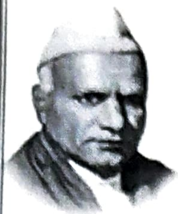

"Education for Knowledge, Science, and Culture"
- Shikshanmaharshi Dr. Bapuji Salunkhe

Shri Swami Vivekanand Shikshan Sanstha's

Vivekanand College, Kolhapur

(Empowered Autonomous)

KOLHAPUR

DEPARTMENT OF MATHEMATICS

Date: 10/09/2024

Notice

B.Sc. II (Sem III)

Unit Test: 2024-25

All the students of B.Sc. II Sem III (Major Mathematics) are hereby informed that their unit test will be conducted on, **Wednesday 18/09/2024.** Syllabus and timetable for the unit test will be mentioned in following table. All students are directed to present for unit test on time.

Syllabus for Unit test B.Sc. II Sem III:

Sr. No.	Name of the paper	Units	Time
1	Multivariable calculus	i)Jacobian ii)Vector Differential Calculus	01:00 PM to 02:00 PM

Nature of Question Paper

Time :- 1 Hour **Total Marks: 20**

Q.1) Choose the correct alternative for each of the following. **[04]**

Four questions

Q.2) Attempt any one **[08]**

Two questions

Q.3) Attempt any two **[08]**

Three questions

VIVEKANAND COLLEGE KOLHAPUR (Empowered Autonomous) ESTD JUNE 1964

Thorat

(Prof. S. P. Thorat)

HEAD
DEPARTMENT OF MATHEMATICS
VIVEKANAND COLLEGE, KOLHAPUR
(EMPOWERED AUTONOMOUS)

Vivekanand College Kolhapur (Empowered Autonomous)
Department of Mathematics
B.Sc. II (Sem III)
Multivariable Calculus
Unit Test 2024-25

Time & date: 18/09/2024 **Total Marks: 20**

Sr. No.	Roll No.	Name of student	Sign.	Marks
1	7744	CHOUGALE ARPITA BALAVANT	Chougale	
2	7745	HARGE SANCHIT GURUPRASAD	Ab	
3	7746	KHADE RIYA BALKRISHNA	R.B.Khade	
4	7747	MAYEKAR PRACHI PUNDLIK	[illegible]	
5	7748	PATIL ARIHANT KIRAN	AKPatil	
6	7749	PATIL RAJNANDINI KUNDALIK	Ab	
7	7750	TONAPE SAHIL SHIVAJI	SahilTonape	
8	7751	KAMBLE SAMRAT SURESH	KambleSS	
9	7960	BANDUKE SIDDHANT SACHIN	Banduke	
10	7972	LAD ALOK BAJARANG	Ab	

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Khorat
HEAD
DEPARTMENT OF MATHEMATICS
VIVEKANAND COLLEGE, KOLHAPUR
(EMPOWERED AUTONOMOUS)

Vivekanand College, Kolhapur (Empowered Autonomous)
B.Sc. II (Semester-III)
Unit test: September 2024

Course Name: Multivariable Calculus
Course Code: DSC03MAT31
Day & Date: Wednesday, 18/09/2024
Time:01.00 PM to 02.00 PM
Total Marks: 20
Instructions:
1. All questions are compulsory
2. Figures in right side indicates full marks.

Q. 1 Select the correct alternative. [04]

i) If $x = r\cos\theta, y = r\sin\theta$ then $\frac{\partial(x,y)}{\partial(r,\theta)}\frac{\partial(r,\theta)}{\partial(x,y)} =$ ___ .

A) r^2 B) r C) 1 D) 0

ii) If u = $\sin\theta\cos\varphi$ and v = $\sin\theta\sin\varphi$ then $\frac{\partial(x,y)}{\partial(\theta,\varphi)} =$ ___ .

A) $\sin\theta$ B) $\cos\varphi$ C) $\sin\theta\cos\theta$ D) $\sin\varphi\cos\varphi$

iii) If $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ then $grad(r) =$...

A) $\frac{\bar{r}}{r}$ B) $\frac{r}{\bar{r}}$ C) r D) $\bar{r}$

iv) If $\bar{f}$ is a constant vector, then....

A) $div\bar{f} = 0$ and $curl\bar{f} \neq 0$ B) $div\bar{f} \neq 0$ and $curl\bar{f} \neq 0$
C) $div\bar{f} = 0$ and $curl\bar{f} = 0$ D) $div\bar{f} \neq 0$ and $curl\bar{f} = 0$

Q.2 Attempt any One. [08]

i) If p, q are functions of u, v and u, v are functions of x, y then show that $\frac{\partial(p,q)}{\partial(x,y)} = \frac{\partial(p,q)}{\partial(u,v)}\frac{\partial(u,v)}{\partial(x,y)}$.

ii) a) Find the directional derivative of $x^2 + y^2 + z^2$ at $(1, 2, 3)$ in the direction of $3\hat{\jmath} + 4\hat{k}$.

b) Find i) $\nabla^2 \log(x^2 + y^2)$

Q.3 Attempt any Two. [08]

i) If $u = 3x + 2y - z, v = x - 2y + z, w = x + 2y - z$ then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

ii) If $\bar{r}$ is position vector of point (x, y, z) and r is $|\bar{r}|$ then, prove that $div(r^n \cdot \bar{r}) = (n+3)r^n$

iii) If $v = 2x^2\hat{\imath} - 3yz\hat{\jmath} + xz^2\hat{k}$ and $\emptyset = 2z - x^3y$ find $v \cdot \nabla\emptyset$ at $(1, -1, 1)$

Name :- ~~Axpita~~ Arpita Balavant Chougale.

।। ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ।।

- शिक्षणमहर्षी डॉ. बापूजी साळुंखे

39180

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor: Amar

Suppliment No. : 1

Roll No. : 7744

Class : B.Sc. II

Subject : Multivariable Calculus.

Test / Tutorial No. : 1

Div. : A

03 + 08 + 08 = $\frac{19}{20}$ A

3

Q.1

03

i) c) 1

ii) c) $\sin\theta\cos\theta$

iii) D) $\bar{r}$

iv) c) $\operatorname{div}\bar{F} = 0$ & $\operatorname{curl}\bar{F} = 0$.

Q.2 Statement :-

If p, q are functions of u, v & u, v are functions of x, y then show that $\frac{\partial(p,q)}{\partial(x,y)} = \frac{\partial(p,q)}{\partial(u,v)} \cdot \frac{\partial(u,v)}{\partial(x,y)}$

Proof :-

let, p, q be functions of u & v

$$\therefore J_1 = \frac{\partial(p,q)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial p}{\partial u} & \frac{\partial p}{\partial v} \\ \frac{\partial q}{\partial u} & \frac{\partial q}{\partial v} \end{vmatrix}$$

Let u, v be functions of x & y

$$\therefore J_2 = \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$

8

Let, p, q be function of x, y

$$\therefore J_3 = \frac{\partial(p,q)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial p}{\partial x} & \frac{\partial p}{\partial y} \\ \frac{\partial q}{\partial x} & \frac{\partial q}{\partial y} \end{vmatrix}$$

To prove that $J_3 = J_1 \cdot J_2$
we will consider
p, q composite function of x, y

we get,

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial p}{\partial v} \cdot \frac{\partial v}{\partial x}.$$

$$\frac{\partial p}{\partial y} = \frac{\partial p}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial p}{\partial v} \cdot \frac{\partial v}{\partial y}.$$

$$\frac{\partial q}{\partial x} = \frac{\partial q}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial q}{\partial v} \cdot \frac{\partial v}{\partial x}$$

$$\frac{\partial q}{\partial y} = \frac{\partial q}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial q}{\partial v} \cdot \frac{\partial v}{\partial y}$$

Now,

consider,

$$J_1 \times J_2 = \begin{vmatrix} \frac{\partial p}{\partial u} & \frac{\partial p}{\partial v} \\ \frac{\partial q}{\partial u} & \frac{\partial q}{\partial v} \end{vmatrix} \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$

$$= \begin{vmatrix} \frac{\partial p}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial p}{\partial v} \cdot \frac{\partial v}{\partial x} & \frac{\partial p}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial p}{\partial v} \cdot \frac{\partial v}{\partial y} \\ \frac{\partial q}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial q}{\partial v} \cdot \frac{\partial v}{\partial x} & \frac{\partial q}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial q}{\partial v} \cdot \frac{\partial v}{\partial y} \end{vmatrix}$$

$$= \begin{vmatrix} \frac{\partial p}{\partial x} & \frac{\partial p}{\partial y} \\ \frac{\partial q}{\partial x} & \frac{\partial q}{\partial y} \end{vmatrix}$$

$\therefore \ J_3 = J_1 \cdot J_2$

Q.3 Statement :-

ii) If $\bar{r}$ is position vector of point (x, y, z) & r is $|\bar{r}|$ then, prove that $\operatorname{div}(r^n \cdot \bar{r}) = (n+3) r^n$.

Proof :-

Let, $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$r = \sqrt{x^2+y^2+z^2}$$

$$= (x^2+y^2+z^2)^{1/2}$$

$$r^n = (x^2+y^2+z^2)^{n/2}.$$

Now,

consider,

$$\nabla \cdot (r^n \cdot \bar{r}) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \cdot \left[(x^2+y^2+z^2)^{n/2}\right] \cdot \left[x\hat{i} + y\hat{j} + z\hat{k}\right]$$

OY

$$= \hat{i}\frac{\partial}{\partial x}\left[(x^2+y^2+z^2)^{n/2} \cdot (x\hat{i}+y\hat{j}+z\hat{k})\right]$$

$$+ \hat{j}\frac{\partial}{\partial y}(x^2+y^2+z^2)^{n/2} \cdot (x\hat{i}+y\hat{j}+z\hat{k})$$

$$+ \hat{k}\frac{\partial}{\partial z}(x^2+y^2+z^2)^{n/2} \cdot (x\hat{i}+y\hat{j}+z\hat{k})$$

$$= \hat{i}(x^2+y^2+z^2)^{n/2}(\hat{i}) + (x\hat{i}+y\hat{j}+z\hat{k})\frac{n}{2}(2x)(x^2+y^2+z^2)^{n/2}$$

$$+ \hat{j}(x^2+y^2+z^2)^{n/2}(\hat{j}) + (x\hat{i}+y\hat{j}+z\hat{k})\frac{n}{2}(2y)(x^2+y^2+z^2)^{n/2}$$

$$+ \hat{k}(x^2+y^2+z^2)^{n/2}(\hat{k}) + (x\hat{i}+y\hat{j}+z\hat{k})\frac{n}{2}(2z)(x^2+y^2+z^2)^{n/2}$$

Name :- Arpita Balavant Chougale.

॥ ज्ञान, विज्ञान आणि सुसंस्कार यांसाठी शिक्षण प्रसार ॥

– शिक्षणमहर्षी डॉ. बापूजी साळुंखे

39162

Shri Swami Vivekanand Shikshan Sanstha Kolhapur's

VIVEKANAND COLLEGE, KOLHAPUR (AUTONOMOUS)

SUPPLIMENT

Signature of Supervisor

Suppliment No. : 2

Roll No. : 7744

Class : B.Sc. II

Subject : Multivariable calculus.

Test / Tutorial No. : I.

Div. : A

ii) $= \hat{i}(x^2+y^2+z^2)^{n/2}(1) + \cancel{2}x\left(\frac{n}{\cancel{2}}\right)(x^2+y^2+z^2)^{n/2-1}$

$+\hat{j}(x^2+y^2+z^2)^{n/2}(1) + \cancel{2}y\left(\frac{n}{\cancel{2}}\right)(x^2+y^2+z^2)^{n/2-1}$

$+\hat{k}(x^2+y^2+z^2)^{n/2}(1) + \cancel{2}z\left(\frac{n}{\cancel{2}}\right)(x^2+y^2+z^2)^{n/2-1}$

$= 3(x^2+y^2+z^2)^{n/2} + n(x\hat{i}+y\hat{j}+z\hat{k})(x^2+y^2+z^2)^{n/2-1}$

$= 3(x^2+y^2+z^2)^{n/2} + n(x^2+y^2+z^2)^{n/2}.$

$= (3+n)(x^2+y^2+z^2)^{n/2}.$

$= (3+n)(r^n).$

$= 3+n(r^n).$

$\therefore \text{div}(r^n.\bar{r}) = (n+3)r^n$

$\therefore$ Hence proved.

iii) let,

$$v = 2x^2\hat{i} - 3yz\hat{j} + xz^2\hat{k}$$

$$\phi = 2z - x^3y$$

$$\nabla\phi = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot(2z - x^3y)$$

$$= \hat{i}\frac{\partial}{\partial x}(2z - x^3y) + \hat{j}\frac{\partial}{\partial y}(2z - x^3y) + \hat{k}\frac{\partial}{\partial z}(2z - x^3y)$$

$$= \hat{i}(-3x2zy) + \hat{j}(2z - x^3) + \hat{k}(2 - x^3y).$$

$$= (3x2zy)\hat{i} + (2z - x^3)\hat{j} + (2 - x^3y)\hat{k}$$

$$\nabla\phi\Big|_{(1,-1,1)} = [3(1)\,2(-1)(-1)]\hat{i} + [2(1) - (1)^3]\hat{j} + [2 - (1)^3(-1)]\hat{k}$$

$$= 6\hat{i} + 2\hat{j} + 2\hat{k}$$

$$\nabla\cdot\nabla\phi\cdot\frac{\nabla\psi}{|\nabla\psi|} = 6\hat{i} + 2\hat{j} + 2\hat{k}\cdot\frac{2z - x^3y}{\sqrt{4+1}}$$

Given,

$$u = 3x + 2y - z,\ v = x - 2y + z,\ w = x + 2y - z$$

$$\frac{\partial u}{\partial x} = 3 \qquad \frac{\partial u}{\partial y} = 2 \qquad \frac{\partial u}{\partial z} = -1$$

$$\frac{\partial v}{\partial x} = 1 \qquad \frac{\partial v}{\partial y} = -2 \qquad \frac{\partial v}{\partial z} = 1$$

$$\frac{\partial w}{\partial x} = 1 \qquad \frac{\partial w}{\partial y} = 2 \qquad \frac{\partial w}{\partial z} = -1$$

Now,

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix} = \begin{vmatrix} 3 & 2 & -1 \\ 1 & -2 & 1 \\ 1 & 2 & -1 \end{vmatrix}$$

04

$$= 3(-2)(2) - 2(-1)(-1) + (-1)(2) + (-2)$$

$$= 3(-2)(2) - 2(-1)(-1) + (-1)[2+2]$$

$$= 3(0) - 2(-2) + (-)4$$

$$= -4 + 4$$

$$= 0$$

$$\therefore \frac{\partial(u,v,w)}{\partial(x,y,z)} = 0.$$