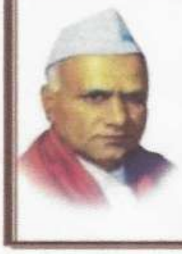

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Vivekanand College, Kolhapur

(Empowered Autonomous)

KOLHAPUR

DEPARTMENT OF MATHEMATICS

Date: 17/09/2024

M. Sc. I Sem. I and M.Sc. II Sem III
Internal Examination 2024-25

All the students of M.Sc. I and M.Sc. II are hereby informed that their Internal Examination of Mathematics will be conducted on **as given below timetable.** The examination will be conducted only one time, students are directed to attend the examination without fail. Syllabus and timetable for examination will be as mentioned in following table.

Syllabus for M. Sc. I Sem. I

Sr. No.	Name of Paper	Topics
1	DSC13MAT11: Modern Algebra	Unit I
2	DSC13MAT12: Ordinary Differential Equation	Unit I
3	DSC13MAT13: Measure and Integration	Unit I
4	DSC13MAT14: Numerical analysis	Unit I
5	DSE13MAT11: Operational Research	Unit I and Unit II (Upto Simplex method)
6	RMD13MAT11: Research Methodology	Unit I

Syllabus for M. Sc. II Sem. III

Sr. No.	Name of Paper	Topics
1	DSC13MAT31: Functional Analysis	Unit I
2	DSC13MAT32: Classical Mechanics	Unit I
3	DSC13MAT33: Complex Analysis	Unit I
4	DSC13MAT34: Advanced Discrete Mathematics	Unit I
5	DSE13MAT31: Lattice Theory	Upto Direct product of lattices

Timetable

Day and Date	Class	Time	Subject
Monday, 23/09/2024	M.Sc. I	12:00PM to 01:00PM	Modern Algebra
	M.Sc. II	12:00PM to 01:00PM	Functional Analysis
Tuesday, 24/09/2024	M.Sc. I	12:00PM to 01:00PM	Ordinary Differential Equation
	M.Sc. II	12:00PM to 01:00PM	Classical Mechanics
Wednesday, 25/09/2024	M.Sc. I	12:00PM to 01:00PM	Measure and Integration
	M.Sc. II	12:00PM to 01:00PM	Complex Analysis
Thursday, 26/09/2024	M.Sc. I	12:00PM to 01:00PM	Numerical analysis
	M.Sc. II	12:00PM to 01:00PM	Advanced Discrete Mathematics
Friday, 27/09/2024	M.Sc. I	12:00PM to 01:00PM	Operational Research
	M.Sc. II	12:00PM to 01:00PM	Lattice Theory
Saturday, 28/09/2024	M.Sc. I	12:00PM to 01:00PM	Research Methodology

Nature of question paper

Time:-1 Hour **Total Marks: 20**

Q.1) Choose the correct alternative for the following question. [04]

i)
ii)
iii)
iv)

Q.2) Attempt any two [08]

i)
ii)
iii)

Q.3) Attempt any One [08]

i)
ii)

(Prof. S. P. Thorat)

HEAD
DEPARTMENT OF MATHEMATICS
VIVEKANAND COLLEGE, KOLHAPUR
(EMPOWERED AUTONOMOUS)

Vivekanand College (Empowered Autonomous), Kolhapur

M.Sc.-I (Sem-I)Internal Examination 2024-25

Modern Algebra (DSC13MAT11)

Day & Date: Monday,23/09/2024 **Time: 12:00PM to 1:00PM** **Marks: 20**

Q.1. Select the correct alternative from each of the following. [04]

i) Consider the following Statements:

I) Every principle series is composition series.

II) Every normal series is principle series.

A) Only I true B) Only II true

C)Both I and II are true D) Both I and II are false

ii) If $|G| = p \ or \ p^2$ where p is prime then G is....

A) Non-abelian B) Cyclic C) Solvable D) Both B) and C)

iii) If H is normal subgroup of G then index of H in G is...

A) 0 B) 1

C) 2 D) 3

iv) A subnormal series of group G is said to be composition series if all factor groups of series are...

A) Abelian B) Simple C) Non-abelian D) None of these

Q.2. Attempt any one. [08]

i) Let G' be a commutator subgroup of group G then show that,

a) G' is normal subgroup of G.

b) $\frac{G}{G'}$ is abelian group.

c) $\frac{G}{N}$ is abelian group iff G' is subgroup of N.

ii) State and prove Schrier Refinement Theorem.

Q.3. Attempt any two. [08]

i) Show that the group of integers Z with respect to addition has no composition series.

ii) If $\emptyset: G_1 \rightarrow G_2$ is onto homomorphism and if G_1 is solvable then show that $G_2 = \emptyset(G_1)$ is solvable.

iii) State and prove Jordan Holder Theorem.

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Vivekanand College, Kolhapur (Empowered Autonomous)
M.Sc. I (Sem-I) Internal Examination 2024-25
Ordinary Differential Equations

Course Code: DSC13MAT12 **Total Marks: 20**
Time: 12.00 pm to 1.00 pm **Date : 24/09/2024**

Q. 1) Select the correct alternative. **[04]**

1) If $\emptyset_1(x)$ and $\emptyset_2(x)$ are two solutions of $L(y) = y'' + a_1 y' + a_2 y = 0$ on an interval I containing point x_0 then $w(\emptyset_1, \emptyset_2)(x) = \ldots \cdot w(\emptyset_1, \emptyset_2)(x_0)$

A) $e^{-a_1(x-x_0)}$ B) $e^{a_1(x-x_0)}$ C) $e^{-a_0(x_0-x)}$ D) $e^{a_0(x_0-x)}$

2) The Order and degree of the equation $\left(\frac{d^2y}{dx^2}\right) = \left(y + \left(\frac{dy}{dx}\right)^6\right)^{\frac{1}{6}}$ is … respectively.

A) 2, 1 B) 2, $\frac{1}{6}$ C) 2, 6 D) 2, 3

3) Wronskian of the two solutions of the differential equation $y'' + a_1(x)y' + a_2(x)y = 0$ o an interval I is ……..

A) Identically Zero B) Never Zero C) Always Constant D) Either identically zero or never zero

4) The functions $\emptyset_1(x) = \sin x$ and $\emptyset_2(x) = e^{ix}$ are…on interval $[-\infty, \infty]$.

A) Linearly dependent B) Linearly independent C) Both A and B D) None of these

Q.2) Attempt any One. **[08]**

1) Define Wronskian and Find all solutions of $y'' + 4y = \cos x$.

2) If $b(x)$ be the continuous function on an interval I every solution φ of $L(y) = b(x)$ on I can be written as $\varphi = \varphi_p + c_1\emptyset_1 + c_2\emptyset_2$ Where φ_p is particular solution and $\emptyset_1$ and $\emptyset_2$ are Linearly Independent solutions of $L(y) = 0$ and c_1 and c_2 are constants. A particular solution φ_p is given By, $\varphi_p = \int_{x_0}^{x} \frac{\emptyset_1(t)\,\emptyset_2(x) - \emptyset_2(t)\emptyset_1(x)}{W(\emptyset_1,\emptyset_2)(x)} b(t)dt$, conversely Every such solution φ is a solution of $L(y) = b(x)$

Q.3) Attempt any Two. **[08]**

1) Check whether the functions e^x, e^{2x}, e^{3x} are linearly independent or not

2) Find the solution of initial Value problem $y'' - 5y' + 6y = 0$, with $y(0) = 0,\ y'(0) = 1$.

3) Show that every solution of constant coefficient equation $L(y) = y'' + a_1 y' + a_2 y = 0$ tends to zero as $x \to \infty$ iff the real part of the roots of the characteristic polynomial are negative.

Vivekanand College Kolhapur (Empowered Autonomous)

Department of Mathematics [M.Sc. I]

Internal Exam

Subject-Measure and Integration Date-25/09/2024

Time-1hr Total Marks-20

Q.1) Choose the correct alternative for each of the following question. [04]

i) **Statement I)**Every closed set is Borel set.

Statement II) Every countable set is Borel set.

a) only (I)true b) only (II)true c) both (I) & (II) true d) both (I) & (II) false.

ii)If A is singleton set then $m^*(A) = \cdots$

a) 0 b) 1 c) 2 d) -1

iii)Which of the following is/are measurable sets

a) any finite set b)any countable set c) [1,2] d) all

iv) If A is measurable set then its complement $A^C$is ...

a)Non measurable b) measurable c) finite d) uncountable.

Q. 2) Attempt any two. [08]

i) Prove that intersection of any finite collection of open sets is open.

ii) Prove that outer measure is translation invarient.

iii) Prove that union of finite collection of measurable sets is measurable.

Q. 3) Attempt any one. [08]

i)Prove that outer measure of an interval is its length.

ii) Prove that collection μ of all measurable sets is $\sigma -$ algebra.

Vivekanand college (Empowered Autonomous) Kolhapur

M.Sc. I (Sem I) Internal Examination :2024-2025

Numerical Analysis (DSC13MAT14)

Day and Date: Thursday,26/09/2024 Time: 12.00pm -1.00 pm Total marks: 20

Q1. Select the correct alternative. **[04]**

1)The rate of convergence of Newton Raphson Method is________

A) Linear B) Quadratic C) Superliner D) cubic

2) In Regula Falsi method, the next approximation to the root is obtained by

A) The midpoint of the interval

B) The intersection of the x-axis with the secant line through two points

C) taking the derivative of the function

D) Averaging the function values at the interval boundaries

3) How many initial guesses are required to start the Secant Method?

A) 1 B) 2 C) 3 D) 4

4) In the Bisection Method, what condition must be satisfied by the function f(x) on the interval [a, b]?

A) $f(a)=f(b)$ B) $f(a)>f(b)$ C) $f(a)\cdot f(b)<0$ D) $f(a)\cdot f(b)=0$

Q2. Attempt any one **[08]**

1)Determine the rate of convergence for Newton Raphson Method.

2)Determine the rate of convergence for Regula Falsi method.

Q3. Attempt any two. **[08]**

1)Find the root of equation $x^3 - 18 = 0$ lying between 2.4 and 3 and correct upto two decimal places usingbisection method.

2)Find the root of equation $f(x) = \cos x - xe^x = 0$ using secant method and correct upto four decimalplaces

3)Find the root of equation $f(x) = x^3 - 2x - 5 = 0$ using Regula Falsi method.

Vivekanand college (Empowered Autonomous) Kolhapur

M.Sc. I (Sem I) Internal Examination 2024-2025 Operation Research (DSE13MAT11)

Day and Date: Friday,27/09/2024 Time: 12.00 PM -1.00 PM Total marks: 20

Q1. Select the correct alternative. [04]

1)If $y \subset R^n$ then the smallest convex set containing y is called__

A) convex function B) convex set C) convex hull D) convex combination

2) Which of the following is a property of a convex set?

A) The set contains all possible linear combinations of its elements.

B) The set contains all possible convex combinations of its elements.

C)Every point in the set is a boundary point.

D)The set contains no interior points.

3) In the matrix form of an LPP, $AX \leq b$, what does the matrix A represent?

A) The matrix of decision variables. B) The matrix of cost coefficients.

C) The matrix of constraint coefficients. D) The matrix of slack variables

4) The extreme points of cube are

A) 4 B) 2 C) 8 D)16

Q2. Attempt any one. [08]

1)Define Convex combination and show that the set of all convex combination of finite number ofpoints of $S \in R^n$ is convex set.

2)Define feasible solution and show that set of feasible solution to LPP is convex set.

Q3. Attempt any two. [08]

1) Show that $S = \{(x1, x2, x3), 2x1 - x2 + x3 \leq 4\}$ is convex set.

2)Find the basic feasible solution

$$\text{Max } (z) = x1 - 12x2$$

Subject to: $x1 + x2 \leq 10, 2x1 - x2 \leq 40; x1, x2 \geq 0$

3)Rewrite in standard form the following LPP

$$\text{Min}(z) = 2x1 + x2 + 4x3$$

Subject to: $-2x1 + 4x2 \leq 4,\ x1 + x3 \geq 5, 2x1 + 3x3 \leq 2; x1, x2 \geq 0, x3$ is unrestricted in sign.

VIVEKANAND COLLEGE, KOLHAPUR (EMPOWERED AUTONOMOUS)

M.SC. Part- I (Mathematics) (Sem-I)

Internal Examination Date:-28/09/2024

Research Methodology

Q.1) Choose correct alternative [04]

i) A corollary is a direct or easy consequence of ________

A) lemma B) theorem C) proposition D) All of the above

ii) The purpose of….. is to summarize the concept of paper.

B) Definition B) Title C) Key words D) Abstract

iii) ….. is the person who did a greatest part.

A) first B) last C) third D) senior person

iv) In mathematics the use of article…. Is inappropriate when the object to which it refers is not unique.

B) An B) A C) The D) none of these

Q.2) Attempt any one [08]

i) What are the Dos and Don'ts of mathematical writing

ii) While writing a paper how to finalize title and Author list.

Q.3) Attempt any two [08]

i) Write short on what is theorem?

ii) How to use notations in mathematical writing

iii) How should you determine Audience while writing a paper?

Vivekanand College, Kolhapur (Empowered Autonomous)
M.Sc. II Semester-III Internal Examination :2024-25
MATHEMATICS

Subject : Functional Analysis | **Date: 23/09/2024**
Time : 12:00 Pm – 01:00 Pm | **Total Marks:20**

Q.1 . Choose correct Alternative for the following. **(4)**

1) Consider following two statements :
   I) Every normed linear space is a metric space.   II) Every metric space is normed linear space.
   A) Only II is true.   B) I is true and II is false   C)Only I is false   D) II is true and I is false.
2) Quotient space N/M = {x + M/ x in N} is norm linear space with respect to norm_____.
   A) $\|x + M\| = \inf\{x + M / x \text{ in } N\}$ B)$\|x + M\| = \inf\{x M / x \text{ in } N\}$
   C) $\|x + M\| = \sup\{x + M / x \text{ in } N\}$ D) $\|x + M\| = \{x + M / x \text{ in } N\}$
3) Identity map I from $(N, \|\ \|_1)$ to $(N, \|\ \|_2)$ is ____then two norms are equivalent .
   A) Homeomorphism   B) Homomorphism   C) Onto   D) Always
4) For any Finite dimensional Normed linear space N all norms are on N are___.
   A)Equal   B) Exactly same   C) Equivalent   D) Different
5) Partial ordering is relation which is____.
   A)Reflexivity, symmetry, transitivity   B) Reflexivity, antisymmetry
   C) Reflexivity, transitivity   D) Reflexivity, antisymmetry, transitivity

Q.2) Attempt any two of the following. **(8)**

1) Show that l_∞ (space of all bounded sequences of scalars) which is normed linear space with $\|\ \|_\infty$ given by $\|x\|_\infty = \sup|x_i|$ for all x in l_∞ is banach space.
3) If N and N' are norm linear space then show that the set B(N,N') of all continuous linear transformation of N into N' is norm linear space with respect to norm $\|T\| = \sup\{\|T(x)\|, x \text{ is in } N \text{ and } \|x\| \leq 1\}$
4) If $\{T_n\}$ and $\{S_n\}$ are sequences in B(N) such that $T_n \to T$ and $S_n \to S$ as $n \to \infty$ then show that, a) $T_n + S_n \to T + S$ b) $kT_n \to kT$ for k in F c) $T_nS_n \to TS$ as $n \to \infty$

Q3) Solve any ONE of the following. **(8)**

1) Define normed linear space. If N and N' are normed linear spaces ,T is linear transformation from N into N' then show that following conditions are equivalent a)T is continuous on N b)T is continuous at origin
   c)there exist a real number $k \geq 0$ with property $\|T(x)\| \leq k\|x\|$ for all x in N
   d)If $s = \{x \text{ in } N \text{ such that } \|x\| \leq 1\}$ is closed unit sphere in N then T(S) is bounded in N'
2) State and prove Riesz theorem.

Vivekanand college (Empowered Autonomous) Kolhapur

Department of Mathematics

M.Sc. II (Sem III) Internal Examination :2024-2025

Subject code: Classical Mechanics Day and Date: Tuesday,24th september2024

Total marks: 20 Time: 12.00pm -1.00pm

Q1. Select the correct alternative. (4)

i) Expression for the Rayleigh's dissipation function is ....

A) $R=\Sigma\lambda_i(\dot{\bar{r}_i})^2$ B) $R=2\Sigma\lambda_i(\dot{\bar{r}_i})^2$

C) $R=\frac{1}{2}\Sigma\lambda_i(\dot{\bar{r}_i})^2$ D) $R=\lambda_i(\dot{\bar{r}_i})^2$

ii) Equation of constraints that contain time as explicit variable are referred as ....

A) holonomic constraints B) non holonomic constraints

C) rheonomic constraints D) scleronomic constraints

iii) The number of generalized co-ordinates in simple pendulum is.......

A) 1 B) 2 C) 3 D) 4

Iv) If the system is conservative then ....

A) $p_j = \frac{\partial L}{\partial \dot{q}_j}$ B) $p_j = \frac{\partial T}{\partial \dot{q}_j}$

C) $p_j = -\frac{\partial L}{\partial \dot{q}_j}$ D) $p_j = \frac{\partial V}{\partial \dot{q}_j}$

Q2. Attempt any one (8)

i) Obtain Lagrange's equations of motion from D'Alembert's Principle for conservative system.

ii) If the cyclic generalized co-ordinate q_j is such that dq_j represents the rotation of the system around some axis $\hat{n}$, then prove that the total angular momentum is conserved along $\hat{n}$.

Q3. Attempt any two of following (8)

i) Show that the Lagrange's equation of motion can also be written as $\frac{\partial L}{\partial t} - \frac{d}{dt}\left(L - \sum_j \dot{q}_j \frac{\partial L}{\partial \dot{q}_j}\right) = 0$

ii) Show that the generalized momentum corresponding to cyclic co-ordinate is conserved

iii) Find the equation of motions for simple pendulum.

Vivekanand College Kolhapur (Empowered Autonomous)

Department of Mathematics (M. Sc. II)

Internal Exam

Subject-Complex Analysis — Date-25/09/2024

Time-1hr — Total Marks-20

Q.1) Choose the correct alternative for each of the following question. [04]

i) Consider the power series $\sum_{n\geq 0} z^{n!}$ then the radius of convergence of the power series around the origin is ....

a) 0 b) 1 c) 3 d) 4

ii) The series $\sum_{n=0}^{\infty} \frac{z^n}{n^n}$ converges on ...

a) $|z| \leq 3$ b) $|z| \leq 4$ c) $|z| < 3$ d) Whole complex plane.

iii) Consider the following statements

I) Mobius transformation is invertible.

II)A Mobius transformation takes circles onto lines.

a) only (I)true b) only (II)true c) both (I) & (II) true d) both (I) & (II) false.

iv) If S & T are Mobius transformation then $S \circ T$ is also ... ...

a) Bilinear Transformation b)Non Bilinear Transformation

c) Mobius Transformation d)None.

Q. 2) Attempt any two. [08]

i) Prove that every Mobius transformation can have at most two fixed points.

ii) Prove that Cross ratio is invarient under any Mobius mapping.

iii) If S is Mobius transformation then prove that S is composition of translation , dilation and inversion.

Q. 3) Attempt any one. [08]

i) If $f(z) = \sum_{n=0}^{\infty} a_n(z-a)^n$ have radius convergence $R > 0$ then prove that for each $k \geq 1$ the series $\sum_{n=k}^{\infty} n(n-1)\dots(n-k+1)a_n(z-a)^{n-k}$ has radius of convergence R.

ii) For given power series $\sum_{n=0}^{\infty} a_n(z-a)^n$ define a number $0 \leq R \leq \infty$ by $\frac{1}{R} = \limsup|a_n|^{\frac{1}{n}}$ then prove that

a) If $|z-a| < R$, the series converges absolutely.

b) If $|z-a| > R$, the terms of series become unbounded and so series diverges.

Vivekanand College (Empowered Autonomous) Kolhapur
M.Sc.-II (Sem-III) Internal Examination 2024-25
Subject: Advanced Discrete Mathematics (DSC13MAT34)

Day & Date: Thursday, 26/09/2024 **Time: 12:00 PM to 1:00 PM** **Total Marks: 20**

Q.1. Select the correct alternative from each of the following. **[04]**

i) Complete graph k_n is.... regular graph.
A) n B) $n-1$ C) $n+1$ D) $2n$

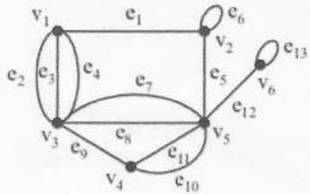
ii) Complete bipartite graph $k_{m,n}$ has.... number of edges.
A) $m+n$ B) m C) mn D) $m-n$

iii) A subgraph H of graph G is said to be spanning subgraph of G if...
A) $V(H) \subset V(G)$ B) $V(H) = V(G)$ C) $V(G) \subset V(H)$ D) None Of these

iv) The join of two vertex disjoint complete graphs is a.......
A) Simple graph B) Complete graph C) Complete bipartite graph D) Bipartite Graph

Q.2. Attempt any One. **[08]**

i) Define simple Graph and complement of Graph.
If G is a simple graph with n-vertices and $\bar{G}$ be its complement, then
a) Prove that for each vertex v in G, $d_G(v) + d_{\bar{G}}(v) = n - 1$.
b) If G has exactly one even vertex, then how many odd vertices does $\bar{G}$ have?



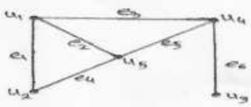
ii) Find graphs
a) G-U and G-F where, $U = \{v_1, v_2, v_3\}$ and $F = \{e_1, e_2, e_6, e_8, e_9\}$
b) G[U], G[F] and also find their union $G[G[U] \cup G[F]]$
where, $U = \{v_2, v_3, v_5\}$ and $F = \{e_1, e_3, e_5, e_7, e_9\}$

Q.3. Attempt any Two. **[08]**

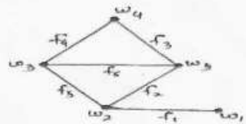
i) Prove that if G is self complementary graph of n vertices then, n is equal to either $4t$ or $4t+1$ for some integer t i.e. $t \in z$.

ii) If G be a k-regular graph where k is odd number then prove that number of edges in G is multiple of k.

iii) Examine whether the following graphs are isomorphic or not.



G_1



G_2

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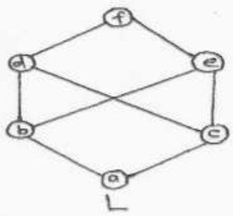
Vivekanand College (Empowered Autonomous) Kolhapur
Internal Examination 2024-25
M.Sc.-II (Sem-III)
Subject: Lattice Theory

Subject Code: DSE13MAT31 Day & Date: Friday, 27/09/2024
Time: 12:00PM to 1:00 PM Marks: 20

Q.1. Select the correct alternative from each of the following. [04]

i) The Hasse Diagram given below is an example of......



A) Lattice B) Semilattice C) Non-lattice poset D) None of these

ii) Consider the statements.
I) Homomorphism in lattices is a isotone map.
II) Every antichain is a lattice.
A) Only I true B) Only II true
C) Both I & II are true D) Both I & II are False

iii) If Sup {a, b} exists $\forall a, b \in P$ in a poset $(P \quad \leq)$ then the poset is ......
A) Meet semilattice B) Semilattice C) Join semilattice D) Lattice

iv) Which of the following is not partial ordered relation?
A) The relation 'less than equal to ($\leq$)' defined on set of natural numbers.
B) The relation $\subseteq$ of a set inclusion defined on a non-empty set.
C) The relation 'Divides (|)' defined on set of natural numbers.
D) The relation $\subset$ of set inclusion defined on a non-empty set.

Q.2. Attempt any one. [08]

i) Prove that a poset $\langle L \quad \leq \rangle$ is a lattice if and only if Sup H and Inf H exists for any non-empty subset H of L.

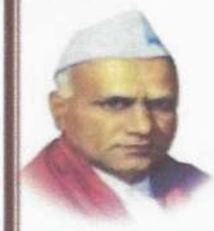
ii) Prove that I is prime ideal if and only if there is homomorphism of L onto C_2 with $I = \emptyset^{-1}(0)$, $I = \{x \in L | \emptyset(x) = 0\}$.

Q.3. Attempt any two. [08]

i) Prove that a finite lattice L can be embedded in $I(L)$ and also in $I_0(L)$.

ii) Prove that if θ be a congruence relation on lattice L then for every $a \in L$, $[a]_\theta$ is convex sublattice of L.

iii) If $D_{36} = \{1,2,3,4,6,9,12,18,36\}$ be the set of divisors of 36 ordered by divisibility then draw the Hasse diagram and find the length and width of the poset D_{36}.



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KOLHAPUR

DEPARTMENT OF MATHEMATICS

Date: 27/02/2025

Notice

M.Sc. I (Sem II) & M.Sc. II (Sem IV)

Internal Examination : 2024-25

All the students of M.Sc. I and II (Mathematics)are hereby informed that their internal examination will be conducted on as given below timetable. The examination will be conducted only one time, students are directed to attend the examination without fail. Syllabus and timetable for examination will be mentioned in following table.

Syllabus for M.Sc. I Sem II :

Sr. No.	Name of the Paper	Topics
1	DSC13MAT21: Linear Algebra	UNIT 1
2	DSC13MAT22: General topology	UNIT 1
3	DSC13MAT23: Advance calculus	UNIT 1&2
4	DSC13MAT24: Numerical Analysis II	UNIT 1
5	DSE13MAT21: Number Theory	UNIT 1

Syllabus for M.Sc. II Sem IV :

Sr. No.	Name of the Paper	Topics
1	DSC13MAT41: Field Theory	UNIT 1 & 2
2	DSC13MAT42: Integral Equation	UNIT 1
3	DSC13MAT43: Partial Differential Equations	UNIT 1
4	DSE13MAT41 Combinatorics	Upto De-arrangements

Timetable:

Day and Date	Class	Time	Subject
Monday, 10/03/2025	M.Sc. I	12:00 PM to 01:00 PM	Linear Algebra
	M.Sc. II	12:00 PM to 01:00 PM	Integral Equations
Tuesday, 11/03/2025	M.Sc. I	12:00 PM to 01:00 PM	General Topology
	M.Sc. II	12:00 PM to 01:00 PM	Partial Differential Equations
Wednesday, 12/03/2025	M.Sc. I	12:00 PM to 01:00 PM	Numerical Analysis II
	M.Sc. II	12:00 PM to 01:00 PM	Combinatorics
Thursday, 13/03/2025	M.Sc. I	12:00 PM to 01:00 PM	Advance Calculus
	M.Sc. II	12:00 PM to 01:00 PM	Field Theory
Saturday, 15/03/2025	M.Sc. I	12:00 PM to 01:00 PM	Number Theory

***Note: All the lectures on the internal exam day will be conducted at 02:00PM to 4:00 PM. Everyone should attend the lectures.**

Nature of Question Paper

Time :- 1 Hour **Total Marks: 20**

Q.1) Choose the correct alternative for each of the following. **[04]**

Four questions

Q.2) Attempt any one **[08]**

Two questions

Q.3) Attempt any two **[08]**

Three questions

Sthorat

(Prof. S. P. Thorat)

HEAD
DEPARTMENT OF MATHEMATICS
VIVEKANAND COLLEGE, KOLHAPUR
(EMPOWERED AUTONOMOUS)

VIVEKANAND COLLEGE, KOLHAPUR (EMPOWERED AUTONOMOUS)

M.Sc. Part-I (Mathematics) (Sem-II) Internal Examination

Course Code: DSC13MAT21

Linear Algebra

Date: 10/03/2025 Time: 12.00 to 1.00 PM Marks: 20

Q.1) Choose the correct alternative for each of the following. **[04]**

i) If V is Finite dimensional vector space, W is subspace of V then dim(A(W)) =…

a) $dim(W) - \dim(V)$ b) $dim(V) - \dim(W)$ c) $dim(W)$ d) $\dim(V)$

ii) Basis of $P_2(\mathbb{R}) = \cdots$

a) $\{x^2\}$ b) $\{1, x^2\}$ c) $\{1, x, x^2\}$ d) $\{1,x\}$

iii) The value of $L(S \cup T) = \cdots$..

a) $L(S) \cup L(T)$ b) $L(S) \cap L(T)$ c) $L(S) - L(T)$ d) $L(S) + L(T)$

iv) If W = {0} then A(W) =….

a) $\hat{V}$ b) 0 c) $\hat{W}$ d) W

Q.2) Attempt any one of the following **[08]**

i) If W is finite dimensional subspace of finite dimensional vector space V then show that $\dim \frac{V}{W} = \dim V - \dim W$.

ii) Prove that if T be a homomorphism of $T: U \to V$ with kernel W then U is isomorphic to $\frac{U}{W}$. Conversely prove that U is a vector space and W is subspace of U then there exists homomorphism U onto $\frac{U}{W}$.

Q.3) Attempt any two of the following **[08]**

i) If $v_1, v_2, \dots . v_n \in V$ then prove that they are either linearly independent or some v_k is a linear combination of preceding once $v_1, v_2, \dots . v_{k-1}$.

ii) If W_1 and W_2 are two subspaces of vector space V which is finite dimensional then prove that $A(W_1 + W_2) = A(W_1) \cap A(W_2)$.

iii) If $\{v_1, v_2, \dots . v_n\}$ be basis of V. Define $\hat{v}_i: V \to F$ such that, $\hat{v}(\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n) = \alpha_i$ $\forall$ i = 1,2,…n. then show that $\hat{v}_i$ is linear Transformation and $\{\hat{v}_1, \hat{v}_2 \dots \hat{v}_n\}$ forms a basis of $\hat{V}$.

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Vivekanand College, Kolhapur (Empowered Autonomous Institute)

M.Sc. I (Semester-II), Internal Examination: February 2025

Course Name: General Topology | Course Code: DSC13MAT22

Day & Date: Tuesday, 11/03/2025 | Time: 12.00 noon to 01.00 pm | Total Marks: 20

Instructions:
1. All questions are compulsory
2. Figures in right side indicates full marks.

Q.1. Select the correct alternative for each of the following. **[04]**

i)Out of the following ....defines a topology on X = {a, b}

a) $\{\emptyset, \{a\}, \{b\}\}$ b) {X, {a}, {b}} c)$\{\emptyset, \{a\}\}$ d) $\{\emptyset, X\}$

ii) Consider statements:

I. Intersection of two topologies on X is again a topology on X.

II. Union of two topologies on X is again a topology on X.

a) only I is true b) only II is true c) both statements are true d) both statements are false

iii) Consider statements:

I. Every neighbourhood of x contains x.

II. Every set containing neighbourhood of x is a neighbourhood of x.

b) only I is true b) only II is true c) both statements are true d) both statements are false

iv) In discrete topology, set of limit point of any subset A of X is............

a) $\emptyset$ b) A c) X – A d) none of them

Q.2 Attempt any One of the following. **[08]**

i) If (X, τ) is topological space then prove that,

a) $\emptyset, X$ are closed sets.

b) Arbitrary intersection of closed sets is closed.

c) Finite union of closed set is closed.

ii) Define topology on a non-empty set X.

If X be a non-empty set and $\tau = \{A \subseteq X \mid X - A \text{ is}$ countable or all of $X\}$ then show that τ is topology on X.

Q.3 Attempt any Two of the following. **[08]**

i) If τ is topology on X then prove that $\mathcal{B}$ is base for τ if and only if every open set in X can be expressed as union of some elements of $\mathcal{B}$.

ii) Let $C: P(X) \to P(X)$ be function defined on $P(X)$ such that $C(A) = \begin{cases} \emptyset, & A = \emptyset \\ X, & A \neq \emptyset \end{cases}$.

Then show that C is Kuratowski closure operator.

iii) Let X ={a, b, c, d} and let $\tau = \{ \emptyset, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}, \}$ find the closure set of each of the following sets

a) {b, c} b) {c} c) {c, d} d) {d}

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Vivekanand College (An Empowered Autonomous Institute), Kolhapur

Department of Mathematics

M.Sc.-I(Sem-II) Internal Examination:2024-25

Subject: Advance Calculus (DSC13MAT23)

Day &Date: Thursday,13/03/2025 Time: 1 hr Total Marks:20

Q.1. Select the Correct Alternatives. [04]

1) Consider the following statements

I. The uniform convergence is sufficient condition for preserving continuity.
II. The uniform convergence is necessary condition for preserving continuity.

A) Only I True. B) Only II True. C) Both (I) &(II) True D) Both (I) &(II) False.

2) If $g(p,q) = \frac{p}{p+q}$ then

A) Double limit exists. B) Double limit does not exist

C) Both iterated limits exist D) Both B) & C)

3) The series $\sum_{n=1}^{\infty} \frac{x}{n^{\beta}(1+nx^2)}$

A) Converges uniformly on any finite interval if $\beta > 1/2$

B) Converges uniformly on any finite interval if $\beta \leq 1/2$

C) Converges uniformly on any finite interval if $\beta < 1/2$

D) Always Divergent.

4)Uniform convergence implies pointwise convergence.

A) Complete statement is true. B) Partial statement is true.

C) Complete statement is false. D) None of these.

Q.2.Attempt any One. [08]

1) Let $f_n(x) = \frac{nx}{1+n^2x^2}$ $x \in [0,1]$

show that $f_n \to f$ on [0,1] & the convergence is pointwise but still

$$\int_0^1 f(x)dx = \lim_{n\to\infty} \int_0^1 f_n(x)dx.$$

2) State & Prove Cauchy Condition for uniform convergence.

Q.3.Attempt any Two. [08]

1) With suitable example show that pointwise convergence of sequences of functions need not preserve continuity.

2)Discuss the existence of two iterated limits & double limit of the sequence $g(p,q) = \frac{pq}{p^2+q^2}$ $p, q \in \mathbb{Z}$

3)Show that the sequence $\{f_n\}$ of functions converges pointwise but not uniformly

where $f_n(x) = \frac{1}{nx+1}$, $0 < x < 1$.

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Vivekanand College Kolhapur (An Empowered Autonomous Institute)
M.Sc. I (Semester II) Internal Examination: March 2025

Course Name: Numerical Analysis-II **Course Code: DSC13MAT24**
Day & Date: Wednesday, 12/03/2025 **Time 12.00 pm to 01.00 pm** **Total Marks: 20**

Q.1. Select the correct alternative for each of the following. **[04]**

i) Shift operator is defined as….

A) $Ef(x) = f(x+h)$ B) $Ef(x) = f(x-h)$

C) $Ef(x) = f(x+h) - f(x)$ D) $Ef(x) = f(x-h) - f(x)$

ii) The truncation error of Lagrange linear interpolation is…

A) $Ef(x) \leq \frac{1}{2}\frac{(x_1+x_0)^2}{4}M_2$ B) $Ef(x) \leq \frac{1}{3}\frac{(x_1-x_0)^2}{4}M_2$

C) $Ef(x) \leq \frac{1}{2}\frac{(x_1-x_0)^2}{4}M_2$ D) $Ef(x) \leq \frac{1}{3}\frac{(x_1+x_0)^3}{4}M_2$

iii) Which of the following is true.

A) $\Delta = E - 1$ B) $\Delta = E + 1$ C) $\nabla = \frac{1}{E} - 1$ D) $\nabla = 1 + E^{-1}$

iv) If $f(2) = 4, f(2.5) = 5.5$ then the Lagrange linear interpolating polynomial is….

A) $9x - 15$ B) $13x - 2$ C) $25x - 7$ D) $3x - 2$

Q.2. Attempt any One of the following. **[08]**

i) Define Interpolating polynomial and Derive Lagrange Linear Interpolation formula.

ii) Derive the numerical integration formula using Newtons Cotes method.

Q.3. Attempt any Two of the following. **[08]**

i) The following data for the function $f(x) = x^4$ is given

x	0.2	0.3	0.4
$f(x)$	0.0016	0.0081	0.0256

Find $f'(0.4)$ and $f''(0.4)$ using quadratic interpolation. Compare the results with exact solution. Obtain the bound on the truncation error.

ii) Evaluate the integral $\int_{x=0}^{x=1}\frac{dx}{1+x}$ using I)Trapezoidal rule II) Composite Simpon's Rule with 2,4 & 8 subintervals.

iii) If $f(0) = 1, f(1) = 3, f(3) = 55$. Find the unique polynomial of degree 2 or less which fits the given data using Newton's Divided difference interpolation.

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Vivekanand College, Kolhapur (An Empowered Autonomous Institute)
M.Sc. I (Sem II) Internal Examination :2024-2025

Subject: Number Theory — Day and Date: Monday,17th March
Total marks: 20 — Time: 12.00pm -1.00 pm

Q1. Select the correct alternative. **(4)**

i) If $\gcd(a,b) = d$ then $\left(\frac{a}{d},\frac{b}{d}\right) =$.......

a) 2 b) 1 c) 3 d) 4

ii) If $\gcd(a,b) = d$ then gcd $(2a + b, a+2b) =$......

a) 1 or 2 b) 2 or 3 c) 1 or 3 d) None of this

iii) Which of the following Diophantine equation cannot be solved?

a) $18x + 42y = 96$ b) $23x + 31y = 105$ c) $45x + 27y = 63$ d) $7x + 56y = 79$

iv) (-100, -200) is particular solution of $7x + 4y = 100$ then general solution is......

a) $x = -100 + 4t, y = 200 - 7t$ b) $x = 100 + 4t, y = 200 + 7t$

c) $x = -100 - 4t, y = 200 - 7t$ a) $x = -100 + 4t, y = -200 - 7t$

Q2. Attempt any one. **(8)**

(i) Prove that for any integers a, b, c following properties holds

i) If $a/b,\ b/c$ then a/c .

ii) If a/b and b/a then $a = \pm b$.

iii) If $a/b,$ and $b \neq 0$ then $|a| \leq |b|$.

iv) If a/b and a/c then $a/bx + cy$ for arbitrary integer x and y.

(ii) Determine all solution in Positive integers of the Diophantine equation $18x + 5y = 48$.

Q3. Attempt any two. **(8)**

i) By using mathematical induction prove that $15/2^{4n-1}$.

ii) State and prove Euclid lemma.

iii) For any integer prove that $\frac{n(n-1)(2n+1)}{6}$ is an integer.

Vivekanand College (An Empowered Autonomous Institute), Kolhapur

Department of Mathematics

M.Sc.-II(Sem-IV) Internal Examination:2024-25

Subject: Field Theory (DSC13MAT41)

Day &Date: Thursday,13/03/2025 Time: 1 hr Total Marks:20

Q.1. Select the Correct Alternatives. [04]

1) If $p(x) = x^2 + x + 1$ be a polynomial in F[x] & b is root of $p(x)$ then [F(b): F] =____

A) 1 B) 2 C)3 D) Either A) or B)

2) Every Algebraic extension is finite extension.

A) Complete statement is true. B) Partial statement is true.

C) Complete statement is false. D) None of these.

3)The minimal polynomial of $\sqrt{2} + 5$ over $\mathbb{Q}$ is

A) $x^2 - 10x + 23$ B) $x^2 + 10x + 23$ C) $x^2 - 10x - 23$ D) $x^2 + 10x - 23$

4) The number of proper fields between $\mathbb{R}$ &$\mathbb{C}$.

A) 1 B)2 C) 3 D) Infinite.

Q.2.Attempt any One. [08]

1) Let $F \subseteq L \subseteq K$ be fields if [L: F] is finite & [K: L] is finite then prove that

[K: F] is finite & [K: F] = [K: L]. [L: F]

2) Let $F \subseteq E$ be fields & $u \in E$ be algebraic over F, then prove that there exists a unique monic irreducible polynomial p(x) ∈ F[x] such that p(u) = 0.

Q.3.Attempt any Two. [08]

1) Prove that every finite extension is algebraic.

2)Let p(x) be an irreducible poynomial in F[x] then prove that there exist an extension E of F in which p(x) ha root.

3)Find the number a such that $\mathbb{Q}(\sqrt{3}, \sqrt{5}) = \mathbb{Q}(a)$.

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Vivekanand College, Kolhapur (An Empowered Autonomous Institute)
M.Sc. II (Semester-IV), Internal Examination: March 2025

Course Name: Integral Equations **Course Code: DSC13MAT42**

Day & Date: Monday, 10/03/2025 **Time: 12.00 pm to 01.00 pm** **Total Marks: 20**

Instructions:

1. All questions are compulsory
2. Figures in right side indicates full marks.

Q.1. Select the correct alternative for each of the following. **[04]**

i) If 'a' is constant and $n \in \mathbb{N}$ then $\int_a^x \int_a^x \dots \int_a^x f(t)dt^n$ = ...

a) $\frac{1}{(n-1)!}\int_a^x (x-t)^{n-1} f(t)dt$

b) $\frac{1}{(n-1)!}\int_x^a (x-t)\ f(t)dt$

c) $\frac{1}{(n-1)!}\int_x^a (x-t)^{n-1} f(t)dt$

d) $\frac{1}{(n-1)!}\int_a^x (x-t)\ f(t)dt$

ii) If $g(s) =$ .... is the solution of integral equation $g(s) = 1 + \int_0^s g(t)dt$.

a) s b) e^{2s} c) e^{s} d) e^{3s}

iii) The following integral equation $x(t) = \sin t + \lambda\int_0^{2\pi} \sin(t+s)\ x(s)ds$ is.........

a) Fredholm integral equation of second kind

b) Fredholm integral equation of first kind

c) Volterra integral equation of second kind

d) Volterra integral equation of first kind

iv) An integral equation is called .... if one or both the limits are infinite or the kernel of equation becomes infinite at one or more points of interval of integration.

a) convolution type of integral equation

b) singular integral equation

c) integro differential equation

d) non-singular integral equation

Q.2 Attempt any One of the following. **[08]**

i) Describe the procedure of finding eigen values and eigen functions for the homogeneous FIE of second kind with separable kernel.

ii) Convert the BVP $y'' + y = x,\ \ y(0) = 1,\ y(\pi) = \pi - 1,\ 0 \le x \le \pi$ into integral equation and obtain the original BVP from the integral equation that you obtain.

Q.3 Attempt any Two of the following. **[08]**

i) Convert the following IE to BVP

$$y(x) = \lambda\int_0^1 K(x,t)y(t)dt\text{, where, } K(x,t) = \begin{cases}(1-t)x, & 0 \le x \le t\\(1-x)t, & t \le x \le 1.\end{cases}$$

ii) Find the eigen values and eigen function of $g(s) = \lambda\int_0^1 st\ g(t)dt$.

iii) Convert the following IVP to IE and conversely derive the original IVP from IE that you obtain $y'' + y' = 0,\ \ \ y(1) = 0,\ y'(1) = 1$.

Vivekanand College, Kolhapur (An Empowered Autonomous Institute)

M.Sc. II (Semester-IV) Internal Examination: March 2025

Course Name: Partial Differential Equations **Course Code: DSC13MAT43**
Day & Date: Tuesday, 11/03/2025 **Time: 12.00 pm to 01.00 pm** **Total Marks: 20**
Instructions:
1. All questions are compulsory.
2. Figures in right side indicates full marks.

Q.1. Select the correct alternative for each of the following. [04]

i) The equation …… represents the set of all right circular cones with z-axis as the axis of symmetry.

A) $x^2 + y^2 = (z - c)^2 tan^2\alpha$ B) $z^2 + y^2 = (y - c)^2 tan^2\alpha$
C) $z^2 + y^2 = (x - c)^2 tan^2\alpha$ D) $x^2 - y^2 = (z - c)^2 tan^2\alpha$

ii) The equation $(x^2 + z^2)p - xyq = z^3x$ is ……

A) linear B) semi-linear C) quasi-linear D) non-linear

iii) The singular integral of the equation $z = ax + by + a^2 + b^2$ is …….

A) $4z + (x^2 - y^2)$=0 B) $z - (x^2y^2)$=0
C) $z - (x^2 + y^2)$=0 D) $4z + (x^2 + y^2)$=0

iv) The partial differential equation obtained from the family $z = (x + a)(y + b)$ is ……

A) $z = p^2q$ B) $z = pq$ C) $z = p - q$ D) $z = \frac{p}{q}$

Q.2 Attempt any One of the following. [08]

i) If $u(x, y)$ and $v(x, y)$ be two functions of x *and* y such that $\frac{\partial v}{\partial y} \neq 0$ if further $\frac{\partial(u,v)}{\partial(x,y)} = 0$ then show that $\exists$ a relation $F(u, v) = 0$ between u and v not involving x *and* y explicitely

ii) Derive the necessary and sufficient condition for the integrability of Pfaffian Differential Equation.

Q.3 Attempt any Two of the following. [08]

i) Find the general integral of the equation $(x^2 + y^2)p + 2xyq = (x + y)z$.

ii) Show that $(x - a)^2 + (y - b)^2 + z^2 = 1$ is complete integral of $z^2(1 + p^2 + q^2) = 1$ by taking A)$b = 2a$, B)$b = a$. Show that envelope of subfamilies are respectively $(y - 2x)^2 + 5z^2 = 5$ and $(x - y)^2 + 2z^2 = 2$ which are particular integrals.

iii) Obtain the partial differential equation by eliminating the arbitrary constant and functions from the following relations.

A) $z = F\left(\frac{x}{\ }\right)$ B) $F(x + y, x - \sqrt{z}) = 0$

Vivekanand College Kolhapur (An Empowered Autonomous Institute)

M.Sc. II (Semester IV) Internal Examination: March 2025

Course Name: Combinatorics · **Course Code: DSE13MAT41**

Day & Date: Wednesday, 12/03/2025 **Time 12.00 pm to 01.00 pm** **Total Marks: 20**

Q.1. Select the correct alternative for each of the following. **[04]**

i) The number of derangements of 6 distinct objects are...

A) 3 B) 24 C) 265 D) 20

ii) The coefficient of $x_1^2x_2^3x_3^3x_4^2x_5^4$ in the expansion of$(x_1 + x_2 + x_3 + x_4 + x_5)^{10}$ is....

A) 510 B) 1050 C) 225 D) 100

iii) In how many ways can 7 girls form a ring.

A) 6 B) 7 C) 720 D) 120

iv) The number of proper divisors of 2500 are...

A) 10 B) 12 C) 25 D) 13

Q.2. Attempt any One of the following. **[08]**

i) Among the integers 1 to 300 find how many are not divisible by 3 nor by 5. Also find how many are not divisible by 7 but divisible by 3 and also find how many are not divisible by 3,5,7.

ii) Define Ramsey Number.

a) Show that $R(p \quad q) = R(q \quad p)$

b) Show that $R(2 \quad p) = p$.

Q.3. Attempt any Two of the following. **[08]**

i) Using combinatorial argument prove that $C(m + n \quad 2) = C(m \quad 2) + C(n \quad 2) + mn$.

ii) In how many ways can one select a cricket team of 11 players from 17 players in which only 5 players can bowl if each cricket team of 11 players must include exactly 4 bowlers.

iii) If there are m pigeons and n pigeonholes then prove that at least one pigeonhole contains p+1 pigeons where $p = \left\lfloor \frac{m-1}{n} \right\rfloor$. Show that 9 colours are used to paint 100 houses then at least 12 houses will be of same colour.

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1203, M.Sc. I
Advance Calculus

25334

(अधिकारप्रदत्त स्वायत्त)
कोल्हापूर

Signature of Jr. Super.

विवेकानंद कॉलेज, कोल्हापूर. (अधिकरप्रदत्त स्वायत्त)

______ परीक्षेच्या ______ या विषयाच्या प्रयोग परीक्षा

Practical Examination in, ______

at the ______ Examination

उमेदवराचा आसन क्रमांक (Candidate's Seat No.) ______ विभाग (Section) ______

उमेदवारांना सूचना

१. प्रश्न काळजीपूर्वक वाचा आणि त्याप्रमाणे विचारलेला प्रयोग करा.
२. उपकरणांच्या वापराबाबत तुम्हांला काही माहीत नसेल तर परीक्षक किंवा प्रयोगशाळा सहाय्यक यांना तुम्हाला मदत करण्याविषयी विनंती करा.
३. कोणताही विद्युतप्रयोग करण्यापूर्वी, प्रत्यक्ष पुरविलेली सर्व उपकरणे आणि सर्व 'कनेक्शन' नीट पाहून घेऊन संबंधित कामाची नीटनेटकी कार्ययोजना करण्याची नितांत आवश्यकता आहे आणि ह्यानंतर पुढे काम चालू करण्याविषयी परीक्षकांची परवानगी मिळविणे आवश्यक आहे.
४. सर्व निरीक्षणे कोटकवजा तक्त्यात भरावी. मधल्या सर्व गणना आणि निर्णय हे क्य तितक्या सुवाच्चपणे आणि स्पष्टपणे नोंदविलेले असणे हे हितावह आहे.
५. प्रारंभिक किंवा अंतिम निरीक्षणात संख्यावाचक आकडे एकावर एक लिहू नयेत. जर लिहिलेला कोणताही आकडा नको असेल तर त्यावर एक रेघ ओढून पाहिजे असलेला आकडा त्याच्याजवळ लिहा. प्रयोगशाळेतून बाहेर पडण्यापूर्वी आपले टेबल चांगल्या स्थितीत आहे याची खात्री करा.

INSTRUCTIONS TO CANDIDATES

1. Read the question carefully and perform the experiment as required.
2. If there by anything the apparatus that you do not know, ask the examiner or the laboratory assistant to help you.
3. Before doing any electrical experiment, it is absolutely essential that you make a neat working sketch of all apparatus actually provided and of the necessary connection and obtain the examiner's permission to proceed.
4. Express all observations in a tabular form. It is also desirable that all intermediate calculations and results should be entered as neatly and clearly as possible.
5. No numerical figures should be written over either in the preliminary or final observations. If any figure is shought to be discarded it should be run through and the desired figure written near to it.
6. Please see that your table is in good order before you leave the laboratory.

(येथून लेखनास सुरवात करा.) (Begin writing here.)

प्र. क्र.
Q. No.

17/20

Q. 1)

i) B) only II True.

ii) A) Double limit exist.

iii) B) Converges uniformly on any finite interval $\leq 1/2$.

iv) A) Complete statement is true.

02	Section	Q. No.										
		Marks										

प्र. क्र.
Q. No.

Q. 3)

1) Consider an example, $f_n(x) = \frac{x^{2n}}{1+x^{2n}}$

$f_1(x) = \frac{x^2}{1+x^2}$,

$f_2(x) = \frac{x^4}{1+x^4}$ and so on

We observed that,
$f_n(x) = \frac{x^{2n}}{1+x^{2n}}$ is continous.

$f(x) = \sum_{n\to\infty} f_n(x)$

$f(x) = \sum_{n\to\infty} \frac{x^{2n}}{1+x^{2n}}$

Case 1: ⇒ $|x| < 1$

$f(x) = \frac{x^{2n}}{1+x^{2n}}$

$\therefore f(x) = 0$

Section	Q. No.													03
	Marks													

प्र. क्र. Q. No.

Case ii) $|x| > 1$

$f(x) = \frac{x^{2n}}{1+x^{2n}}$

by taking derivative

$\therefore f(x) = 1$

Case iii) $|x| = 1$

$f(x) = \frac{x^{2n}}{1+x^{2n}}$

$\therefore f(x) = 1/2$.

By the cases i), ii) and iii)

$f_n(x) = \begin{cases} 0, & \text{if } |x|<1 \\ 1, & \text{if } |x|>1 \\ 1/2, & \text{if } |x|=1 \end{cases}$

the the function f is given as

$f(x) = \begin{cases} 0, & \text{if } |x|<1 \\ 1, & \text{if } |x|>1 \\ 1/2, & \text{if } |x|=1 \end{cases}$

f(x) is not continous convergent at $x=1$ and $x=-1$.

Hence, Pointwise convergence of sequences of functions need not preserve continuity.

08

04

04 Section	Q. No.												
	Marks												

प्र. क्र. Q. No.

Q. 3)

iii) Given - $f_n(x) = \frac{1}{nx+1}$, $0 < x < 1$.

$f(x) = \sum_{n \to \infty} f_n(x)$

$= \frac{1}{nx+1}$

$f(x) = x$

$\{f_n(x)\} = f(x) =$ continous.

$\lim_{n \to \infty} f(x) = \lim_{n \to \infty} f_n(x)$

$= \lim_{n \to \infty} \frac{1}{nx+1}$

$= 0$

ok

$\therefore$ Sequence $\{f_n\}$ of functions converges pointw
but not uniformly.

02	Section	Q. No.										
		Marks										

प्र. क्र.
Q. No.

Q.2) i) Given - $f_n(x) = \frac{nx}{1+n^2x^2}$, $x \in [0,1]$.

$|f_n(x)| = \left| \frac{nx}{1+n^2x^2} \right|$

$= \frac{1}{1+n^2+2n}$ $x \in [0,1]$

$|f_n(x)| = \frac{1}{(1+n)^2} \Rightarrow 0$

$\Rightarrow f_n$

$\lim_{n \to \infty} f_n(x) = \lim_{n \to \infty} \frac{1}{(1+n)^2}$, $x \in [0,1]$

$= \left[\frac{1}{(1+n)^2} \right]_0^1$

$\lim_{n \to \infty} f_n(x) = 0$

$\lim_{n \to \infty} f(x) = \lim_{n \to \infty} 0 \cdot dx = 0.$

$\therefore \lim_{n \to \infty} f_n(x) = \lim_{n \to \infty} f(x).$

Section	Q. No.													
	Marks													

प्र. क्र. Q. No.

$$\lim_{n\to\infty} f_n(x) = 0$$

$$\int_0^1 f(x)\,dx = \int_0^1 0\,dx$$

$$\therefore \int_0^1 f(x)\,dx = 0$$

$$\therefore \lim_{n\to\infty} f_n(x) = \int_0^1 f(x)\,dx.$$

2)

i) Given :- $f_n(x) = \dfrac{nx}{1+n^2x^2}$, $x \in [0,1]$

$$f(x) = \lim_{n\to\infty} f_n(x)$$

$$= \lim_{n\to\infty} \frac{nx}{1+n^2x^2}$$

$$f(x) = 0.$$

$\therefore f_n \to f$ on $x \in [0,1]$.

$$\lim_{n\to\infty} f_n(x) = \frac{nx}{1+n^2x^2}.$$

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04	Section	Q. No.											
		Marks											

प्र. क्र.
Q. No.

$$\lim_{n\to\infty} f_n(x)dx = 0$$

$$\int_0^1 f(x) = 0 \cdot dx = 0$$

$$\therefore \lim_{n\to\infty} f_n(x) = \int_0^1 f(x) = 0.$$

$\therefore f_n \to f$ on $[0,1]$ and the convergence is point
but still $\int_0^1 f(x)\,dx = \lim_{n\to\infty} f_n(x)\,dx$.

08

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(अधिकारप्रदत्त स्वायत्त)
कोल्हापूर

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विवेकानंद कॉलेज

(अधिकारप्रदत्त स्वायत्त)

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Q.1	Q.2	Q.3	Total
04	08	08	20

$0 = x + 2a$

$0 = y + 2b$

$a = -\frac{x}{2}$ $b = -$

प्र. क्र.
Q. No.

Q 1.

i) A] $x^2+y^2 = (z-c)^2 \tan^2\alpha$ — 01

ii) C] Quasi - linear. — 01

iii) D) $4z + (x^2+y^2) = 0$. — 01

iv) B) $z = pq$. — 01

04

02 Section	Q. No.										
	Marks										

प्र. क्र.
Q. No.

Q2.

i] Here the nessary condition
Let there exist between two fun
a relation of the type

$F(u,v) = 0$ ——— (1)

where the functions are $u(x,y)$ an
not involving x or y explicitly
Differentiate eqn (1) partially w.r.t
we get

$F_u u_x + F_v v_x = 0$ ——— (2)

08

Differentiate eqn (1) partially w.
we get,

$F_u u_y + F_v v_y = 0$ ——— (3)

From eqn (2) and (3) Eliminate F_v

$$\frac{F_u u_x}{F_u u_y} = \frac{-F_v v_x}{-F_v v_y}$$

$$\frac{u_x}{u_y} = \frac{v_x}{v_y}$$

$$u_x v_y - v_x u_y = 0$$

$$\begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix} = 0$$

$$\frac{\partial(u,v)}{\partial(x,y)} = 0$$

The sufficient condition.

Section	Q. No.												
	Marks												

प्र. क्र.
Q. No.

Let $u(x,y)$ and $v(x,y)$ be two functions of x and y such that $\frac{\partial v}{\partial x} \neq 0$ and $\frac{\partial v}{\partial y} \neq 0$

let $\frac{\partial(u,v)}{\partial(x,y)} = 0$

We claim that $F(u,v)=0$ not involving x explicitely.

Eliminating 'y' from two function $u(x,y)$ and $v(x,y)$ we get a relation

$$F(u,v,x) = 0 \quad \text{——} \quad (4)$$

Differentiate eqn (4), w.r.t. x and y partially we get

$$\left.\begin{aligned} F_x + F_u u_x + F_v v_x &= 0 \\ F_u u_y + F_v v_y &= 0 \end{aligned}\right\} \quad (5)$$

Eliminating F_v from above eqns we ge

$$\frac{F_x + F_u u_x}{F_u u_y} = \frac{-F_v v_x}{-F_v v_y}$$

$$\frac{F_x + F_u u_x}{F_u u_y} = \frac{v_x}{v_y}$$

$$F_x v_y + F_u u_x v_y - F_u u_y v_x = 0$$

$$F_x v_y + F_u [u_x v_y - u_y v_x] = 0$$

$$F_x v_y + F_u \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix} = 0$$

$$F_x v_y + F_u \frac{\partial(u,v)}{\partial(x,y)} = 0$$

$$\Rightarrow F_x v_y = 0$$

$$\Rightarrow F_x = 0$$

04	Section	Q. No.										
		Marks										

प्र. क्र.
Q. No.

Rough.

$z = ax + by + a^2 + b^2$

$0 = x + 2a$

$0 = y + 2b$

$a = \frac{-x}{2}$

$b = \frac{-y}{2}$

$z = -\frac{x}{2} \cdot x - \frac{y}{2} \cdot y + \frac{x^2}{4} + \frac{y^2}{4}$.

$= \frac{-(x^2+y^2)}{2} + \frac{(x^2+y^2)}{4}$

$= \frac{-2(x^2+y^2) + (x^2+y^2)}{4}$

$4z = -(x^2+y^2)$

$4z + x^2 + y2$

$p = (y+b)$

$q = (x+a)$

$z = pq$.

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– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

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प्र. क्र. Q. No.

⟹ The function does not contain variab 'x' explicitely

From eqn (4), we get

$F(u,v) = 0$

Hence proved.

Q3.

ii] Let Given eqn is

$(x-a)^2 + (y-b)^2 + z^2 = 1$ —(1)

Claim: $(x-a)^2 + (y-b)^2 + z^2 = 1$ is complete integral of $z^2(1+p^2+q^2) = 1$.

$$M = \begin{pmatrix} F_a & F_{xa} & F_{xb} \\ F_b & F_{ya} & F_{yb} \end{pmatrix}$$

$$= \begin{pmatrix} -2(x-a) & 1 & 0 \\ -2(y-b) & 0 & 1 \end{pmatrix}$$

$\therefore |M| \neq 0$

$\therefore$ Rank is 2

$\therefore \rho(M) = 2$.

02	Section	Q. No.										
		Marks										

प्र. क्र.
Q. No.

$(x-a)^2+(y-b)^2+z^2=1$ is comp[illegible]
integral of $z^2(1+p^2+q^2)=1$

A] $b=2a$.

Then parameter family (1) becor[illegible]

$(x-a)^2+(y-2a)^2+z^2=1$ —

Diff. eqn (1) w.r.t a.

$$2(x-a)(-1)+2(y-2a)(-2)=0$$

$$-2(x-a)-4(y-2a)=0$$

$$-2x+2a-4y+8a=0$$

$$10a=2x+4y$$

$$a=\frac{x+2y}{5}$$

$$\left[x-\left(\frac{x+2y}{5}\right)\right]^2+\left(y-2\left(\frac{x+2y}{5}\right)\right)^2+z^2$$

$$x^2-\frac{2x(x+2y)}{5}+\frac{(x+2y)^2}{25}+y^2-\frac{4y(}{}$$

$$+4\frac{(x+2y)^2}{25}+z^2=$$

$$x^2+y^2-\left(\frac{2x(x+2y)}{}\right)-\left(\frac{x+2y}{5}\right)\left[2x+4\right.$$

$$+\frac{(x+2y)^2}{5}+z^2=1$$

$$x^2+y^2-2\frac{(x+2y)^2}{5}+\frac{(x+2y)^2}{5}$$

$$x^2+y^2-\frac{(x+2y)^2}{5}+z^2=1$$

Section	Q. No.											
	Marks											

प्र. क्र.
Q. No.

$$5x^2+5y^2-[x^2+4xy+4y^2]+5z^2=5.$$

$$5x^2+5y^2-x^2-4xy-4y^2+5z^2=5.$$

$$4x^2-4xy+y^2+5z^2=5.$$

$$(2x-y)^2+5z^2=5$$

i.e $(y-2x)^2+5z^2=5.$

which is envelop of eqn (2).

B] $b=a$

Then parameter family (1) becomes

$$(x-a)^2+(y-a)^2+z^2=1 \quad \text{---} (3)$$

Diff. eqn (3) w.r.t. a

$$2(x-a)(-1)+2(y-a)(-1)=0$$

$$-2x+2a-2y+2a=0$$

$$4a=2x+2y$$

$$a=\frac{x+y}{2}.$$

Eqn (3) becomes

$$\left(x-\left(\frac{x+y}{2}\right)\right)^2+\left(y-\left(\frac{x+y}{2}\right)\right)^2+z^2=1$$

$$x^2-\frac{2x(x+y)}{2}+\frac{(x+y)^2}{4}+y^2-\frac{2y(x+y)}{2}+\frac{(x+y)^2}{4}$$

$$x^2+y^2-(x+y)^2+\frac{2(x+y)^2}{4}+z^2=1$$

$$x^2+y^2-(x+y)^2+\frac{(x+y)^2}{2}+z^2=1$$

$$x^2+y^2-\frac{(x+y)^2}{2}+z^2=1$$

04	Section	Q. No.										
		Marks										

प्र. क्र.
Q. No.

$2x^2+2y^2-[x^2+y^2+2xy]+2z^2=2$

$2x^2+2y^2-x^2-y^2-2xy+2z^2=2$

$x^2+y^2-2xy+2z^2=2$

$(x-y)^2+2z^2=2$

Which is required envelope of sub (3).

iii)

A] Given equation is

$z=F\left(\frac{x}{y}\right)$ —— (1)

Diff. eqn (1) partially w.r.t. x.

$p=F'\left(\frac{x}{y}\right)\frac{1}{y}$ —— (2)

Diff. eqn (1) partially w.r.t y

$q=F'\left(\frac{x}{y}\right)\frac{-1}{y^2}$ —— (3)

04

Divide eqn (2) and (3)

$\frac{p}{q}=\frac{F'(x/y)\ 1/y}{F'(x/y)\ -1/y^2}$

$\frac{p}{q}=-y$

$p=-qy$

$p+qy=0$

Which is required partial differentia equation.

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Q. No.

ii] B] Given equation is ~~in the~~

$F(x+y, x-\sqrt{z}) = 0$ ——— (1)

Given eqn is in the form of

$F(u,v) = 0$ ——— (2)

where,

$u = x+y$, $v = x - \sqrt{z}$.

$u_x = 1$ $v_x = 1 - \left(\frac{1}{2\sqrt{z}} \cdot p\right)$

$u_y = 1$

$v_y = -\frac{1}{2\sqrt{z}} q$.

Diff. eqn (2) partially w.r.t. x

$F_u u_x + F_v v_x = 0$ ——— (3)

Diff. eqn (2) partially w.r.t y

$F_u u_y + F_v v_y = 0$ ——— (4)

From (3) and (4) we get

$\frac{F_u u_x}{F_u u_y} = \frac{-F_v v_x}{-F_v v_y}$

$\frac{u_x}{u_y} = \frac{v_x}{v_y}$

02	Section	Q. No.											
		Marks											

प्र. क्र.
Q. No.

$$u_x v_y - v_x u_y = 0$$

$$1\left(\frac{-1}{2\sqrt{2}} q\right) - (1)\left[1 - \frac{1}{2\sqrt{2}} p\right] = 0$$

$$\frac{-q}{2\sqrt{2}} - 1 + \frac{p}{2\sqrt{2}} = 0$$

$$\frac{p-q}{2\sqrt{2}} = 1$$

$$p - q = 2\sqrt{2}$$

Which is required partial d.e.

i) $(x^2+y^2)p + 2xyq = (x+y)z$.

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