



"Education for Knowledge, Science, and Culture"

- Shikshanmaharshi Dr. Bapuji Salunkhe

Shri Swami Vivekanand Shikshan Sanstha's

Vivekanand College, Kolhapur

(Empowered Autonomous)



DEPARTMENT OF MATHEMATICS

Date: 13/09/2025

Notice

M.Sc. I (Sem I) & M.Sc. II (Sem III)

Internal Examination: 2025-26

All the students of M.Sc. I (Sem I) and M.Sc. II (Sem III) are hereby informed that their internal examination will be conducted from **24/09/2025 to 30/09/2025**. The examination will be conducted only one time, students are directed to attend the examination without fail. Syllabus, timetable & Question paper pattern for examination will be mentioned in following table.

Syllabus for M.Sc. I Sem I:

Sr. No.	Name of the Paper	Topics
1	DSC13MAT11: Modern Algebra	UNIT 1
2	DSC13MAT12: Ordinary Differential Equations	UNIT 1
3	DSC13MAT13: Measure & Integration	UNIT 1&2
4	DSC13MAT14: Numerical Analysis I	UNIT 1
5	DSE13MAT11: Operational Research	UNIT 1
6	RMD13MAT11: Research Methodology	UNIT 1

Syllabus for M.Sc. II Sem III:

Sr. No.	Name of the Paper	Topics
1	DSC13MAT31: Functional Analysis	UNIT 1&2
2	DSC13MAT32: Classical Mechanics	UNIT 1
3	DSC13MAT33: Complex Analysis	UNIT 1&2
4	DSC13MAT34: Advanced Discrete Mathematics	UNIT 1
5	DSE13MAT31: Lattice Theory	UNIT 1&2

Timetable:

Day and Date	Class	Time	Subject
Wednesday, 24/09/2025	M.Sc. I	12:00 PM to 01:00 PM	Modern Algebra
	M.Sc. II	12:00 PM to 01:00 PM	Functional Analysis
Thursday, 25/09/2025	M.Sc. I	12:00 PM to 01:00 PM	Ordinary Differential Equations
	M.Sc. II	12:00 PM to 01:00 PM	Classical Mechanics
Friday, 26/09/2025	M.Sc. I	12:00 PM to 01:00 PM	Measure & Integration
	M.Sc. II	12:00 PM to 01:00 PM	Complex Analysis
Saturday, 27/09/2025	M.Sc. I	12:00 PM to 01:00 PM	Numerical Analysis I
	M.Sc. II	12:00 PM to 01:00 PM	Advanced Discrete Mathematics
Monday, 29/09/2025	M.Sc. I	12:00 PM to 01:00 PM	Operational Research
	M.Sc. II	12:00 PM to 01:00 PM	Lattice Theory
Tuesday, 30/09/2025	M.Sc. I	12:00 PM to 01:00 PM	Research Methodology

***Note: All the lectures on the internal exam day will be conducted at 02:00PM to 4:00 PM. Everyone should attend the lectures.**

Nature of Question Paper

Time :- 1 Hour

Total Marks: 20

Q.1) Choose the correct alternative for each of the following.

[04]

i)

a) b) c) d)

ii)

a) b) c) d)

iii)

a) b) c) d)

iv)

a) b) c) d)

Q.2) Attempt any one

[08]

i)

ii)

Q.3) Attempt any two

[08]

i)

ii)

iii)

S. P. Thorat

(Dr. S. P. Thorat)

HEAD

DEPARTMENT OF MATHEMATICS
VIVEKANAND COLLEGE, KOLHAPUR
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Q.1. Select the correct alternative from each of the following.

[04]

i) Consider the following Statements:

I) Every principle series is composition series.

II) Every normal series is principle series.

A) Only I true

B) Only II true

C) Both I and II are true

D) Both I and II are false

ii) If $|G| = p$ or p^2 where p is prime then G is....

A) Non-abelian

B) Cyclic

C) Solvable

D) Both B) & C)

iii) Which of the following group of integers is not simple group?

A) \mathbb{Z}_5

B) \mathbb{Z}_2

C) \mathbb{Z}_6

D) \mathbb{Z}_{11}

iv) A subnormal series of group G is said to be composition series if all factor groups of series are...

A) Abelian

B) Simple

C) Non-abelian

D) None of these

Q.2. Attempt any one.

[08]

i) Let G' be a commutator subgroup of group G then show that,

a) G' is normal subgroup of G .

b) $\frac{G}{G'}$ is abelian group.

c) $\frac{G}{N}$ is abelian group iff G' is subgroup of N .

ii) State and prove Schrier Refinement Theorem.

Q.3. Attempt any two.

[08]

i) If H is subgroup of group G with index 2 then show that H is normal subgroup of G .

ii) If $\phi: G_1 \rightarrow G_2$ is onto homomorphism and if G_1 is solvable then show that $G_2 = \phi(G_1)$ is solvable.

iii) Show that the symmetric group S_n is a group w. r. to mapping composition.

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Department of Mathematics

Internal Examination :2025-2026

M.Sc. I (Sem I)

Course Name: Ordinary Differential Equation

Course Code:DSC13MAT12

Day and Date: Thursday, 25/09/2025

Time : 12:00PM to 01:00PM

Total marks : 20

Q.1. Select the correct alternative.

i) If $\phi_1(x)$ and $\phi_2(x)$ are two solutions of $L(y) = y'' + a_1(y') + a_2(y) = 0$ on an interval I containing point x_0 then $w(\phi_1, \phi_2)(x) = \dots w(\phi_1, \phi_2)(x_0)$.

- A) $e^{-a_1(x-x_0)}$ B) $e^{a_1(x-x_0)}$ C) $e^{-a_0(x_0-x)}$ D) $e^{a_0(x_0-x)}$

ii) The order and degree of equation $\left(\frac{d^2y}{dx^2}\right) = \left(y + \left(\frac{dy}{dx}\right)^6\right)^{1/6}$ is.... respectively.

- A) 2,1 B) $2, \frac{1}{6}$ C) 2,6 D) 2,3

iii) Wronskian of the two solutions of differential equation $y'' + a_1(x)y' + a_2(x)y = 0$ on an interval I is..

- A) Identically zero B) Never zero
C) Always constant D) either identically zero or never zero

iv) The functions $\phi_1(x) = \sin(x)$ and $\phi_2(x) = e^{ix}$ are on interval $[-\infty, \infty]$.

- A) Linearly dependent B) Linearly independent
C) Both A & B D) None of these

Q.2. Attempt Any One of the following.

1) Define Wronskian and find all the solutions of $y'' + 4y = \cos x$.

2) If $b(x)$ be continuous function on an interval I every solution ϕ of $L(y) = b(x)$ on I can be written as $\phi = \phi_p + C_1\phi_1 + C_2\phi_2$ where ϕ_p is particular solution and ϕ_1 and ϕ_2 are linearly independent solution $L(y) = 0$ and C_1, C_2 are constants and a particular solution ϕ_p is given by

$$\phi_p = \int_{x_0}^x \frac{\phi_1(t)\phi_2(x) - \phi_2(t)\phi_1(x)}{W(\phi_1, \phi_2)(x)} b(t) dt, \text{ conversely Every such solution is } \phi \text{ is the solution of } L(y) = b(x)$$

Q.3. Attempt Any Two of the following.

i) Check whether functions e^x, e^{2x}, e^{3x} are linearly independent or not.

ii) Find the solutions of initial value problem $y'' - 5y' + 6y = 0$ with $y(0) = 0, y'(0) = 1$.

iii) Show that every solution of constant coefficient equation $L(y) = y'' + a_1y' + a_2y = 0$ tends to zero as $x \rightarrow \infty$ iff the real part of roots of characteristics polynomial are negative.

Vivekanand college Kolhapur (An Empowered Autonomous Institute)
M.Sc. I (Sem I) Internal Examination :2025-2026
Operation Research

Course code: DSE13MAT11
Total marks: 20

Day and Date: Monday 29/09/2025
Time: 12:00PM to 1:00 PM

Q1. Select the correct alternative.

[04]

- (i) If $y \subset R^n$ then the smallest convex set containing y is called__
(a) convex function (b)convex set (c) convex hull (d) convex combination
- (ii) Which of the following is a property of a convex set?
(a) The set contains all possible linear combinations of its elements.
(b) The set contains all possible convex combinations of its elements.
(c) Every point in the set is a boundary point.
(d) The set contains no interior points.
- (iii) In the matrix form of an LPP, $AX \leq b$, what does the matrix A represent?
(a) The matrix of decision variables.
(b) The matrix of cost coefficients.
(c) The matrix of constraint coefficients.
(d) The matrix of slack variables
- (iv)The extreme points of cube are__
(a)4 (b)2 (c)8 (d)16

Q2. Attempt any one.

[08]

- (i) Define Convex combination and let S and T be convex set in R^n , then $\alpha S + \beta T$ is also convex set.
(ii) Define feasible solution and show that set of feasible solution to LPP is convex set.

Q3. Attempt any two.

[08]

- (i) Show that $S = \{(x_1, x_2, x_3), 2x_1 - x_2 + x_3 \leq 4\}$ is convex set.
(ii) Find the basic feasible solution:
Max (z) = $x_1 - 12x_2$
Subject to: $x_1 + x_2 \leq 10$
 $2x_1 - x_2 \leq 40; x_1, x_2 \geq 0$
- (iii) Rewrite in standard form the following LPP
Min(z) = $2x_1 + x_2 + 4x_3$
Subject to: $-2x_1 + 4x_2 \leq 4$
 $x_1 + x_3 \geq 5$
 $2x_1 + 3x_3 \leq 2; x_1, x_2 \geq 0, x_3$ is unrestricted in sign

Vivekanand College Kolhapur (An Empowered Autonomous Institute)
Department of Mathematics

M.Sc.-I (Sem-I) Internal Examination 2025-26

Course Name: Research Methodology

Course Code: ~~DSC13MAT~~

Day & Date: Tuesday, 30/09/2025

Marks: 20

RMD13MAT11

Time: 01:00PM to 02 :00PM

Q.1. Select the correct alternative from each of the following.

[04]

- i) A corollary is easy consequences of.....
A) lemma B) preposition C) theorem D) All the above
- ii) The purpose.....is to summarise the concept of the paper.
A) Definition B) Abstract C) Title D) Keywords
- iii) is the person who did the greatest part.
A) First B) last C) third D) Senior person.
- iv) In mathematics the use of article.....is unappropriated when the object to which it refers is not unique.
A) A B) An C) the D) None of these

Q.2. Attempt any one.

[08]

- i) While writing paper how to finalize title author list.
ii) What are the Do's and Don'ts of mathematical writing.

Q.3. Attempt any two.

[08]

- i) How to use notations in mathematical writing?
ii) How should you determine Audience while writing a paper?
iii) Write short note on what is theorem.

Vivekanand College Kolhapur (An Empowered Autonomous Institute)
Department of Mathematics
Internal Examination 2025-26
M.Sc.-I (Sem-I)

Course Name: Numerical Analysis-I
 Day & Date: Saturday, 27/09/2025
 Time: 12:00PM to 1:00PM

Course Code: DSC13MAT14
 Marks: 20

Q.1. Select the correct alternative from each of the following.

[04]

i) In Bairstow method $\Delta q = \text{-----}$

A) $\Delta q = \frac{b_n c_{n-2} + b_{n-1} (b_{n-1} - c_{n-1})}{c^2_{n-2} + c_{n-3} (b_{n-1} - c_{n-1})}$

B) $\Delta q = \frac{b_{n-2} c_n + b_{n-1} (b_n - c_n)}{c^2_{n-2} + c_{n-3} (b_n - c_n)}$

C) $\Delta q = \frac{b_n c_n + b_{n-1} (b_{n-2} - c_{n-2})}{c^2_n + c_{n-3} (b_{n-1} - c_{n-1})}$

D) $\Delta q = \frac{b_{n-1} c_{n-2} - b_n c_{n-3}}{c^2_{n-2} + c_{n-3} (b_{n-1} - c_{n-1})}$

ii) If $f(x)$ is continuous function in the interval $[a, b]$, $f(a) \cdot f(b) < 0$ then the equation $f(x) = 0$ has atleast one real root or an odd number of real roots in (a, b) is called.....

- A) Bisection Method
 B) Direct method

- B) Iterative method
 D) Intermediate Value theorem

iii) The rate of convergence of Bisection method is...

A) 0

B) 1

C) 3

D) 2

iv) An iterative method is said to be of order p , if p is the largest positive real number for which there exists a finite constant $c \neq 0$ such that...

- A) $|\epsilon_{k+1}| \geq c |\epsilon_k|^p$
 B) $|\epsilon_{k+1}| \leq c |\epsilon_k|^p$

- B) $|\epsilon_k| \neq c |\epsilon_k|^p$
 D) $|\epsilon_k| > c |\epsilon_k|^p$

Q.2. Attempt any one.

[08]

i) Determine the Rate of convergence of Secant method.

ii) Perform two iterations of Bairstow method to extract quadratic factor

$x^2 + px + q$ from $P_3(x) = x^3 + x^2 - x + 2$. Use initial approximations $p_0 = -0.9$ and $q_0 = 0.9$

Q.3. Attempt any two.

[08]

i) Perform 4 iterations of the Newton Raphson Method to find the smallest positive root of the equation $f(x) = x^3 - 5x + 1 = 0$. (Taking smallest positive root lies in the interval $(0,1)$).

ii) Use Secant method to determine roots of equation $\cos x - xe^x = 0$. Do four iterations taking initial approximations $x_0 = 0$ and $x_1 = 1$.

iii) Determine the Rate of convergence of Regula Falsi method.

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Department of Mathematics

Internal Examination 2025-26

M.Sc.-I (Sem-I)

Course Name: Measure & Integration

Day & Date: Friday, 26/09/2025

Time: 12:00PM to 1:00PM

Course Code: DSC13MAT13

Marks: 20

Q.1. Select the correct alternative from each of the following.

[04]

i) If A is singleton set the $m^*(A) = \dots$

A) 0

B) 1

C) 2

D) -1

ii) Consider the following statements:

I) Every countable set is Borel set.

II) A Set of Real number a is Borel set.

A) Only I true

B) Only II true

C) Both I & II true D) Both I & II are false.

iii) If A is measurable set then the complement A^c is.....

A) non-measurable

B) measurable

C) finite

D) uncountable

iv) A set F is F_σ if it is.....

A) Countable union of open sets

B) Countable intersection of open sets

C) Countable intersection of closed sets

D) Countable union of closed sets

Q.2. Attempt any one.

[08]

i) Prove that outer measure of an interval is equal to its length.

ii) Define σ -algebra. Prove that there exists a smallest σ -algebra containing a given collection of subsets.

Q.3. Attempt any two.

[08]

i) Show union of finite collection of measurable sets is measurable

ii) Show that outer measure is translation invariant.

iii) Give an example of uncountable set with outer measure zero.

Samruddhi Gunda Magdum



॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥

- शिक्षणमहर्षी डॉ. बापूजी साळुंखे

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Student's Sign :

Seat No./ Roll No. : 1205

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In words zero five

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VIVEKANAND COLLEGE, KOLHAPUR

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Assignment

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Q1.

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Q. No.

1) A) $e^{-a_1(x-x_0)}$

2) C) 2.6

3) D) Either identically zero or never zero

4) B) Linearly independent

04

02	Section	Q. No.																	
		Marks																	

Q2

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Q. No.

1) Wronskian :

Let, $\phi_1(x)$ and $\phi_2(x)$ are two elements the interval I , the wronskian can be defined as

$$W(\phi_1, \phi_2)(x) = \begin{vmatrix} \phi_1(x) & \phi_2(x) \\ \phi_1'(x) & \phi_2'(x) \end{vmatrix}$$

$$= \phi_1(x) \cdot \phi_2'(x) - \phi_1'(x) \cdot \phi_2(x)$$

The solutions are linearly independent

⇒ Given,

$$y'' + 4y = \cos x \quad \text{--- (I)}$$

the given eqⁿ is in the form of $L(y) = b(x)$

The char. poly eqⁿ of is,

$$r^2 + 4 = 0$$

put,

$$r^2 + 4 = 0$$

$$r^2 + 4 = 0$$

$$r = \pm 2i$$

The particular eqⁿ of $L(y) = 0$ is,

$$\psi_p(x) = C_1 \cos 2x + C_2 \sin 2x$$

$$\therefore \psi_p(x) = C_1 \phi_1(x) + C_2 \phi_2(x),$$

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Q. No.

$$\text{Here, } \phi(x) = c_1 \phi_1(x) + c_2 \phi_2(x)$$

$$\Rightarrow \phi(x) = c_1 \cos 2x + c_2 \sin 2x$$

$$\therefore \phi_1(x) = \cos 2x \quad \text{and} \quad \phi_2(x) = \sin 2x$$

The general particular solⁿ of $L(y)=0$

$$\psi_p(x) = u_1 \phi_1(x) + u_2 \phi_2(x)$$

Here,

$$u_1 = - \int \frac{\phi_2(x) \cdot b(x)}{w(\phi_1, \phi_2)(x)} \quad \text{and} \quad u_2 = \int \frac{\phi_1(x) \cdot b(x)}{w(\phi_1, \phi_2)(x)}$$

Here

$$w(\phi_1, \phi_2)(x) = \begin{vmatrix} \phi_1'(x) & \phi_2'(x) \\ \phi_1(x) & \phi_2(x) \end{vmatrix}$$

$$= \begin{vmatrix} \cos 2x & \sin 2x \\ -2 \sin 2x & 2 \cos 2x \end{vmatrix}$$

$$= 2 \cos^2 2x + 2 \sin^2 2x$$

$$= 2(1)$$

$$w(\phi_1, \phi_2)(x) = 2$$

$$\text{Now, } u_1 = - \int \frac{\sin 2x \cdot \cos x}{2} dx$$

$$= -\frac{1}{2} \int \sin 2x \cdot \cos x dx$$

$$= -\frac{1}{4} \int 2 \sin 2x \cdot \cos x dx$$

04

Section

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Q. No.

$$u_1 = -\frac{1}{4} \int [\sin(2x+x) + \sin(2x-x)] dx$$

$$= -\frac{1}{4} \int (\sin 3x + \sin x) dx$$

$$u_1 = -\frac{1}{4} \left[\frac{-\cos 3x}{3} + (-\cos x) \right]$$

$$u_1 = \frac{1}{4} \left[\frac{\cos 3x}{3} + \cos x \right]$$

Now,

$$u_2 = \int \frac{\cos 2x \cdot \cos x}{2} dx$$

$$= \frac{1}{4} \int 2 \sin 2x \cdot \cos x dx$$

$$= \frac{1}{4} \int [\cos 3x + \cos x] dx$$

$$u_2 = \frac{1}{4} \left[\frac{\sin 3x}{3} + \sin x \right]$$

Now,

$$\psi_p(x) = u_1 \phi_1(x) + u_2 \phi_2(x)$$

$$= \frac{1}{4} \left[\frac{\cos 3x + \cos x}{3} \right] \cos 2x + \frac{1}{4} \left[\frac{\sin 3x + \sin x}{3} \right]$$

$$= \frac{1}{4} \left[\frac{\cos 3x \cdot \cos 2x}{3} + \cos x \cdot \cos 2x \right]$$

$$+ \left[\frac{\sin 3x \cdot \sin 2x}{3} + \sin x \cdot \sin 2x \right]$$

Samruddhi Gunda Madum

॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥

- शिक्षणमहर्षी डॉ. बापूजी सालुंखे



VIVEKANAND COLLEGE, KOLHAPUR

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Assignment

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Seat No./ Roll No. : 1205

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Q2.

क्र. No.

$$1) \psi_p(x) = \frac{1}{4} \left[\frac{\cos 3x \cdot \cos 2x + \sin 3x \cdot \sin 2x}{3} + \frac{\cos x \cdot \cos 2x + \sin x \cdot \sin 2x}{3} \right]$$

$$= \frac{1}{4} [\cos(3x) + \cos(5x)]$$

$$= \frac{1}{4} \left[\frac{\cos x + \cos 2x}{3} \right]$$

$$= \frac{1}{4} \left[\frac{4 \cos x}{3} \right]$$

$$\psi_p(x) = \frac{\cos x}{3}$$

The particular solution of eqⁿ is,

$$\psi_p(x) = \psi_p(x) + C_1 \phi_1(x) + C_2 \phi_2(x)$$

$$= \frac{\cos x}{3} + C_1 \cos 2x + C_2 \sin 2x$$

8

02	Section	Q. No.												
		Marks												

Q3.

प्र. क्र.
Q. No.

↳ let, $\phi_1(x) = e^x$, $\phi_2(x) = e^{2x}$, $\phi_3(x) = e^{3x}$

Here,

$$W(\phi_1, \phi_2, \phi_3)(x) = \begin{vmatrix} \phi_1(x) & \phi_2(x) & \phi_3(x) \\ \phi_1'(x) & \phi_2'(x) & \phi_3'(x) \\ \phi_1''(x) & \phi_2''(x) & \phi_3''(x) \end{vmatrix}$$

$$= \begin{vmatrix} e^x & e^{2x} & e^{3x} \\ e^x & 2e^{2x} & 3e^{3x} \\ e^x & 4e^{2x} & 9e^{3x} \end{vmatrix}$$

$$= e^x (18e^{2x} \cdot e^{3x} - 12e^{2x} \cdot e^{3x}) - e^{2x} (9e^{3x} - 3e^x \cdot e^{3x}) + e^{3x} (4e^x \cdot e^{2x} - 2e^x \cdot e^{2x})$$

$$= e^x (6e^{2x} \cdot e^{3x}) - e^{2x} (6e^x \cdot e^{3x}) + e^{3x} (2e^x \cdot e^{2x})$$

$$= 6e^{6x} - 6e^{6x} + 2e^{6x}$$

$$= 2e^{6x}$$

$$W(\phi_1, \phi_2, \phi_3)(x) \neq 0$$

Here, the given functions are linearly independent.

04	Section	Q. No.												
		Marks												

प्र. क्र.
Q. No.

Here, $C_1 = 1$
and, $C_2 = -1$

The particular eqⁿ is,

$$\phi(x) = C_1 e^{3x} + C_2 e^{2x}$$

$$\phi(x) = 1e^{3x} - e^{2x}$$

$$\phi(x) = e^{3x} - e^{2x}$$

4

$$\begin{aligned} \cos A \cdot \cos B + \sin A \cdot \sin B &= \cos(A+B) \\ \cos A \cdot \cos B - \sin A \cdot \sin B &= \cos(A-B) \end{aligned}$$

. Attempt any two of the following.

[8]

If $\{T_n\}$ and $\{S_n\}$ are sequences in $B(N)$ such that $T_n \rightarrow T$ and $S_n \rightarrow S$ as $n \rightarrow \infty$ then show that,

a) $T_n + S_n \rightarrow T + S$ b) $kT_n \rightarrow kT$ for k in F c) $T_n S_n \rightarrow TS$ as $n \rightarrow \infty$

ii) State and prove that Riesz Lemma

iii) Prove that nls N is separable if it's Conjugate Space N^* is.

iv) If N is a normed linear space and x_0 is non zero vector in N then show that there exist a functional f_0 in N^* such that $f_0(x_0) = \|x_0\|$ and $\|f_0\| = 1$

Vivekanand College Kolhapur (An Empowered Autonomous Institute)

Department of Mathematics

Internal Examination 2025-26

M.Sc.-II (Sem-III)

Course Name: Lattice Theory

Course Code: DSE13MAT31

Day & Date: Monday, 29/09/2025

Marks: 20

Time: 12:00PM to 1:00PM

Q.1. Select the correct alternative from each of the following.

[04]

i) Consider the statements.

I) Every Maximal antichain is maximum.

II) Every Maximum element is maximal.

A) Only I true

B) Only II true

C) Both I & II are true

D) Both I & II are False

ii) Which of the following is not partial ordered relation?

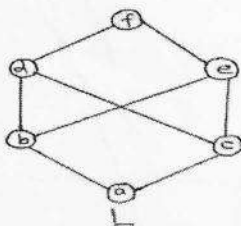
A) The relation 'less than equal to (\leq)' defined on set of natural numbers.

B) The relation \subseteq of a set inclusion defined on a non-empty set.

C) The relation 'Divides ($|$)' defined on set of integers.

D) The relation strictly less than ($<$) defined on a set of natural numbers

iii) The Hasse Diagram given below is an example of.....



A) Lattice

B) Semilattice

C) Non-lattice poset

D) None of these

iv) In the poset $\langle \mathbb{Z}^+, | \rangle$ where \mathbb{Z}^+ is the set of positive integers & $|$ is divides relation then 3 & 18 are.....

A) Comparable

B) Parallel

C) Both A) & B)

D) Neither A nor B)

Q.2. Attempt any one.

[08]

i) Prove that I is prime ideal if and only if there is homomorphism of L onto C_2 with $I = \phi^{-1}(0)$, $I = \{x \in L | \phi(x) = 0\}$.

ii) Define Poset. Prove that if a poset satisfies ACC then it has a maximal element.

Q.3. Attempt any two.

[08]

i) Prove that every homomorphic image of lattice L is isomorphic to a suitable quotient lattice of L .

ii) Show that $I(L)$ is a lattice under set inclusion

iii) Prove that the algebra $\langle L, \wedge, \vee \rangle$ be a lattice and $a \leq b$ if and only if $a = a \wedge b$ then $\langle L, \leq \rangle$ is a poset and as a poset it is a lattice.

Vivekanand College Kolhapur (An Empowered Autonomous Institute)

Department Of Mathematics

Internal Examination 2025-26

M.Sc. II (Sem III)

Course Name: Complex Analysis

Day & Date : Friday, 26/09/2025

Course Code: DSC13MAT33

Marks: 20

Q.1. Select the correct alternative for each of the following.

[04]

i) The series $\sum_{n=0}^{\infty} \frac{z^n}{n^n}$ converges on.....

- A) $|z| \leq 3$ B) $|z| < 1$ C) $|z| < 1$ D) Whole Complex plane

ii) Consider the following statements:

I) Every Mobius transformation can have atmost 2 fixed points.

II) If $S(z) = az$ then S is inversion.

- A) Only I true B) Only II true C) Both I & II true D) Both I & II are false

iii) Consider the power series $\sum_{n \geq 0} z^{n!}$ then the radius of convergence of power series around the origin is....

- A) 0 B) 1 C) 3 D) 4

iv) If S and T mobius transformations then $S \circ T$ is....

- A) Bilinear Transformation B) Non-Bilinear Transformation
C) Mobius Transformation D) None.

Q.2. Attempt any one of the following.

[08]

i) State and prove Liouville's theorem.

ii) If z_1, z_2, z_3, z_4 be four distinct points in C_{∞} then show that cross ratio (z_1, z_2, z_3, z_4) is real number if and only if all four points lie on a circle.

Q.3. Attempt any two of the following.

[08]

i) Prove that $\int_0^{2\pi} \frac{e^{it}}{e^{it}-z} dt = 2\pi$ when $|z| < 1$.

ii) If S is Mobius transformation then prove that 'S' is composition of translation, dilation, and inversion.

iii) Evaluate the following integral

$$\int_{\vartheta} \frac{\sin z}{z^3} dz \quad \text{where } \vartheta(t) = re^{it}, \quad 0 \leq t \leq 2\pi$$

Vivekanand College Kolhapur (An Empowered Autonomous Institute)
Department of Mathematics
Internal Examination 2025-26
M.Sc.-II (Sem-III)

Course Name: Advanced Discrete Mathematics
 Day & Date: Saturday, 27/09/2025
 Time: 12:00PM to 1:00PM

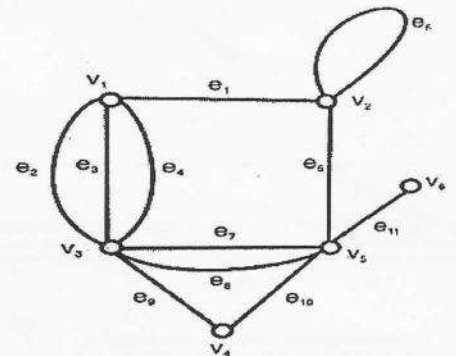
Course Code: DSC13MAT34
 Marks: 20

Q.1. Select the correct alternative from each of the following. [04]

- i) The join of two vertex disjoint complete graphs is a.....
 A) Simple graph B) Complete graph C) Complete bipartite graph D) Bipartite Graph
- ii) The radius and diameter of wheel graph $W_n, n \geq 5$ is.....respectively.
 A) 1,3 B) 1,1 C) 1,2 D) 2,2
- iii) Consider the statements
 I) Every path need not be a trail.
 II) Every complete graph is regular graph.
 A) Only I true B) Only II true C) Both I & II true D) Both I & II are false
- iv) A non-trivial closed trail is called.....
 A) path B) cycle C) tree D) walk

Q.2. Attempt any one. [08]

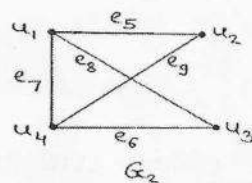
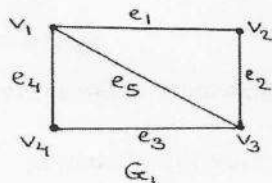
- i) Find the graphs
 a) $G-U, G-F$, where $U = \{v_4, v_5\}$ and $F = \{e_2, e_4, e_8, e_9, e_{10}\}$
 b) $G[U], G[F]$ and also find their union $G[G[U] \cup G[F]]$ where, $U = \{v_2, v_3, v_5\}$ and $F = \{e_1, e_3, e_7, e_9\}$



- ii) Define Underlying simple graph.
 a) Prove that in any graph G , there is even number of odd vertices.
 b) Prove that for any vertices u, v, w in $G, d(u, v) \leq d(u, w) + d(w, v)$.

Q.3. Attempt any two. [08]

- i) Prove that in any connected graph $G, \text{rad}G \leq \text{diam}G \leq 2 \text{rad}G$
 ii) Define Path & Trail. Prove that any two vertices of the graph G every $u-v$ walk contains a $u-v$ path.
 iii) Verify whether following two graphs are isomorphic or not.



Vivekanand college (An Empowered Autonomous Institute) Kolhapur

Department of Mathematics

M.Sc. II (Sem III) Internal Examination :2025-2026

Subject code: Classical Mechanics

Day and Date: Thursday, 25th september 2025

Total marks: 20

Time: 12.00pm -1.00pm

Q1. Select the correct alternative.

(4)

i) Kinetic energy of a particle of mass m and position vector \vec{r} in polar form is

A) $T = m (\dot{r}^2 + r^2 \dot{\theta}^2)$

B) $T = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2)$

C) $T = 2m (\dot{r}^2 + r^2 \dot{\theta}^2)$

D) $T = \dot{r}^2 + r^2 \dot{\theta}^2$

ii) Equation of constraints that do not contain time as explicit variable are referred as

A) holonomic constraints

B) non holonomic constraints

C) rheonomic constraints

D) scleronomic constraints

iii) The number of generalized co-ordinates in simple pendulum is.....

A) 1

B) 2

C) 3

D) 4

iv) If the system is conservative then

A) $p_j = \frac{\partial L}{\partial \dot{q}_j}$

B) $p_j = \frac{\partial T}{\partial \dot{q}_j}$

C) $p_j = -\frac{\partial L}{\partial \dot{q}_j}$

D) $p_j = \frac{\partial v}{\partial \dot{q}_j}$

Q2. Attempt any one

(8)

- i) Obtain Lagrange's equations of motion from D'Alembert's Principle.
- ii) If the cyclic generalized co-ordinate q_j is such that dq_j represents the translation of the system, then prove that the total linear momentum is conserved.

Q3. Attempt any two of following

(8)

- i) Show that the Lagrange's equation of motion can also be written as
$$\frac{\partial L}{\partial t} - \frac{d}{dt} \left(L - \sum_j \dot{q}_j \frac{\partial L}{\partial \dot{q}_j} \right) = 0$$
- ii) Show that the generalized momentum corresponding to cyclic co-ordinate is conserved
- iii) Find the equation of motions for Atwood machine.
- iv) Show that gravitational force is conservative.



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- शिक्षणमहर्षी डॉ. बापूजी साळुंखे

VIVEKANAND COLLEGE, KOLHAPUR

(An Empowered Autonomous Institute)

Assignment

Student's Sign : Pallavi

Seat No./ Roll No. : 2202

Seat No./ Roll No. Two two
In words

zero two 03082

$$03 + 08 + 08 = 19$$

प्र. क्र.
Q. No.

Q. 1)

~~i) B) only II true.~~

~~ii) D) The relation strictly less than ($<$) defined on set of natural numbers.~~

~~iii) A) Lattice.~~

~~iv) A) Comparable.~~

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02	Section	Q. No.	2											
		Marks	08											

प्र. क्र.
Q. No.

Q. 2)

ii) Let, $\langle P, \leq \rangle$ is partial ordered set (POSET) it satisfies following properties.

i) Anti-symmetry -

Let, $a, b \in P$ then,
 $a \leq b$ and $a \geq b \Rightarrow a = b. \quad \forall a, b \in P$

ii) Reflexivity -

$a \in P$ then
 $a \leq a \Rightarrow a \quad \forall a \in P$

iii) Transitivity -

$a, b, c \in P$ then
 $a \leq b$ and $b \leq c$ then $a \leq c \quad \forall a, b, c \in P$.

To prove - If poset satisfies Acc then it maximal element.

Let, $\langle P, \leq \rangle$ be poset. which satisfies Acc.

Let, $x_0 \in P$ be any element.

If x_0 is maximal element then we are done.

If x_0 is not maximal element then,

$\exists x_1 \in P$ such that
 $x_0 \leq x_1$.

If x_1 is maximal element then we are done.

If x_1 is not maximal element then

$\exists x_2 \in P$ such that
 $x_0 \leq x_1 \leq x_2$

Section	Q. No.																			
	Marks																			

Fr. No.

By continuing this process
 We get increasing chain of elements

$$x_0 \leq x_1 \leq x_2 \leq \dots \leq x_n$$

this increasing chain terminates the, $j \in W$,

$$x_i = x_{i+1} = x_{i+2} = \dots$$

If x_i is not equal to all remaining elements

i.e. x_i covers all the element of P then

x_i is maximal. and if not $\forall x_i$ is upper bound. then it has $x_i \leq y_0$

If not, there exist $y_0 \in P$, $x_i \neq y_0$ then.

If y_0 is maximal then we are done.

If y_0 is not maximal then

$\exists y_1 \in P$ such that

$$x_i \leq y_0 \leq y_1$$

If y_1 is maximal then we stop.

If y_1 is not maximal then $\exists y_2 \in P$ s.t. $x_i \leq y_0 \leq y_1 \leq y_2$.

By continuing this process.

We get increasing chain of elements.

$$x_i \leq y_0 \leq y_1 \leq y_2 \leq \dots \leq y_m$$

this increasing chain terminates the, $j \in W$, such that

$$y_j = y_{j+1} = y_{j+2} = \dots$$

If all the elements covered by y_j then y_j is maximal element and if not then it has upper bound.

Do the same argument for all possible chains of POSET.

By Zorn's lemma.

all poset satisfies ACC then it has maximal element

04	Section	Q. No.	3															
		Marks	$3\frac{1}{2}$	for	$= 7\frac{1}{2} = 08$													

प्र. क्र.
Q. No.

Q. 3)

ii)

Let, L be lattice and $I(L)$ be set of all ideals of lattice.
Claim - $I(L)$ is lattice under set inclusion.
as, ϕ , $\{0\}$ and L are always lattice.

Consider,

$I_1, I_2 \in I(L)$ be any two elements.

Case I):- $I_1 \subseteq I_2$.

$$\sup \{I_1, I_2\} = I_1 \cup I_2 = I_2$$

$$\inf \{I_1, I_2\} = I_1 \cap I_2 = I_1$$

Case II):- $I_2 \subseteq I_1$

$$\sup \{I_1, I_2\} = I_1 \cup I_2 = I_1$$

$$\inf \{I_1, I_2\} = I_1 \cap I_2 = I_2$$

In both the case $I(L)$ is lattice.

Case III):- Neither $I_1 \subseteq I_2$ nor $I_2 \subseteq I_1$

$$\inf \{I_1, I_2\} = I_1 \cap I_2$$

$$\inf \{I_1, I_2\} = I_1 \cap I_2 \text{ is exist } (\because \phi \text{ is lattice}).$$

$$\sup \{I_1, I_2\} = I_1 \cup I_2$$

By Absorption property,

$$\sup \{I_1, I_2\} = I_1 \cup I_2 \in L.$$

We define ϕ and L are always lattice

$$\sup \{I_1, I_2\} \text{ exist.}$$



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- शिक्षणमहर्षी डॉ. बापूजी साळुंखे

VIVEKANAND COLLEGE, KOLHAPUR

(An Empowered Autonomous Institute)

Assignment

Student's Sign : *Pallavi*

Seat No./ Roll No. : 2202

Seat No./ Roll No. Two two
In words

zero two 03049

from (iii) $I(L)$ is lattice.

By case I, II and III

$I(L)$ is lattice under set inclusion.

प्र. क्र.

Q. No.

02	Section	Q. No.																
		Marks																

प्र. क्र.
Q. No.

Q. 3) i) Let, L and L' be lattices.

L is isomorphic to L/θ (quotient lattice).

claim:- $L \cong L/\theta$.

$$\exists \{ [x]_\theta \} = \phi(x).$$

Define :- $\gamma : L \rightarrow \frac{L}{\theta}$

To prove, γ is isomorphic

i) We have to prove γ is well defined and one-to-one.
Let, $[x]_\theta, [y]_\theta \in L/\theta$.

$$[x]_\theta = [y]_\theta$$

$$\Leftrightarrow x = y$$

$$\Leftrightarrow \phi(x) = \phi(y)$$

$$\Leftrightarrow \gamma \{ [x]_\theta \} = \gamma \{ [y]_\theta \} \quad \dots (\because \text{def of } \gamma).$$

γ is well defined and one-to-one function.

ii)

To prove γ is onto.

Let, $[x]_\theta \in L/\theta$

$\phi(x) [x]_\theta = a \in L$ be any element.

$$\gamma \{ [x]_\theta \} = \phi(x) \quad \dots (\because \text{by def}^n \text{ of } \gamma)$$

$$\gamma \{ [x]_\theta \} = \phi(x) = a.$$

γ is onto.



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Shri Swami Vivekanand Shikshan Sanstha, Kolhapur.

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Notice

Internal Exam 2025-26

M.Sc. I & II (Sem II & IV)

All the students of M.Sc. I & II (Sem II & IV) are hereby informed that your internal examination of semester VI will be start from 16/03/2026 to 23/02/2026. The examination will be conducted only once. Attendance is mandatory, and no exceptions will be permitted. The syllabus, timetable, and pattern of question paper for the Internal will be mentioned as follows:

M.Sc. I

Sr. No.	Name of the Paper	Syllabus	Day & Date	Time
1	Linear Algebra	Unit 1	Monday, 16/03/2026	12:00 PM to 01:00 PM
2	General Topology	Unit 1	Tuesday, 17/03/2026	12:00 PM to 01:00 PM
3	Advance Calculus	Unit 1	Wednesday, 18/03/2026	12:00 PM to 01:00 PM
4	Numerical Analysis - II	Unit 1	Friday, 20/03/2026	12:00 PM to 01:00 PM
5	Number Theory	Unit 1	Monday, 23/03/2026	12:00 PM to 01:00 PM

M.Sc. II

Sr. No.	Name of the Paper	Syllabus	Day & Date	Time
1	Field Theory	Unit 1	Monday, 16/03/2026	12:00 PM to 01:00 PM
2	Integral Equations	Unit 1	Tuesday, 17/03/2026	12:00 PM to 01:00 PM
3	Partial Differential equations	Unit 1	Wednesday, 18/03/2026	12:00 PM to 01:00 PM
4	Combinatorics	Unit 1	Friday, 20/03/2026	12:00 PM to 01:00 PM

Question Paper Pattern

20 Marks

Q.1) Select the correct alternative.

4 Mcqs

[04]

Q.2) Attempt any one of the following.

2 Questions

[08]

Q.3) Attempt any two of the following.

3 Questions

[08]



S. P. Thorat

(Dr. S. P. Thorat)

HEAD

DEPARTMENT OF MATHEMATICS
VIVEKANAND COLLEGE, KOLHAPUR
(EMPOWERED AUTONOMOUS)

Vivekanand College Kolhapur (An Empowered Autonomous Institute)
Department of Mathematics
Internal Examination 2025-26
M.Sc.-II (Sem-III)

Course Name: Linear Algebra
Day & Date: Monday, 16/03/2026
Time: 12:00PM to 1:00PM

Course Code: DSC13MAT21
Marks: 20

Q.1. Select the correct alternative from each of the following.

[04]

- i) Which of the following is not a vector space?
A) $C(R)$ B) $R(Q)$ C) $R(R)$ D) $Q(R)$
- ii) If $W = \{0\}$ then $A(W) = \dots$
A) \hat{V} B) 0 C) \hat{W} D) W
- iii) Consider the following statements
I) A zero vector is always linearly dependent.
II) A non-zero vector is not always linearly independent.
A) Only I true B) Only II true
C) Both I and II true D) Both I and II false
- iv) $A(A(W))$ is a subspace of
A) V B) \hat{V} C) \hat{V} D) W

Q.2. Attempt any two of the following.

[08]

- i) If $v_1, v_2, \dots, v_n \in V$ are linearly independent vectors then prove that every element in their linear span has a unique representation in the form
 $\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n, \quad \alpha_i \in F.$
- ii) If V is finite dimensional vector space then show that, for $0 \neq v$ there is element f in dual space such that $f(v) \neq 0$.
- iii) If W_1 and W_2 be two subspaces of vector space which is finite dimensional then describe $A(W_1 + W_2)$ in terms of $A(W_1)$ and $A(W_2)$.

Q.3. Attempt any one of the following.

[08]

- i) Prove that if T be a homomorphism of $T: U \rightarrow V$ with W then U is isomorphic to $\frac{U}{W}$. Conversely U is a vector space and W is subspace of U then there exists homomorphism U onto $\frac{U}{W}$.
- ii) If W is finite dimensional subspace of finite dimensional vector space V then show that $\dim\left(\frac{V}{W}\right) = \dim V - \dim W$.

Vivekanand College (An Empowered Autonomous Institute) Kolhapur,
M.Sc. I (Sem II) Internal Examination 2025-26

Course Name : General Topology

Subject Code: DSC13MAT22

Day and Date:

Time: 12:00 to 1:00 PM

Total Marks: 20

Instruction:

1. All the questions are compulsory.
2. Figure to the right indicates full marks.

Q.1. Select the correct alternative for each of the following.

[04]

- i) Out of the followingdefines a topology on $X = \{a, b\}$
- A) $\{\emptyset, \{a\}, \{b\}\}$ B) $\{X, \{a\}, \{b\}\}$ C) $\{\emptyset, \{a\}\}$ D) $\{\emptyset, X\}$
- ii) In a topology, every open set can be expressed as.....
- A) union of some member of bases B) union of intersection of some member of subbases
C) intersection of bases D) intersection of union of some member of subbase
- iii) A is open if and only if
- A) $A^0 \neq X$ B) $A^0 = \emptyset$ C) $A^0 = A$ D) none of these
- vi) In..... topology every set is closure itself.
- A) Discrete B) Indiscrete C) Co-finite D) Co-countable

Q.2. Attempt any two of the following.

[08]

- i) Define Topology. If X be a non-empty set and τ consist $\{\emptyset\}$ and all those subset of X whose complement is countable $\tau_{cc} = \{\emptyset, G \subseteq X : G^c \text{ is countable}\}$.
- ii) Define the Limit point. If $X = \{a, b, c\}$ and let $\tau = \{\emptyset, X, \{b\}, \{b, c\}\}$, $A = \{b, c\}$ Find the limit point of A .
- iii) Define the following terms:
- a) Discrete topology b) Indiscrete topology c) Weaker and stronger topology

Q.3. Attempt any one of the following.

[08]

- i) Prove that let X be any non – empty set and B be family of some subset of τ . Then B base for τ on X if and only if
- a) $X = \cup \{B_i : B_i \in B\}$
b) $\forall B_1, B_2 \in B, \forall x \in B_1 \cap B_2 \exists B_3 \in B$ such that $x \in B_3 \subseteq B_1 \cap B_2$
- ii) State and prove the Left ray topology.

Q.1. Select the Correct Alternatives.

[04]

1) Consider the following statements

- I. The uniform convergence is sufficient condition for preserving continuity.
 II. The uniform convergence is necessary condition for preserving continuity.

A) Only I True. B) Only II True. C) Both (I) & (II) True D) Both (I) & (II) False.

2) If $g(p, q) = \frac{p}{p+q}$ then

- A) Double limit exists. B) Double limit does not exist
 C) Both iterated limits exist D) Both B) & C)

3) The series $\sum_{n=1}^{\infty} \frac{x}{n^{\beta}(1+nx^2)}$

- A) Converges uniformly on any finite interval if $\beta > 1/2$
 B) Converges uniformly on any finite interval if $\beta \leq 1/2$
 C) Converges uniformly on any finite interval if $\beta < 1/2$
 D) Always Divergent.

4) Uniform convergence implies pointwise convergence.

- A) Complete statement is true. B) Partial statement is true.
 C) Complete statement is false. D) None of these.

Q.2. Attempt any One.

[08]

1) Let $f_n(x) = \frac{nx}{1+n^2x^2}$ $x \in [0,1]$

show that $f_n \rightarrow f$ on $[0,1]$ & the convergence is pointwise but still

$$\int_0^1 f(x) dx = \lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx.$$

2) State & Prove Cauchy Condition for uniform convergence.

Q.3. Attempt any Two.

[08]

1) With suitable example show that pointwise convergence of sequences of functions need not preserve continuity.

2) Discuss the existence of two iterated limits & double limit of the sequence $g(p, q) = \frac{pq}{p^2+q^2}$ $p, q \in \mathbb{Z}$

3) Show that the sequence $\{f_n\}$ of functions converges pointwise but not uniformly

where $f_n(x) = \frac{1}{nx+1}$, $0 < x < 1$.

Vivekanand College (An Empowered Autonomous Institute), Kolhapur

Department of Mathematics

M.Sc.-II(Sem-IV) Internal Examination:2025-26

Subject: Field Theory (DSC13MAT41)

Day & Date: Monday, 16/03/2026

Time: 1 hr

Total Marks: 20

Q.1. Select the Correct Alternatives.

[04]

1) If $p(x) = x^2 + x + 1$ be a polynomial in $F[x]$ & b is root of $p(x)$ then $[F(b): F] = \underline{\hspace{2cm}}$

- A) 1 B) 2 C) 3 D) Either A) or B)

2) Every Algebraic extension is finite extension.

A) Complete statement is true. B) Partial statement is true.

C) Complete statement is false. D) None of these.

3) The minimal polynomial of $\sqrt{2} + 5$ over \mathbb{Q} is

- A) $x^2 - 10x + 23$ B) $x^2 + 10x + 23$ C) $x^2 - 10x - 23$ D) $x^2 + 10x - 23$

4) The number of proper fields between \mathbb{R} & \mathbb{C} .

- A) 1 B) 2 C) 3 D) Infinite.

Q.2. Attempt any One.

[08]

1) Let $F \subseteq L \subseteq K$ be fields if $[L: F]$ is finite & $[K: L]$ is finite then prove that

$[K: F]$ is finite & $[K: F] = [K: L] \cdot [L: F]$

2) Let $F \subseteq E$ be fields & $u \in E$ be algebraic over F , then prove that there exists a unique monic irreducible polynomial $p(x) \in F[x]$ such that $p(u) = 0$.

Q.3. Attempt any Two.

[08]

1) Prove that every finite extension is algebraic.

2) Let $p(x)$ be an irreducible polynomial in $F[x]$ then prove that there exist an extension E of F in which $p(x)$ has a root.

3) Find the number a such that $\mathbb{Q}(\sqrt{3}, \sqrt{5}) = \mathbb{Q}(a)$.

Q.1. Select correct alternative.

[04]

- i) If $\gcd(a, b) = d$ then $\left(\frac{a}{d}, \frac{b}{d}\right) = \dots\dots$
a) 2 b) 1 c) 3 d) 4
- ii) If $\gcd(a, b) = d$ then $\gcd(2a + b, a + 2b) = \dots\dots$
a) 1 or 2 b) 2 or 3 c) 1 or 3 d) None of this
- iii) Which of the following Diophantine equation cannot be solved?
a) $18x + 42y = 96$ b) $23x + 31y = 105$
c) $45x + 27y = 63$ d) $7x + 56y = 79$
- iv) $(-100, -200)$ is particular solution of $7x + 4y = 100$ then general solution is.....
a) $x = -100 + 4t, y = 200 - 7t$ b) $x = 100 + 4t, y = 200 + 7t$
c) $x = -100 - 4t, y = 200 - 7t$ d) $x = -100 + 4t, y = -200 - 7t$

Q.2. Attempt any One of the following.

[08]

- i) State and Prove Division Algorithm.
ii) Prove that every positive integer $n > 1$ can be expressed as product of primes and this representation is unique apart from the order in which the factor occurs.

Q.3. Attempt any Two of the following.

[08]

- i) By using mathematical induction prove that $15/2^{4n-1}$
ii) State and Prove Euclid's theorem.
iii) Prove that for given integers a and b not both zero there exists integers x and y such that $\gcd(a, b) = ax + by$.

VIVEKANAND COLLEGE, KOLHAPUR (An Empowered Autonomous Institute)
M.SC. Part- II (Mathematics) Internal examination
Subject: Integral equations

Date : 17/03/2026

Marks : 20

Q. 1] Select the correct alternative for each of the following [4]

i) The type of integral equation $g(s) = f(s) + \lambda \int_a^s K(s,t)g(t)dt$ is -----

- a) Fredholm integral equation of 1st kind
- b) Volterra integral equation of 1st kind
- c) Homogeneous Volterra integral equation of 2nd kind
- d) Non-homogeneous Volterra integral equation of 2nd kind

ii) The eigen values of non-zero symmetric kernel are -----

- a) real
- b) zero
- c) only imaginary
- d) none of these

iii) If $\{\phi_k\}$ is orthonormal set, then $\langle \phi_i, \phi_i \rangle =$ ----- for all i .

- a) 0
- b) 1
- c) -1
- d) ∞

iv) Using Cauchy formula, $\int_0^x \int_0^x y(t)dt^2 =$ -----

- a) $\int_0^x y(t)dt$
- b) $\int_0^x xy(t)dt$
- c) $\int_0^x ty(t)dt$
- d) $\int_0^x (x-t)y(t)dt$

Q.2] Attempt any one [08]

i) Prove that the transposed equation $\psi(s) = f(s) + \lambda \int_a^b K(t,s)\psi(t)dt$ possesses unique solution whenever the integral equation $g(s) = f(s) + \lambda \int_a^b K(s,t)g(t)dt$ does.

ii) Solve the integral equation $g(s) = 1 + \lambda \int_{-\pi}^{\pi} e^{iw(s-t)}g(t)dt$, $w \neq 0$ is a parameter, by discussing all possible cases.

Q.3] Attempt any two [08]

i) Convert the following initial value problem to an integral equation.

$$y'' + y' = 0, y(1) = 0, y'(1) = 1$$

ii) Find the eigen values and eigen functions of the homogeneous integral equation

$$g(s) = \lambda \int_0^1 (6s - 2t)g(t)dt$$

iii) Convert the following boundary value problem to integral equation $y''(x) + \lambda y(x) = 0, y(0) = y(l) = 0, 0 \leq x \leq l$.

iv) prove that the eigenfunction $g(s)$ and $\psi(s)$ corresponding to distinct eigenvalues λ_1 and λ_2 respectively of the homogeneous integral equation and its transpose are orthogonal.

Instructions:

1. All questions are compulsory.
2. Figures in right side indicates full marks.

Q.1. Select the correct alternative for each of the following.

[04]

- i) The number of proper divisors of 1260...
A) 35 B) 30 C) 36 D) 34
- ii) The number of arrangements of the letters in the word 'FAILURE' so that vowels are always come together is...
A) 576 B) 575 C) 570 D) None of these
- iii) The number of ways in which 7 boys sit in a round table so that two particular boys may sit together are...
A) 200 B) 240 C) 120 D) 14
- iv) The coefficient of $x_1 x_2^2 x_4 x_5^3$ in the expansion of $(x_1 + x_2 + x_3 + x_4 + x_5)^7$ is...
A) 340 B) 4320 C) 420 D) 355

Q.2. Attempt any one of the following.

[08]

- i) Define Ramsey number.
a) Show that $R(p, q) = R(q, p)$ b) Show that $R(2, p) = p$
- ii) If there are 'm' pigeons and 'n' pigeonholes then prove that at least one pigeonhole contains $p+1$ pigeons where $p = \left\lfloor \frac{m-1}{n} \right\rfloor$. Show that 9 colours are used to paint 100 houses then at least twelve houses will be of same colour.

Q.3. Attempt any two of the following.

[08]

- i) In how many ways can one select cricket team of 11 players from 17 players in which only 5 players can bowl if each cricket team of 11 players must include exactly 4 bowlers.
- ii) Using combinatorial argument prove the following. $C(n, r) = C(n-1, r) + C(n-1, r-1)$
- iii) If 7 points are chosen at random in interior of regular hexagon. Each side of which 2 units long. Show that at least one pair of points has separation of less than 2 units.



॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥

- शिक्षणमहर्षी डॉ. बापूजी साळुंखे

VIVEKANAND COLLEGE, KOLHAPUR

(An Empowered Autonomous Institute)

Assignment

Student's Sign : *Magdum*

Seat No./ Roll No. : 1205

Seat No./ Roll No. one two
In words zero five

12306

02+08+07 = $\frac{17}{20}$

प्र. क्र.

Q. No.

Q1.

1) a) C(R)

2) b) 0

3) a) only I true

4) c) \hat{v}

02

02	Section	Q. No.	2											
		Marks	06	06	= 08									

प्र. क्र.
Q. No.

Q2.

1) Here, $v_1, v_2, \dots, v_n \in V$ are linearly independent.

Now,

$$L(S) = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 + \dots + \alpha_n v_n$$

$$L(S) = \sum_{i=1}^n \alpha_i v_i, \quad \forall v_i \in S \text{ \& } \alpha_i \in F$$

If the possible a , has two linear combination in linear span

Now,

$$\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n = \beta_1 v_1 + \beta_2 v_2 + \dots + \beta_m v_m$$

$$\Rightarrow \alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n - \beta_1 v_1 - \beta_2 v_2 - \dots - \beta_m v_m = 0$$

ok

$$\Rightarrow (\alpha_1 - \beta_1) v_1 + (\alpha_2 - \beta_2) v_2 + \dots + (\alpha_n - \beta_m) v_n = 0$$

$$\Rightarrow \alpha_i - \beta_i = 0$$

$$\Rightarrow \alpha_i = \beta_i$$

So, Every element in linear span has a unique representation.

3) W_1 and W_2 be subspaces of finite dimension.

Here,

$A(W_1+W_2)$, $A(W_1)$, $A(W_2)$ are subspaces of V & \hat{V} resp

Now,

$$W_1 \subseteq W_1+W_2 \quad \text{and} \quad W_2 \subseteq W_1+W_2$$

Now,

$$A(W_1+W_2) \subseteq A(W_1) \quad \text{and} \quad A(W_1+W_2) \subseteq A(W_2)$$

$$\Rightarrow A(W_1+W_2) \subseteq A(W_1) + A(W_2) \quad \text{--- (1)}$$

Here,

Consider,

$$f(w) = 0 \quad f \in A(W_1+W_2)$$

$$f(w) = 0 \quad \forall w \in (W_1 \cap W_2)$$

$$\text{i.e. } f(w) = 0 \quad , \quad w \in W_1 \quad \& \quad f(w) = 0 \quad \forall w \in W_2$$

$$f \in A(W_1) \quad \& \quad f \in A(W_2)$$

$$\text{let, } w = W_1 \cap W_2$$

$$\text{let, } w = u+v$$

$$f(w) = f(u+v)$$

$$= f(u) + f(v)$$

$$= 0$$

Then,

$$f \in A(W_1+W_2)$$

$$A(W_1) \cap A(W_2) \subseteq A(W_1+W_2) \quad \text{--- (2)}$$

from (1) & (2)

we get,

$$A(W_1+W_2) = A(W_1) + A(W_2) \quad //$$

04

Section

Q. No.

3

Marks

07

प्र. क्र.

Q. No.

93.

② If W is FDVS of V , i.e. $W \subseteq V$

then, $\dim W = m$

Here let, the elements of W be the vector span of V is

(w_1, w_2, \dots, w_m) and $\#$ which is in L.I.

Now, (v_1, v_2, \dots, v_n) are in also L.I.

Now,

$(\alpha_1 v_1, \alpha_2 v_2, \dots, \alpha_n v_n, \beta_1 w_1, \beta_2 w_2, \dots, \beta_m w_m)$ are ~~#~~ L.I.

① ^{Prove,} S is L.I.

Now,

element w has,

$$\beta_1 w_1 + \beta_2 w_2 + \dots + \beta_m w_m \in W$$

and $\#$ for V has,

$$\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n \in V$$

Here,

i) ' \cdot ' quotient

~~$\alpha_1 v_1, \alpha_2 v_2, \dots, \alpha_n v_n, \beta_1 w_1, \beta_2 w_2, \dots, \beta_m w_m$ are L.I.~~

$$f \cdot g(w) = g \cdot f(w)$$

ii) '+' quotient,

$$w + a + w + b = w + (a + b)$$

$$\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n = \beta_1 w_1 + \beta_2 w_2 + \dots + \beta_m w_m$$

$$\Rightarrow \alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n - \beta_1 w_1 - \beta_2 w_2 - \dots - \beta_m w_m = 0$$

Here, S is L.I.



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VIVEKANAND COLLEGE, KOLHAPUR

(An Empowered Autonomous Institute)

Assignment

Student's Sign : SyJum.

Seat No./ Roll No. : 1205

Seat No./ Roll No. one two
In words zero five

12317

प्र. क्र.

Q. No.

93

2) a) S spans $\left(\frac{V}{M}\right)$

Here,

$$V = \alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n$$

and,

$$W + V \in \frac{V}{M}$$

$$\begin{aligned} \therefore W + V &= W + (\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n) + \beta_1 w_1 + \beta_2 w_2 + \dots + \beta_m w_m \\ &= W + \alpha_1 v_1 + W + \alpha_2 v_2 + \dots + W + \alpha_n v_n + W + \beta_1 w_1 + \\ &\quad W + \beta_2 w_2 + \dots + W + \beta_m w_m \end{aligned}$$

$$= \overset{W+}{\cancel{W}} + \alpha_1 v_1 + \overset{W+}{\cancel{W}} + \alpha_2 v_2 + \dots + \overset{W+}{\cancel{W}} + \alpha_n v_n + W + 0$$

$$W + V = W + \alpha_1 v_1 + W + \alpha_2 v_2 + \dots + \alpha_n v_n$$

$$W + V = W + \alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n$$

So, S spans $\frac{V}{M}$

form (I) S (II)

02

Section

Q. No.

Marks

Here, S is LI and S spans $\left(\frac{V}{M}\right)$

then,

$$\therefore \dim \left(\frac{V}{M}\right) = n$$

$$\text{BEST} \quad \dim \left(\frac{V}{M}\right) = m+n-m$$

$$\dim \left(\frac{V}{M}\right) = \dim(V) - \dim(W) //$$



VIVEKANAND COLLEGE, KOLHAPUR

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Assignment

Student's Sign : Palay

Seat No./ Roll No. : 2202

Seat No./ Roll No.
In words

12124

प्र. क्र.
Q. No.

9. i)

i) The number of proper divisors of 6300 is 53

ii) The number of arrangement of the letters in the word 'COMMERCE' so that vowels are always come together is 540

iii) The number of ways in which 7 boys sit in a round table so that two particular boys may sit together are 240.

iv) The coefficient of $x_1 x_2^3 x_4 x_5^3$ in the expansion of $(x_1 + x_2 + x_3 + x_4 + x_5)^8$ are 1120.

$$04 + 08 + 08 = \frac{20}{20}$$

01

01

01

01

04

प्र. क्र.

Q. No.

Q. 3)

iii)

Let, Total no. of persons in a party

$$n + n = 2n$$

$$\text{Total no. of person's} = 16.$$

For 1 handshake 2 person's required.

out of 16 people's,

We select ~~two~~ two person's for 1 handshake.

This can be done in, ${}^{16}C_2$ ways.

out of this 8 pairdes will be married couple

$$\text{Total no. of handshake} = {}^{16}C_2 - 8.$$

$$= \frac{16!}{2!(14)!} - 8$$

$$= \frac{16 \times 15 \times 14!}{2 \times 14!} - 8$$

$$= (8 \times 15) - 8$$

$$= 120 - 8$$

$$= 112.$$

\therefore Total no. of handshake.

04



॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥

- शिक्षणमहर्षी डॉ. बापूजी साळुंखे

VIVEKANAND COLLEGE, KOLHAPUR

(An Empowered Autonomous Institute)

Assignment

Student's Sign :

Seat No./ Roll No. :

Seat No./ Roll No.
In words

12078

प्र. क्र.
Q. No.

Q. 3)

- i) Total no. of player's = 17
 No. of player's who are bowler = 5
 No. of player's who are non-bowler = $17 - 5 = 12$.

Each cricket team of 11 player's must include exactly 4 bowler's.

No. of ways to select 4 bowler's from 5 bowler's

$$= {}^5C_4$$

$$= \frac{5!}{4!(1)!}$$

$$= \frac{5 \times 4!}{4!}$$

$$= 5 \text{ ways.}$$

∴ Remaining player's selected from 12 player's.

No. of ways to select non-bowler's =

$$= {}^{12}C_7$$

$$= \frac{12!}{7!(12-7)!}$$

02

Section

Q. No.

Marks

प्र. क्र.

Q. No.

$$= \frac{12 \times 11 \times 10 \times 8}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 7!}$$

प्रति

$$= 792 \text{ ways.}$$

$$\therefore \text{Total no. of ways} = {}^5C_4 \cdot {}^{12}C_7$$

$$= 5 \cdot 792$$

$$\therefore \text{Total no. of ways} = 3960 \text{ ways.}$$

∴