(A \cap B) \subset D(A) \cap D(B)$.

Vivekanand College Kolhapur (An Empowered Autonomous Institute)
Department of Mathematics
Internal Examination 2025-26
B.Sc.-III (Sem-IV)

Course Name: Linear Algebra
Day & Date: Wednesday,25/02/2026
Time: 12:00PM to 1:00PM

Course Code: DSC03MAT62
Marks: 20

Q.1. Select the correct alternative from each of the following. **[04]**

i) $Dim(\{0\}) = \cdots$
a) 1 b) 0 c) 2 d) 3

ii) Inner product space over complex field is…
a) euclidean space b) complex space c) unitary space d) quotient space

iii)If $T: V \longrightarrow W$ is homomorphism then Range(T) is….
a) subspace of V b) subspace of W c) not subspace d) quotient space

iv)If u and v and elements of inner product space of $V(F)$ and $\alpha \in F$ then $\overline{\langle u \quad \alpha v \rangle} = \cdots$
a) $\bar{\alpha}\ \overline{\langle u \quad v \rangle}$ b) $\langle \alpha u \quad \alpha v \rangle$ c) $\bar{\alpha}\langle u \quad v \rangle$ d) $\alpha\ \overline{\langle u \quad v \rangle}$

Q.2 Attempt any one of the following. **[08]**

i) Prove that if S is finite subset of vector space V such that V=L(S) then there exist a subset of S which is basis of V.

ii) Define inner product space. State and Prove Cauchy Schwartz Inequality.

Q.3. Attempt any two of the following. **[08]**

i) Define sum of two subspaces. Prove that sum of two subspaces is again a subspace.

ii) Apply Gram Schmidt process to orthonormalize the set of linearly independent vectors
$(1,\ 0,\ 0), (1,\ 1,\ 1), (1,\ 2,\ 3)$.

iii) Prove that if $T: V \longrightarrow U$ is linear transformation then $\ker(T) = \{0\}$ if and only if T is one-one.

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Vivekanand College, Kolhapur (An Empowered Autonomous Institute)

B.Sc. III (Mathematics) (Semester-VI), Examination:

Course Name: Complex Analysis | **Course Code: DSC03MAT63**

Day & Date: 26-02-2026 | **Time:** 12pm-1pm | **Total Marks: 20**

Instructions:

1.All the questions are compulsory.

2.Figure to the right indicates full marks.

Q.1. Select the correct alternative for each of the following. **[08]**

1. If $f(z) = u(x,y) + iv(x,y)$is analytic, then the necessary condition is

A) $u_x = v_x$ B) $u_y = v_y$ C) $u_x = v_y$and $u_y = -v_x$ D) $u_x = -v_y$

2. If $z = x + iy$, then $e^z =$

A) $e^x(\cos y + i\sin y)$ B) $e^y(\cos x + i\sin x)$ C) $e^x(\cosh y + i\sinh y)$ D) $e^y(\cosh x + i\sinh x)$

3. Which function is entire?

A) $\frac{1}{z}$ B) e^z C) $\log z$ D) $\frac{1}{z-1}$

4. The singular points of $f(z) = \frac{1}{(z-1)(z+2)}$ are

A) $z = 0$ B) $z = -2$ C) $z = 1$ D) $z = 1, -2$

Q.2. Attempt any one of the following. **[08]**

1.If f(z) = $u + iv$ be analytic function and $z = re^{i\theta}$ where u, v, r, θ are all real numbers then show that

$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{dv}{d\theta}$, $\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{du}{d\theta}$ i.e. $ru_r = v_\theta$, $u_\theta = -rv_r$

2. If $z = re^{it}$, show that Log $z = \ln r + i(\theta + 2n\pi)$, $n \in Z$

Q.3. Attempt any two of the following. **[08]**

1.Using C-R equation show that $f'(z)$ and its derivative $f''(z)$ exist everywhere and find $f''(z)$ when $f(z) = z^3$.

2.If z = -1-$\sqrt{3}$ i find exponential logarithmic function of this complex number.

3.Show that $u(x,y)$ is harmonic in some domain and find a harmonic conjugate $v(x,y)$ when $u(x,y) = 2x - x^2 + 3xy^2$

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Vivekanand College, Kolhapur (An Empowered Autonomous Institute).

Department of Mathematics

B.Sc. III (Sem VI) Internal Examination

Course Code: DSE03MAT11 (Optimization Techniques) Day & Date: 27/02/2026 Total Marks: 20

Q.1. Select the correct alternative for each of the following. [04]

i) In the optimal simplex table the value $z_j - c_j = 0$ indicates

A) unbounded solution B) Optimal solution C) alternative solution D) infeasible solution

ii) For maximization problem in Big-M method, coefficient for an artificial variable in the objective function is....

A) +M B) -M C) Zero D) 1

iii) The solution to a transportation problem with m rows (supplies) and n columns(destinations) is non-degenerate if number of positive allocations are..........

A) $m+n$ B) mxn C) $m+n-1$ D) $m+n+1$

iv) Transportation problem having Minimization of objective function has an alternate optimal solution whenever in optimal TP table .....for any empty cell.

A) $c_{ij} - (u_i + v_j) > 0$ B) $c_{ij} - (u_i + v_j) < 0$ C) $c_{ij} - (u_i + v_j) \geq 0$ D) $c_{ij} - (u_i + v_j) \leq 0$

Q.2. Attempt any one. [08]

i) Find the optimal solution of Transportation problem by using Modified Distribution method.

	D1	D2	D3	D4	Supply
S1	11	13	17	14	250
S2	16	18	14	10	300
S3	21	24	13	10	400
Demand	200	225	275	250	

ii) Solve the following L.P.P. by Big-M Method.

Min $Z = 7x_1 + 15x_2 + 20x_3$, subject to $2x_1 + 4x_2 + 6x_3 \geq 24, 3x_1 + 9x_2 + 6x_3 \geq 30,\ x_1, x_2, x_3 \geq 0$

Q.3. Attempt any One. [04]

i) Find the initial basic feasible solution of the Transportation problem by using least cost method.

	D1	D2	D3	D4	D5	Supply
S1	5	8	6	6	3	8
S2	4	7	7	6	5	5
S3	8	4	6	6	4	9
Demand	4	4	5	4	8	

ii) Solve the following L.P.P. by Simplex Method.

Max $Z = x_1 + x_2$, subject to $x_1 + 2x_2 \leq 20, x_1 + x_2 \leq 15,\ x_1, x_2 \geq 0$

Vivekanand College, Kolhapur (An Empowered Autonomous Institute).

Department of Mathematics

B.Sc. III (Sem VI) Internal Examination

Course Code: MIN03MAT11 (Basics of Operations Research) Day & Date: 28/02/2026 Total Marks: 20

Q.1. Select the correct alternative for each of the following. **[04]**

i) Transportation problem having Minimization of objective function has no alternate optimal solution whenever in optimal TP table .....for any empty cell.

A) $c_{ij} - (u_i + v_j) > 0$ B) $c_{ij} - (u_i + v_j) < 0$ C) $c_{ij} - (u_i + v_j) \geq 0$ D) $c_{ij} - (u_i + v_j) \leq 0$

ii) When are artificial variables NOT required in a Linear Programming Problem?

A) When all constraints are $\leq$ type B) When constraints are $\geq$ type

C) When constraints are $=$ type D) Only in minimization problem

iii) Initial Basic feasible solution of L.P.P. is obtained by assuming the values of ______variables equal to zero.

A) Basic B) non-basic C) Slack D) Surplus

iv) The solution to a transportation problem with m rows (supplies) and n columns(destinations) is degenerate if number of positive allocations are.......... $m + n - 1$.

A) less than B) greater than C) *equal to* D) *None of these*

Q.2. Attempt any one. **[08]**

i) Solve the following L.P.P. by Big-M Method.

Max $Z = 3x_1 + 2x_2 + x_3$,subject to $2x_1 + x_2 + x_3 = 12, 3x_1 + 4x_2 = 11,$ x_1 *unrestricted*, $x_2, x_3 \geq 0$

ii) Find the optimal solution of Transportation problem by using Modified Distribution method.

	D1	D2	D3	D4	Supply
S1	23	27	17	18	30
S2	12	17	20	31	40
S3	22	28	12	32	53
Demand	22	35	25	41	

Q.2. Attempt any one. **[04]**

i) Solve the following L.P.P. by Simplex Method.

Max $Z = x_1 + x_2$, subject to $x_1 + 2x_2 \leq 20, x_1 + x_2 \leq 15,$ $x_1,x_2 \geq 0.$

ii) Find the initial basic feasible solution of the Transformation problem by using Vogel approximation method.

	D1	D2	D3	Supply
F1	5	1	8	15
F2	3	9	6	25
F3	4	2	7	30
F4	7	11	10	20
Demand	18	25	22	

Name : Arihant Kiran Tari

।। ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ।।
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VIVEKANAND COLLEGE, KOLHAPUR

(An Empowered Autonomous Institute)

Assignment

Student's Sign :	Patil
Seat No./ Roll No. :	8235
Seat No./ Roll No. In words	Eight Two Three Five.
1+2=3	11527

04+08+08 = 20/20

प्र. क्र. Q. No.

Q.1

i. b] 0 — 01

ii. c] unitary space — 01

iii. b] subspace of w. — 01

iv. d] $\bar{a}\langle \bar{u}\, v\rangle$. — 01

04

02	Section	Q. No.	03
		Marks	04 + 04 = 08

प्र. क्र. Q. No.

Q 3.
ii) By Gram schmidt process,

Given, $v_1 = (1,0,0)$, $v_2 = (1,1,1)$, $v_3 = (1,2,3)$

Step I : $\omega_1 = v_1 = (1,0,0)$

Step II : $\omega_2 = v_2 - \frac{\langle v_2, v_1 \rangle}{\langle v_1, v_1 \rangle} . v_1$

$= v_2 - \frac{\langle v_2, \omega_1 \rangle}{\langle \omega_1, \omega_1 \rangle} . \omega_1$

$\langle v_2, \omega_1 \rangle = \cancel{(1,1,1)}\ 1(1) + 1(0) + 1(0) = 1$
$\langle \omega_1, \omega_1 \rangle = 1(1) + 0(0) + 0(0) = 1$

$= (1,1,1) - \frac{1}{1}(1,0,0)$

04

$\omega_2 = (0,1,1)$

$\omega_3 = \cancel{\omega_3} - \frac{\langle v_3, v_2 \rangle}{\langle v_2, v_2 \rangle} v_2 - \frac{\langle v_3, \cancel{\omega_1} \rangle}{\langle v_1, v_1 \rangle} . v_1$

$= v_3 - \frac{\langle v_3, \omega_2 \rangle}{\langle \omega_2, \omega_2 \rangle} . \omega_2 - \frac{\langle v_3, \omega_1 \rangle}{\langle \omega_1, \omega_1 \rangle} . \omega_1$

$\langle v_3, \omega_2 \rangle = 1(0) + 2(1) + 3(1) = 5$
$\langle \omega_2, \omega_2 \rangle = 0(0) + 1(1) + 1(1) = 2$

$\langle v_3, \omega_1 \rangle = 1(1) + 2(0) + 3(0) = 1$
$\langle \omega_1, \omega_1 \rangle = 1(1) + 0(0) + 0(0) = 1$

Section	Q. No.													03
	Marks													

क्र. No.

$= (1,2,3) - \frac{5}{2}(0,1,1) - \frac{1}{1}(1,0,0)$

$= (1,2,3) - \left(0, \frac{5}{2}, \frac{5}{2}\right) - (1,0,0)$

$w_3 = \left(0, \frac{-1}{2}, \frac{1}{2}\right)$

$\therefore \{w_1 = (1,0,0), w_2 = (0,1,1), w_3 = \left(0, -\frac{1}{2}, \frac{1}{2}\right)\}$

is a orthagonal set.

$||w_1|| = \sqrt{1^2+0^2+0^2}$

$= \sqrt{1}$

$= 1$

$||w_2|| = \sqrt{0^2+1^2+1^2}$

$= \sqrt{2}$

$||w_3|| = \sqrt{0^2+(-\frac{1}{2})^2+(\frac{1}{2})^2}$

$= \sqrt{\frac{1}{4}+\frac{1}{4}}$

$= \sqrt{2/4}$

$= \frac{1}{\sqrt{2}}$

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		Marks											

प्र. क्र.
Q. No.

$\therefore \omega = \langle 1, \sqrt{2}, 1/\sqrt{2} \rangle$
$\therefore$ is a orthonormal set.

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VIVEKANAND COLLEGE, KOLHAPUR
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Assignment

Student's Sign :	Patil
Seat No./ Roll No. :	8235
Seat No./ Roll No. In words	
	11523

Q. 2
08

प्र. क्र.
Q. No.

Q. 2.
iii. Given that $\mathbb{R}(\varphi)$.

i) Closure property:
$a, b \in \mathbb{R}$ $1, 2 \in \mathbb{R}$.
$\Rightarrow 1 + 2 = 3 \in \mathbb{R}$
$\therefore$ closure property holds.

ii) Associative Property:
$a+(b+c) = (a+b)+c$ &
Now, $1, 2, 3 \in \mathbb{R}$.
$1 + (2+3) = (1+2)+3$
$1+5 = 3+3$
$6 = 6 \in \mathbb{R}$.
$\therefore$ Associativity holds.

iii] Identity Property:
for, $a \in \mathbb{R}$.
such that $a.+0. = a = 0.+a$
$\therefore 0$ is identity element
$\therefore$ Identity holds.

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		Marks												

प्र. क्र.
Q. No.

iv] Existence of inverse:
for $a \in \mathbb{R}$ $\exists$ $-a \in \mathbb{R}$.
such that,
$a + (-a) = 0 = (-a) + a$ $\forall$ $a \in \mathbb{R}$.
$\therefore -a$ is invese of a.
$\therefore$ inverse exist.

v] Commutativity Property:
For, $a, b \in \mathbb{R}$ There exist.
$a + b = b + a \in \mathbb{R}$.

$\therefore$ Commutativity holds.

$\therefore$ From I, II, III, IV, V
$\mathbb{R}(\mathbb{Q})$ is abelian Group.

vi] To Check External Composition:
Now,
$\alpha \in \mathbb{F}$ & $x, y \in \mathbb{R}$

$\therefore$ $\frac{1}{2} \in \mathbb{Q}$ & $2 \in \mathbb{R}$

$\Rightarrow$ $\frac{1}{2} \cdot 2 = 1 \in \mathbb{R}$

vii] To check Internal Composition.

Section	Q. No.													
	Marks													03

प्र. क्र.
Q. No.

i) Let, $\alpha \in \mathbb{F}$ & $x, y \in \mathbb{R}$.

$\therefore \frac{1}{2} \in \mathbb{F}$ and $1, 2 \in \mathbb{R}$.

$\Rightarrow$ L.H.S $= \alpha(x+y) = \frac{1}{2}(1+2)$

$= \frac{3}{2}$

R.H.S $= \alpha(x) + \alpha(y) = \frac{1}{2}(1) + \frac{1}{2}(2)$

$= \frac{1}{2} + 1$

$= \frac{3}{2}$

$\therefore$ L.H.S $=$ R.H.S.

$\therefore \alpha(x+y) = \alpha(x) + \alpha(y)$.

ii) Let, $\alpha, \beta \in \mathbb{F}$ & $x \in \mathbb{R}$.

$\frac{1}{2}, \frac{3}{4} \in \mathbb{F}$ & $4 \in \mathbb{R}$.

$\Rightarrow$ L.H.S. $= (\alpha+\beta)x = \left(\frac{1}{2} + \frac{3}{4}\right) 4$

$= \frac{5}{4} \cdot 4$

$= 5$

R.H.S. $\alpha(x) + \beta(x) = \frac{1}{2}(4) + \frac{3}{4} \cdot (4)$

$= 2 + 3$

$= 5$

04	Section	Q. No.											
		Marks											

प्र. क्र.
Q. No.

$\therefore$ L.H.S = R.H.S.

$\therefore (\alpha+\beta)(x) = \alpha(x) + \beta(x)$.

iii) Let, $\alpha, \beta \in \mathbb{F}$ & $x \in \mathbb{R}$.

such that $\frac{1}{2}, \frac{3}{4} \in \mathbb{F}$ & $2 \in \mathbb{R}$.

$\Rightarrow (\alpha\beta)x = \cancel{\left(\frac{3}{8}\right)} 2 \quad \alpha \quad \beta(x)$

$\left(\frac{3}{8}\right) 2 = \frac{1}{2}\left[\frac{3}{4}(2)\right]$

$\frac{3}{4} = \frac{3}{4}$

$\therefore (\alpha\beta)x = \alpha\ \beta(x)$.

iv) Let, $x \in \mathbb{R}$ such that.

$\therefore 10 \in \mathbb{R}$.

$\Rightarrow 1 \cdot x = x$.

8 $\Rightarrow 1 \cdot 10 = 10$.

$\therefore$ from i, ii, iii, iv, v, vi, vii

$\mathbb{R}(Q)$ is a vector space.

|| ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ||
– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

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Assignment

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	11529

क्र. No.

Q.3.iii) Given,

$T : V \longrightarrow U$ be L.T

by defn

$\ker T = \{ v \in V \mid T(v) = 0 \}$

show that.

T is one-one.

Now, $x, y \in V$.

$\Rightarrow T(x) = T(y)$

$\Rightarrow \cancel{0 = 0}$

$\Rightarrow T(x) - T(y) = 0$

$\Rightarrow T(x-y) = 0$

$\Rightarrow x - y = 0$

04 $\Rightarrow x = y$

$\therefore$ T is one-one.

Conversly,

we have, T is one-one

prove that, $\ker(T) = \{0\}$

Consider, $u \in \ker(T)$

by defn.

$\ker T = \{ u \in V \mid T(u) = 0 \}$

$T(u) = T(0)$

$T(u) = 0$

02	Section	Q. No.										
		Marks										

प्र. क्र.
Q. No.

$\implies u = 0 \quad u \in \ker T.$

$\therefore \ker T = \{0\}.$

hence, proved.