"Education for Knowledge, Science, and Culture"
- Shikshanmaharshi Dr. Bapuji Salunkhe

Shri Swami Vivekanand Shikshan Sanstha's

Vivekanand College, Kolhapur

(Empowered Autonomous)

DEPARTMENT OF MATHEMATICS

Date: 13/09/2025

Notice

M.Sc. I (Sem I) & M.Sc. II (Sem III)

Internal Examination: 2025-26

All the students of M.Sc. I (Sem I) and M.Sc. II (Sem III) are hereby informed that their internal examination will be conducted from **24/09/2025 to 30/09/2025**. The examination will be conducted only one time, students are directed to attend the examination without fail. Syllabus, timetable & Question paper pattern for examination will be mentioned in following table.

Syllabus for M.Sc. I Sem I:

Sr. No.	Name of the Paper	Topics
1	DSC13MAT11: Modern Algebra	UNIT 1
2	DSC13MAT12: Ordinary Differential Equations	UNIT 1
3	DSC13MAT13: Measure & Integration	UNIT 1&2
4	DSC13MAT14: Numerical Analysis I	UNIT 1
5	DSE13MAT11: Operational Research	UNIT 1
6	RMD13MAT11: Research Methodology	UNIT 1

Syllabus for M.Sc. II Sem III:

Sr. No.	Name of the Paper	Topics
1	DSC13MAT31: Functional Analysis	UNIT 1&2
2	DSC13MAT32: Classical Mechanics	UNIT 1
3	DSC13MAT33: Complex Analysis	UNIT 1&2
4	DSC13MAT34: Advanced Discrete Mathematics	UNIT 1
5	DSE13MAT31: Lattice Theory	UNIT 1&2

Timetable:

Day and Date	Class	Time	Subject
Wednesday, 24/09/2025	M.Sc. I	12:00 PM to 01:00 PM	Modern Algebra
	M.Sc. II	12:00 PM to 01:00 PM	Functional Analysis
Thursday, 25/09/2025	M.Sc. I	12:00 PM to 01:00 PM	Ordinary Differential Equations
	M.Sc. II	12:00 PM to 01:00 PM	Classical Mechanics
Friday, 26/09/2025	M.Sc. I	12:00 PM to 01:00 PM	Measure & Integration
	M.Sc. II	12:00 PM to 01:00 PM	Complex Analysis
Saturday, 27/09/2025	M.Sc. I	12:00 PM to 01:00 PM	Numerical Analysis I
	M.Sc. II	12:00 PM to 01:00 PM	Advanced Discrete Mathematics
Monday, 29/09/2025	M.Sc. I	12:00 PM to 01:00 PM	Operational Research
	M.Sc. II	12:00 PM to 01:00 PM	Lattice Theory
Tuesday, 30/09/2025	M.Sc. I	12:00 PM to 01:00 PM	Research Methodology

***Note: All the lectures on the internal exam day will be conducted at 02:00PM to 4:00 PM. Everyone should attend the lectures.**

Nature of Question Paper

Time :- 1 Hour   Total Marks: 20

Q.1) Choose the correct alternative for each of the following. [04]

i)
a) b) c) d)

ii)
a) b) c) d)

iii)
a) b) c) d)

iv)
a) b) c) d)

Q.2) Attempt any one [08]

i)
ii)

Q.3) Attempt any two [08]

i)
ii)
iii)

Thorat

(Dr. S. P. Thorat)

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DEPARTMENT OF MATHEMATICS
VIVEKANAND COLLEGE, KOLHAPUR
(EMPOWERED AUTONOMOUS)

Vivekanand College (Empowered Autonomous), Kolhapur
M.Sc.-I (Sem-I) Internal Examination 2025-26
Modern Algebra (DSC13MAT11)

Day & Date: Wednesday,24/09/2025 **Time: 12:00PM to 1:00PM** **Marks: 20**

Q.1. Select the correct alternative from each of the following. **[04]**

i) Consider the following Statements:

I) Every principle series is composition series.

II) Every normal series is principle series.

A) Only I true B) Only II true
C)Both I and II are true D) Both I and II are false

ii) If $|G| = p\ or\ p^2$ where p is prime then G is….

A) Non-abelian B) Cyclic C) Solvable D) Both B) &C)

iii) Which of the following group of integers is not simple group?

A) $\mathbb{Z}_5$ B) $\mathbb{Z}_2$ C) $\mathbb{Z}_6$ D) $\mathbb{Z}_{11}$

iv) A subnormal series of group G is said to be composition series if all factor groups of series are…

A)Abelian B) Simple C) Non-abelian D) None of these

Q.2. Attempt any one. **[08]**

i) Let G' be a commutator subgroup of group G then show that,

a)G' is normal subgroup of G.

b)$\frac{G}{G'}$ is abelian group.

c) $\frac{G}{N}$ is abelian group iff G' is subgroup of N.

ii) State and prove Schrier Refinement Theorem.

Q.3. Attempt any two. **[08]**

i) If H is subgroup of group G with index 2 then show that H is normal subgroup of G.

ii) If $\emptyset$: $G_1 \rightarrow G_2$ is onto homomorphism and if G_1 is solvable then show that $G_2 = \emptyset(G_1)$ is solvable.

iii) Show that the symmetric group Sn is a group w. r. to mapping composition.

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Vivekanand college Kolhapur (An Empowered Autonomous Institute)

Department of Mathematics

Internal Examination :2025-2026

M.Sc. I (Sem I)

Course Name: Ordinary Differential Equation

Course Code:DSC13MAT12

Day and Date:Thursday,25/09/2025

Time : 12:00PM to 01:00PM

Total marks : 20

Q.1. Select the correct alternative. [04]

i) If $\emptyset_1(x)$ and $\emptyset_2(x)$are two solutions of $L(y) = y'' + a1(y') + a2(y) = 0$ on an interval I containing point x_0 then $w(\emptyset 1, \emptyset 2)(x) =$ --- $w(\emptyset 1, \emptyset 2)(x)$.

A)$e^{-a_1(x-x0)}$ B)$e^{a_1(x-x0)}$ C)$e^{-a_0(x_0-x)}$ D)$e^{a_0(x_0-x)}$

ii) The order and degree of equation $\left(\frac{d^2y}{dx^2}\right) = \left(y + \left(\frac{dy}{dx}\right)^6\right)^{1/6}$ is.... respectively.

A)2,1 B)$2,\frac{1}{6}$ C)2,6 D)2,3

iii) Wronskian of the two solutions of differential equation $y'' + a_1(x)y' + a_2(x)y = 0$ an interval I is..

A) Identically zero

B) Never zero

C) Always constant

D) either identically zero or never zero

iv) The functions $\emptyset_1(x) = \sin(x)$and $\emptyset_2(x) = e^{ix}$ are ......on interval $[-\infty, \infty]$.

A) Linearly dependent

B) Linearly independent

C) Both A & B

D) None of these

Q.2. Attempt Any One of the following. [08]

1) Define Wronskian and find all the solutions of $y'' + 4y = \cos x$.

2) If b(x) be continuous function on an interval I every solution φ of $L(y) = b(x)$ on I can be written as $\varphi = \varphi_p + C_1\emptyset_1 = C_2\emptyset_2$ where φp is particular solution and $\emptyset_1$ and $\emptyset_2$ are linearly independent solution $L(y) = 0$ and C_1, C_2 are constants and a particular solution φ_p is given by

$\varphi p = \int_{x0}^{x} \frac{\emptyset_1(t)\emptyset_2(x) - \emptyset_2(t)\emptyset_1(x)}{W(\emptyset 1, \emptyset 2)(x)} b(t)dt$, conversely Every such solution is $\varphi$is the solution of $L(y) = b(x)$

Q.3.Attempt Any Two of the following. [08]

i) Check whether functions e^x, e^{2x}, e^{3x} are linearly independent or not.

ii) Find the solutions of initial value problem $y'' - 5y' + 6y = 0$ with $y(0) = 0, y'(0) = 1$.

iii) Show that every solution of constant coefficient equation $L(y) = y'' + a_1y' + a_2y = 0$ tends to zero as $x \to \infty$ iff the real part of roots of characteristics polynomial are negative.

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Vivekanand college Kolhapur (An Empowered Autonomous Institute)
M.Sc. I (Sem I) Internal Examination :2025-2026
Operation Research

Course code: DSE13MAT11
Total marks: 20

Day and Date: Monday 29/09/2025
Time: 12:00PM to 1:00 PM

Q1. Select the correct alternative. **[04]**

(i) If $y \subset R^n$ then the smallest convex set containing y is called__
(a) convex function (b)convex set (c) convex hull (d) convex combination

(ii) Which of the following is a property of a convex set?
(a) The set contains all possible linear combinations of its elements.
(b) The set contains all possible convex combinations of its elements.
(c) Every point in the set is a boundary point.
(d) The set contains no interior points.

(iii) In the matrix form of an LPP, $AX \leq b$, what does the matrix A represent?
(a) The matrix of decision variables.
(b) The matrix of cost coefficients.
(c) The matrix of constraint coefficients.
(d) The matrix of slack variables

(iv)The extreme points of cube are__
(a)4 (b)2 (c)8 (d)16

Q2. Attempt any one. **[08]**

(i) Define Convex combination and let S and T be convex set in R^n, then $\alpha S + \beta T$ is also convex set.
(ii) Define feasible solution and show that set of feasible solution to LPP is convex set.

Q3. Attempt any two. **[08]**

(i) Show that $S = \{(x_1, x_2, x_3), 2x_1 - x_2 + x_3 \leq 4\}$ is convex set.
(ii) Find the basic feasible solution:
Max $(z) = x_1 - 12x_2$
Subject to: $x_1 + x_2 \leq 10$
$2x_1 - x_2 \leq 40;\ x_1, x_2 \geq 0$
(iii) Rewrite in standard form the following LPP
$\text{Min}(z) = 2x_1 + x_2 + 4x_3$
Subject to: $-2x_1 + 4x_2 \leq 4$
$x_1 + x_3 \geq 5$
$2x_1 + 3x_3 \leq 2;\ x_1, x_2 \geq 0$, x_3 is unrestricted in sign

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Vivekanand College Kolhapur (An Empowered Autonomous Institute)

Department of Mathematics

M.Sc.-I (Sem-I) Internal Examination 2025-26

Course Name: Research Methodology | Course Code: ~~DSC13MAT~~ RMD13MAT11

Day & Date: Tuesday,30/09/2025 | Marks: 20

Time: 01:00PM to 02 :00PM

Q.1. Select the correct alternative from each of the following. **[04]**

i) A corollary is easy consequences of.....

A) lemma B) preposition C) theorem D) All the above

ii) The purpose......is to summarise the concept of the paper.

A) Definition B) Abstract C) Title D) Keywords

iii) .... is the person who did the greatest part.

A) First B) last C) third D) Senior person.

iv) In mathematics the use of article......is unappropriated when the object to which it refers is not unique.

A) A B) An C) the D) None of these

Q.2. Attempt any one. **[08]**

i)While writing paper how to finalize title author list.

ii) What are the Do's and Don'ts of mathematical writing.

Q.3. Attempt any two. **[08]**

i)How to use notations in mathematical writing?

ii) How should you determine Audience while writing a paper?

iii) Write short note on what is theorem.

Vivekanand College Kolhapur (An Empowered Autonomous Institute)
Department of Mathematics
Internal Examination 2025-26
M.Sc.-I (Sem-I)

Course Name: Numerical Analysis- I
Course Code: DSC13MAT14
Day & Date: Saturday,27/09/2025
Marks: 20
Time: 12:00PM to 1:00PM

Q.1. Select the correct alternative from each of the following. **[04]**

i) In Bairstow method Δq = ----------

A) $\Delta q = \frac{b_n c_{n-2} + b_{n-1}(b_{n-1} - c_{n-1})}{c^2{}_{n-2} + c_{n-3}(b_{n-1} - c_{n-1})}$ B) $\Delta q = \frac{b_{n-2} c_n + b_{n-1}(b_n - c_n)}{c^2{}_{n-2} + c_{n-3}(b_n - c_n)}$

C) $\Delta q = \frac{b_n c_n + b_{n-1}(b_{n-2} - c_{n-2})}{c^2{}_n + c_{n-3}(b_{n-1} - c_{n-1})}$ D) $\Delta q = \frac{b_{n-1} c_{n-2} - b_n c_{n-3}}{c^2{}_{n-2} + c_{n-3}(b_{n-1} - c_{n-1})}$

ii) If $f(x)$ is continuous function in the interval $[a, b]$, $f(a) \cdot f(b) < 0$ then the equation $f(x) = 0$ has atleast one real root or on odd number of real roots in (a, b) is called.....

A) Bisection Method B) Iterative method
B) Direct method D) Intermediate Value theorem

iii) The rate of convergence of Bisection method is...

A) 0 B)1 C) 3 D) 2

iv) An iterative method is said to be of order p, if p is the largest positive real number for which there exists a finite constant $c \neq 0$ such that...

A) $|\epsilon_{k+1}| \geq c|\epsilon_k|^p$ B) $|\epsilon_k| \neq c|\epsilon_k|^p$
B) $|\epsilon_{k+1}| \leq c|\epsilon_k|^p$ D) $|\epsilon_k| > c|\epsilon_k|^p$

Q.2. Attempt any one. **[08]**

i) Determine the Rate of convergence of Secant method.
ii) Perform two iterations of Bairstow method to extract quadratic factor $x^2 + px + q$ from $P_3(x) = x^3 + x^2 - x + 2$. Use initial approximations $p_0 = -0 \cdot 9$ *and* $q_0 = 0.9$

Q.3. Attempt any two. **[08]**

i) Perform 4 iterations of the Newton Raphson Method to find the smallest positive root of the equation $f(x) = x^3 - 5x + 1 = 0$. (Taking smallest positive root lies in the interval (0,1).

ii) Use Secant method to determine roots of equation $cosx - xe^x = 0$ .Do four iterations taking init approximations $x_0 = 0$ *and* $x_1 = 1$.

iii) Determine the Rate of convergence of Regula Falsi method.

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Vivekanand College Kolhapur (An Empowered Autonomous Institute)
Department of Mathematics
Internal Examination 2025-26
M.Sc.-I (Sem-I)

Course Name: Measure & Integration
Day & Date: Friday,26/09/2025
Time: 12:00PM to 1:00PM

Course Code: DSC13MAT13
Marks: 20

Q.1. Select the correct alternative from each of the following. **[04]**

i)If A is singleton set the $m^*(A) =$.....

A) 0 B) 1 C) 2 D) -1

ii) Consider the following statements:

I)Every countable set is Borel set.

II) A Set of Real number a is Borel set.

A) Only I true B) Only II true C) Both I & II true D) Both I & II are false.

iii) If A is measurable set then the complement A^c is......

A) non-measurable B) measurable C) finite D) uncountable

iv) A set F is F_σ if it is......

A) Countable union of open sets B) Countable intersection of open sets
C) Countable intersection of closed sets D) Countable union of closed sets

Q.2. Attempt any one. **[08]**

i) Prove that outer measure of an interval is equal to its length.

ii) Define σ -algebra. Prove that there exists a smallest σ -algebra containing a given collection of subsets.

Q.3. Attempt any two. **[08]**

i) Show union of finite collection of measurable sets is measurable

ii)Show that outer measure is translation invariant.

iii) Give an example of uncountable set with outer measure zero.

Samruddhi Gunda Magdum

|| ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ||
– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

कोल्हापूर
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VIVEKANAND COLLEGE, KOLHAPUR

(An Empowered Autonomous Institute)

Assignment

MSC-I ODE

Student's Sign :	
Seat No./ Roll No. :	1205
Seat No./ Roll No. In words	one two zero five.
	03016

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Q. No.

Q1.

1) A) $e^{-a_1(x-x_0)}$

2) C) 2, 6

3) D) Either identically zero or never zero

4) B] Linearly independent

04

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		Marks											

प्र. क्र.
Q. No.

Q2

1) Wronskian :

Let, $\phi_1(x)$ and $\phi_2(x)$ are two elements the interval I, ~~we~~ wronskian can defined

$$W(\phi_1, \phi_2)(x) = \begin{vmatrix} \phi_1(x) & \phi_2(x) \\ \phi_1'(x) & \phi_2'(x) \end{vmatrix}$$

$$= \phi_1(x) . \phi_2'(x) - \phi_1'(x) . \phi_2(x)$$

The solutions are linearly independent

$\Rightarrow$ Given,

$$y'' + 4y = \cos x \quad \text{——} \quad (I)$$

the given eq^n is in the form of $L(Y) = b(x$

The char. poly eq^n ~~of~~ is,

$$r^2 + 4\cancel{r} = \cos x$$

put,

$$r^2 + 4 = 0$$
$$r^2 + 4 = 0$$
$$r = \pm 2i$$

The particular eq^n of $L(Y) = 0$ is,

$$\cancel{\psi_p(x)} = \cancel{c_1 \cos 2x} + \cancel{c_2 \sin 2x}$$

$$\therefore \psi_p(x) = \cancel{c_1 \phi_1(x)} + c_2 \phi_2(x),$$

Section	Q. No.												
	Marks												

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Q. No.

Here, $\phi(x) = C_1\phi_1(x) + C_2\phi_2(x)$

$\Rightarrow \phi(x) = C_1 \cos 2x + C_2 \sin 2x$

$\therefore \phi_1(x) = \cos 2x$ and $\phi_2(x) = \sin 2x$

The general particular sol^n of $L(y)=0$

$\psi_p(x) = u_1\phi_1(x) + u_2\phi_2(x)$

Here,

$u_1 = -\int \frac{\phi_2(x) . b(x)}{W(\phi_1,\phi_2)(x)}$ and $u_2 = \int \frac{\phi_1(x) . b(x)}{W(\phi_1,\phi_2)(x)}$

Here

$W(\phi_1,\phi_2)(x) = \begin{vmatrix} \phi_1(x) & \phi_2(x) \\ \phi_1'(x) & \phi_2'(x) \end{vmatrix}$

$= \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}$

$= 2\cos^2 2x + 2\sin^2 2x$

$= 2(1)$

$W(\phi_1,\phi_2)(x) = 2$

Now, $u_1 = -\int \frac{\sin 2x \cdot \cos x}{2} dx$

$= \frac{-1}{2}\int \sin 2x . \cos x \, dx$

$= \frac{-1}{4}\int 2\sin 2x . \cos x \, dx$

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$$u_1 = -\frac{1}{4}\int \left[\sin(2x+x) + \sin(2x-x)\right] dx$$

$$= -\frac{1}{4}\int (\sin 3x + \sin x)\,dx$$

$$u_1 = -\frac{1}{4}\left[\frac{-\cos 3x}{3} + (-\cos x)\right]$$

$$u_1 = \frac{1}{4}\left[\frac{\cos 3x}{3} + \cos x\right]$$

Now,

$$u_2 = \int \frac{\overset{\cos 2x}{\cancel{\sin 2x}}\cdot \cos x}{2}\,dx$$

$$= \frac{1}{4}\int 2\,\overset{\cos 2x}{\cancel{\sin 2x}}\cdot \cos x\,dx$$

$$= \frac{1}{4}\int [\cos 3x + \cos x)\,dx$$

$$u_2 = \frac{1}{4}\left[\frac{\sin 3x}{3} + \sin x\right]$$

Now,

$$\psi_p(x) = u_1\phi_1(x) + u_2\phi_2(x)$$

$$= \frac{1}{4}\left[\frac{\cos 3x}{3} + \cos x\right]\cos 2x + \frac{1}{4}\left[\frac{\sin 3x}{3} + \sin x\right]$$

$$= \frac{1}{4}\left\{\left[\frac{\cos 3x\cdot\cos 2x}{3} + \cos x\cdot\cos 2x\right]\right.$$

$$\left. + \left[\sin 3x\cdot\sin 2x + \sin x\cdot\sin 2x\right]\right\}$$

Samruddhi Gunda Magdum

|| ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ||

– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

VIVEKANAND COLLEGE, KOLHAPUR

(An Empowered Autonomous Institute)

Assignment

ODE

Student's Sign :

Seat No./ Roll No. : 1205

Seat No./ Roll No. In words: one two zero five

03017

क्र. No.

Q2.

1) $\psi_p(x) = \frac{1}{4}\left[\left\{\frac{\cos 3x . \cos 2x + \sin 3x \cdot \sin 2x}{3}\right\} + \{\cos x \cdot \cos 2x + \sin x \cdot \sin 2x\}\right]$

$= \frac{1}{4}\left[\cancel{\cos(6x)} + \cos(5x)\right.$

$= \frac{1}{4}\left[\frac{\cos x}{3} + \cos x\right]$

$= \frac{1}{4}\left[\frac{4\cos x}{3}\right]$

$\psi_p(x) = \frac{\cos x}{3}$

The particular solution of eq^n is,

$\psi_p'(x) = \psi_p(x) + c_1\phi_1(x) + c_2\phi_2(x)$

$= \frac{\cos x}{3} + c_1 \cos 2x + c_2 \cancel{\cos} \sin 2x$

8

02	Section	Q. No.											
		Marks											

प्र. क्र.
Q. No.

Q3.

1) let, $\phi_1(x) = e^x$, $\phi_2(x) = e^{2x}$, $\phi_3(x) = e^{3x}$

Here,

$$W(\phi_1,\phi_2,\phi_3)(x) = \begin{vmatrix} \phi_1(x) & \phi_2(x) & \phi_3(x) \\ \phi_1'(x) & \phi_2'(x) & \phi_3'(x) \\ \phi_1''(x) & \phi_2''(x) & \phi_3''(x) \end{vmatrix}$$

$$= \begin{vmatrix} e^x & e^{2x} & e^{3x} \\ e^x & 2.e^{2x} & 3.e^{3x} \\ e^x & 4.e^{2x} & 9e^{3x} \end{vmatrix}$$

$$= e^x(18e^{2x}.e^{3x} - 12.e^{2x}.e^{3x}) - e^{2x}(9e$$
$$-3e^x.e^{3x}) + e^{3x}(4e^x.e^{2x} - 2e^x.e$$

$$= e^x(6.e^{2x}.e^{3x}) - e^{2x}(6e^x.e^{3x})$$
$$+ e^{3x}(2.e^x.e^{2x})$$

4

$$= 6.e^{6x} - 6e^{6x} + 2e^{6x}$$

$$= 2.e^{6x}$$

$W(\phi_1,\phi_2,\phi_3)(x) \neq 0$

Here, the given functions are linearly indep

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	Marks												

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Q3)

2) Given,

$$\left.\begin{array}{l} y'' - 5y' + 6y = 0 \\ y(0) = 0, \ y'(0) = 1 \end{array}\right\} \text{—— (1)}$$

The given is IVP,
the char poly of eq^n (1) is,

$$r^2 - 5r + 6 = 0$$
$$(r-3)(r-2) = 0$$
$$r = 3, 2$$

so,

$$\phi_1(x) = e^{3x} \quad \text{and} \quad \phi_2(x) = e^{2x}$$

The general sol^n is,

$$\phi(x) = c_1 \phi_1(x) + c_2 \phi_2(x)$$

$$\left.\begin{array}{l} \phi(x) = c_1 e^{3x} + c_2 e^{2x} \\ \Rightarrow \phi'(x) = 3c_1 e^{3x} + 2c_2 e^{2x} \end{array}\right\} \text{—— (2)}$$

Now, $y(0) = 0$ and $y'(0) = 1$ put in eq^n (2)

$$0 = c_1 + c_2 \quad \text{—— (3)}$$
$$1 = 3c_1 + 2c_2 \quad \text{—— (4)}$$

~~ex~~ Multiply eq^n (3) by 2

$$\cancel{3c} \quad 2c_1 + 2c_2 = 0$$
$$3c_1 + 2c_2 = 1$$

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Here, $C_1 = 1$
and, $C_2 = -1$

The particular eqⁿ is,

$$\phi(x) = C_1 e^{3x} + C_2 e^{2x}$$

$$\phi(x) = 1 e^{3x} - e^{2x}$$

$$\phi(x) = e^{3x} - e^{2x}$$

4

$= \sin(A+B)$

$\cos A . \cos B + \sin A . \sin B = \cos(A+B) . \cancel{\sin(A-B)}$

$\cos A . \cos B -$ $= \cos(A+B)$

Vivekanand College, Kolhapur (An Empowered Autonomous Institute)
M.Sc. II (Internal Examination) 2025-26

Course Name: Functional Analysis
Total Marks: 20
Day & Date:
Time:

Instructions:
1.All the questions are compulsory.
2.Figure to the right indicates full marks.

Q.1. Select the correct alternative for each of the following. **[4]**

i) Every Banach space is a:

a) Complete normed linear space
b) Hilbert space
c) Finite space
d) Compact space

ii) Open Mapping Theorem is valid for:

a) Compact Spaces
b) Banach Space
b) c) Finite Spaces
d) Metric spaces

iii) If T is a bounded linear operator, then ‖Tx‖ ≤ M for some:

a) $M > 0$ b) $M < 0$ c) $M = 0$ d) None

iv) Which theorem ensures that a bounded operator maps open sets to open sets?

a) Banach–Steinhaus
b) Hahn–Banach
c) Open mapping
d) Closed Graph

Q.2. Attempt any ONE of the following. **[8]**

i) Define normed linear space. If N and N′ are normed linear spaces ,T is linear transformation from N into N′ then show that following conditions are equivalent

a) T is continuous on N

b) T is continuous at origin

c) there exist a real number k ≥ 0 with property | | T(x) | | ≤k | | x | | for all x in N

d) If s = {x in N such that | | x | | ≤ 1 } is closed unit sphere in N then T(S) is bounded in N′

ii) If N is Banach space and M is closed linear subspace of N then show that, quotient space N/M is Banach space.

iii) State and prove Hahn Banach theorem.

. Attempt any two of the following. **[8]**

i) If $\{T_n\}$ and $\{S_n\}$ are sequences in $B(N)$ such that $T_n \to T$ and $S_n \to S$ as $n \to \infty$ then show that,

a) $T_n + S_n \to T + S$ b) $kT_n \to kT$ for k in F c) $T_nS_n \to TS$ as $n \to \infty$

ii) State and proof that Riesz Lemma

iii) Prove that nls N is separable if it's Conjugate Space N* in.

iv) If N is a normed linear space and x_0 is non zero vector in N then show that there exist a functional f_0 in N^* such that $f_0(x_0) = ||x_0||$ and $||f_0|| = 1$

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7280 - Dhiraj D. Sutar

Vivekanand College Kolhapur (An Empowered Autonomous Institute)
Department of Mathematics
Internal Examination 2025-26
M.Sc.-II (Sem-III)

Course Name: Lattice Theory
Day & Date: Monday,29/09/2025
Time: 12:00PM to 1:00PM

Course Code: DSE13MAT31
Marks: 20

Q.1. Select the correct alternative from each of the following. **[04]**

i) Consider the statements.

I) Every Maximal antichain is maximum.

II) Every Maximum element is maximal.

A) Only I true B) Only II true

C) Both I & II are true D) Both I & II are False

ii) Which of the following is not partial ordered relation?

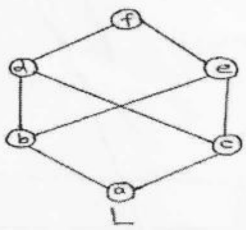
A) The relation 'less than equal to ($\leq$)'defined on set of natural numbers.

B) The relation $\subseteq$ of a set inclusion defined on a non-empty set.

C) The relation 'Divides ($|$)' defined on set of integers.

D) The relation strictly less than ($<$) defined on a set of natural numbers

iii) The Hasse Diagram given below is an example of……



A) Lattice B) Semilattice C) Non-lattice poset D) None of these

iv) In the poset $\langle \mathbb{Z}^+, | \rangle$ where $\mathbb{Z}^+$ is the set of positive integers & $|$ is divides relation then 3 & 18 are……

A) Comparable B) Parallel C) Both A) & B) D) Neither A nor B)

Q.2. Attempt any one. **[08]**

i)Prove that I is prime ideal if and only if there is homomorphism of L onto C_2 with $I = \emptyset^{-1}(0)$, $I = \{x \in L | \emptyset(x) = 0\}$.

ii) Define Poset. Prove that if a poset satisfies ACC then it has a maximal element.

Q.3. Attempt any two. **[08]**

i) Prove that every homomorphic image of lattice L is isomorphic to a suitable quotient lattice of L.

ii) Show that $I(L)$ is a lattice under set inclusion

iii) Prove that the algebra $\langle L \ \wedge \ \vee \rangle$ be a lattice and $a \leq b$ if and only if $a = a \wedge b$ then $\langle L \ \leq \rangle$ is a poset and as a poset it is a lattice.

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Vivekanand College Kolhapur (An Empowered Autonomous Institute)
Department Of Mathematics
Internal Examination 2025-26
M.Sc. II (Sem III)

Course Name: Complex Analysis | **Course Code: DSC13MAT33**
Day & Date : Friday, 26/09/2025 | **Marks: 20**

Q.1. Select the correct alternative for each of the following. **[04]**

i) The series $\sum_{n-0}^{\infty} \frac{z^n}{n^n}$ converges on.....

A) $|z| \leq 3$ B) $|z| < 1$ C) $|z| < 1$ D) Whole Compex plane

ii) Consider the following statements:

I) Every Mobius transformation can have atmost 2 fixed points.

II) If $S(z) = az$ then S is inversion.

A) Only I true B) Only II true C) Both I & II true D) Both I & II are false

iii) Consider the power series $\sum_{n\geq 0} z^{n!}$ then the radius of convergence of power series around the origin is....

A) 0 B) 1 C) 3 D) 4

iv) If S and T mobius transformations then $S \circ T$ is....

A) Bilinear Transformation B) Non-Bilinear Transformation

C) Mobius Transformation D) None.

Q.2. Attempt any one of the following. **[08]**

i) State and prove Liouville's theorem.

ii) If z_1, z_2, z_3, z_4 be four distinct points in C_∞ then show that cross ratio (z_1, z_2, z_3, z_4) is real number if and only if all four points lie on a circle.

Q.3. Attempt any two of the following. **[08]**

i) Prove that $\int_0^{2\pi} \frac{e^{it}}{e^{it}-z} dt = 2\pi$ when $|z| < 1$.

ii) If S is Mobius transformation then prove that 'S' is composition of translation , dilation, and inversion.

iii) Evaluate the following integral

$\int_\vartheta \frac{\sin z}{z^3} dz$ where $\vartheta(t) = re^{it}$, $0 \leq t \leq 2\pi$

Vivekanand College Kolhapur (An Empowered Autonomous Institute)
Department of Mathematics
Internal Examination 2025-26
M.Sc.-II (Sem-III)

Course Name: Advanced Discrete Mathematics Course Code: DSC13MAT34
Day & Date: Saturday,27/09/2025 Marks: 20
Time: 12:00PM to 1:00PM

Q.1. Select the correct alternative from each of the following. **[04]**

i)The join of two vertex disjoint complete graphs is a.......

A) Simple graph B) Complete graph C) Complete bipartite graph D) Bipartite Graph

ii)The radius and diameter of wheel graph W_n , $n \geq 5$ is......respectively.

A)1,3 B) 1,1 C) 1,2 D) 2,2

iii) Consider the statements

I)Every path need not be a trail.

II) Every complete graph is regular graph.

A) Only I true B) Only II true C) Both I & II true D) Both I & II are false

iv) A non-trivial closed trail is called......

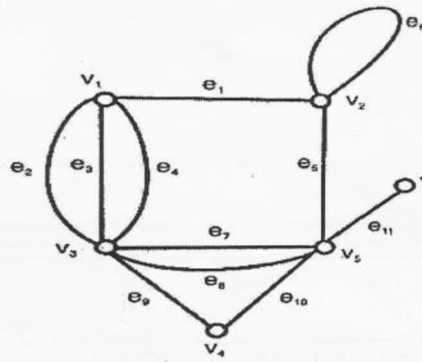
A) path B) cycle C) tree D) walk

Q.2. Attempt any one. **[08]**

i) Find the graphs

a) G-U, G-F, where $U = \{v_4, v_5\}$ and $F = \{e_2, e_4, e_8, e_9, e_{10}\}$

b) G[U], G[F] and also find their union $G[G[U] \cup G[F]]$ where, $U = \{v_2, v_3, v_5\}$ and $F = \{e_1, e_3, e_7, e_9\}$



ii) Define Underlying simple graph.

a) Prove that in any graph G, there is even number of odd vertices.

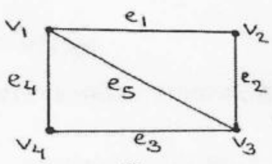
b) Prove that for any vertices u, v, w in G, $d(u, v) \leq d(u, w) + d(w, v)$.

Q.3. Attempt any two. **[08]**

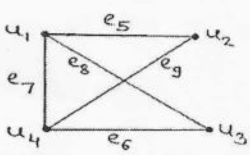
i) Prove that in any connected graph G, radG ≤ diamG ≤ 2 radG

ii) Define Path & Trail. Prove that any two vertices of the graph G every u-v walk contains a u-v path.

iii) Verify whether following two graphs are isomorphic or not.



G_1



G_2

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Vivekanand college (An Empowered Autonomous Institute) Kolhapur

Department of Mathematics

M.Sc. II (Sem III) Internal Examination :2025-2026

Subject code: Classical Mechanics Day and Date: Thursday,25th september2025

Total marks: 20 Time: 12.00pm -1.00pm

Q1. Select the correct alternative. (4)

i) Kinetic energy of a particle of mass m and position vector $\bar{r}$ in polar form is ....

A) T= m $(\dot{r}^2 + r^2\dot{\theta}^2)$ B) T= $\frac{1}{2}$ m $(\dot{r}^2 + r^2\dot{\theta}^2)$

C) T= 2m $(\dot{r}^2 + r^2\dot{\theta}^2)$ D) T= $\dot{r}^2 + r^2\dot{\theta}^2$

ii) Equation of constraints that do not contain time as explicit variable are referred as ....

A) holonomic constraints B) non holonomic constraints

C) rheonomic constraints D) scleronomic constraints

iii) The number of generalized co-ordinates in simple pendulum is.......

A) 1 B) 2 C) 3 D) 4

iv) If the system is conservative then ....

A) $p_j = \frac{\partial L}{\partial \dot{q}_j}$ B) $p_j = \frac{\partial T}{\partial \dot{q}_j}$ C) $p_j = -\frac{\partial L}{\partial \dot{q}_j}$ D) $p_j = \frac{\partial V}{\partial \dot{q}_j}$

Q2. Attempt any one (8)

i) Obtain Lagrange's equations of motion from D'Alembert's Principle.

ii) If the cyclic generalized co-ordinate q_j is such that dq_j represents the translation of the system, then prove that the total linear momentum is conserved.

Q3. Attempt any two of following (8)

i) Show that the Lagrange's equation of motion can also be written as $\frac{\partial L}{\partial t} - \frac{d}{dt}\left(L - \sum_j \dot{q}_j \frac{\partial L}{\partial \dot{q}_j}\right) = 0$

ii) Show that the generalized momentum corresponding to cyclic co-ordinate is conserved

iii) Find the equation of motions for Atwood machine.

iv) Show that gravitational force is conservative.

Pallavi Bhujgonda Gudle

॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥

– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

कोल्हापूर
१९६४

VIVEKANAND COLLEGE, KOLHAPUR

(An Empowered Autonomous Institute)

Assignment

Student's Sign :	Pallavi
Seat No./ Roll No. :	2202
Seat No./ Roll No. In words	Two two zero two
	03082

03 + 08 + 08 = 19

प्र. क्र.
Q. No.

Q. 1)

i) B) only II true.

01

ii) D) The relation strictly less than (<) defined on set of natural numbers.

01

iii) A) Lattice.

00

iv) A) Comparable.

01

03

02	Section	Q. No.	2
		Marks	08

प्र. क्र.
Q. No.

Q. 2)

ii) Let, $< P, \leq >$ is partial ordered set (POSET) it satisfies following properties.

i) Anti-symmetry -
Let, $a, b \in P$ then,
$a \leq b$ and $a \geq b \overset{iff}{\Rightarrow} a = b. \quad \forall a, b. \in P$

ii) Reflexivity -
$a \in P$ then
$a \leq a \Rightarrow a \quad \forall a. \in P$

iii) Transitivity -
$a, b, c \in P.$ then
$a \leq b$ and $b \leq c$ then $a \leq c \quad \forall a, b, c. \in P.$

08

To prove - If poset satisfies ACC then it maximal element.

Let, $< P \leq >$ be poset. which satisfies ACC.
Let, $x_0 \in P$ be any element.
If x_0 is maximal element then we are d
If x_0 is not maximal element then,
$\exists\ x_1 \in P$ such that
$x_0 \leq x_1.$
If x_1 is maximal element then we are don
If x_1 is not maximal element element the
$\exists\ x_2 \in P$ such that
$x_0 \leq x_1 \leq x_2$

Section	Q. No.													03
	Marks													

By continuing this process
we get increasing chain of elements
$x_0 \leq x_1 \leq x_2 \leq \cdots \leq x_n$
this increasing chain terminates the
$i \in W$,

$x_i = x_{i+1} = x_{i+2} = \cdots$
If x_i is not equal to all remaining elements
i.e. x_i covers all the element of P then
x_i is maximal. and if not x_i is upper bound.
then it has
If not, there exist $y_0 \in P$, $x_i \neq y_0$ $x_i \leq y_0$ then.
If y_0 is maximal then we are done.
If y_0 is not maximal then
$\exists\ y_1 \in P$ such that
$x_1 \leq y_0 \leq y_1$
If y_1 is maximal then we stop.
If y_1 is not maximal then $\exists\ y_2 \in P$ s.t. $x_i \leq y_0 \leq y_1 \leq y_2$.
By continuing this process.
We get increasing chain of elements.
$x_i \leq y_0 \leq y_1 \leq y_2 \leq \cdots \leq y_m$.
this increasing chain terminates the
$j \in W$, such that
$y_j = y_{j+1} = y_{j+2} = \cdots$
If all the elements covered by y_j then y_j is maximal element and if not then it has upper bound.
Do the same argument for all possible chains of POSET.
By Zorn's lemma.
all poset satisfies ACC then it has maximal element

04	Section	Q. No.	3										
		Marks	$3\frac{1}{2}$ + 04	$= 7\frac{1}{2} = 08$									

प्र. क्र. Q. No.

Q. 3)

ii) Let, L be Lattice and $I(L)$ be set of all ideals of lattice.
Claim – $I(L)$ is lattice under set inclusion.
as, ϕ, $\{0\}$ and L are always lattice.
Consider,
$I_1, I_2 \in I(L)$ be any two elements.

Case I) :– $I_1 \subseteq I_2$.
$\sup\{I_1, I_2\} = I_1 \cup I_2 = I_2$
$\inf\{I_1, I_2\} = I_1 \cap I_2 = I_1$

Case II) :– $I_2 \subseteq I_1$
$\sup\{I_1, I_2\} = I_1 \cup I_2 = I_1$
$\inf\{I_1, I_2\} = I_1 \cap I_2 = I_2$

~~06~~ $3\frac{1}{2}$

In both the case $I(L)$ is lattice.

Case III) :– Neither $I_1 \subseteq I_2$ nor $I_2 \subseteq I_1$
$\inf\{I_1, I_2\} = I_1 \cap I_2$
$\inf\{I_1, I_2\} = I_1 \cap I_2$ is exist ($\because \phi$ is lattice).

$\sup\{I_1, I_2\} = I_1 \cup I_2$
~~By Absorption property,~~
$\sup\{I_1, I_2\} = I_1 \cup I_2 \in L$.
We define ϕ and L are always lattice
$\sup\{I_1, I_2\}$ exist.

|| ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ||
– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

कोल्हापूर
१९६४

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Assignment

Student's Sign :	Pallavi
Seat No./ Roll No. :	2202
Seat No./ Roll No. In words	Two two zero two
	03049

प्र. क्र.
Q. No.

from (III) $I(L)$ is lattice.

By Case I), II) and III)

$I(L)$ is lattice under set inclusion.

02	Section	Q. No.										
		Marks										

प्र. क्र.
Q. No.

Q. 3) i) Let, L and L' be lattices.
L is isomorphic to L/θ (quotent lattice).
claim:- $L \cong L/\theta$.
$\gamma\{[x]_\theta\} = \phi(x)$.
Define :- $\gamma : L \to \frac{L}{\theta}$

To prove, γ is isomorphic
i) We have to prove γ is well defined and o
Let, $[x]_\theta$, $[y]_\theta \in L/\theta$.
$[x]_\theta = [y]_\theta$
$\Leftrightarrow x = y$
$\Leftrightarrow \phi(x) = \phi(y)$
$\Leftrightarrow \gamma\{[x]_\theta\} = \gamma\{[y]_\theta\}$ -- ($\because$ def of γ).
γ is well defined and one-one function.

2 4

~~iii)~~
ii) To prove γ is onto.
Let, $[x]_\theta \in L/\theta$
$\phi(x)$ ~~$[x]_\theta$~~ $= a \in L$ be any element.

$\gamma\{[x]_\theta\} = \phi(x)$ ... ($\because$ by def^n of γ)
$\gamma\{[x]_\theta\} = \phi(x) = a$.

γ is onto.

Section	Q. No.													03
	Marks													

iii) To prove ϑ is homomorphism.

let, $[x]_\theta$, $[y]_\theta \in L/\theta$ be any elements.

I) Meet homomorphism -

$$\vartheta\{[x]_\theta \wedge [y]_\theta\} = \vartheta[x \wedge y]_\theta \quad \text{--- } (\because \text{def of } L/\theta).$$
$$= \phi[x \wedge y] \quad \text{--- } (\because \text{def of } \vartheta)$$
$$= \phi(x) \wedge \phi(y) \quad \text{-- } (\theta \text{ is homomorphic})$$
$$= \vartheta\{[x]_\theta\} \wedge \vartheta\{[y]_\theta\}. \quad \text{-- } (\because \text{def of } \vartheta)$$

ϑ is meet homomorphism.

II) Join homomorphism -

$$\vartheta\{[x]_\theta \vee [y]_\theta\} = \vartheta[x \vee y]_\theta \quad \text{-- } (\because \text{def}^n \text{ of } L/\theta)$$
$$= \phi[x \vee y] \quad \text{-- } (\because \text{def}^n \text{ of } \vartheta)$$
$$= \phi(x) \vee \phi(y) \quad \text{--- } (\because \theta \text{ is homomorphic})$$
$$= \vartheta\{[x]_\theta\} \vee \vartheta\{[y]_\theta\} \quad \text{-- } (\because \text{def}^n \text{ of } \vartheta).$$

ϑ is Join homomorphism.

By (I) and (II) ϑ is Homomorphic.

and from (i), (ii) and (iii) ϑ is Isomorphic.

Hence, every homomorphic image of lattice L is isomorphic to quotient lattice of L.