2

CHAPTER

CIRCUIT ANALYSIS

2.1 INTRODUCTION

An electronic circuit is a system consists of electronic components such as resistors, transistors, capacitors, inductors, diodes, etc connected by wires through which electric current can flow. Circuit analysis is the mathematical analysis of an electrical or electronic circuit. It is the process of studying and analyzing electrical quantities through calculations. By using analysis, we can find the unknown quantities, such as voltage, current, resistance, impedance, power in circuit component. For this analysis, we need to understand the electrical quantities, relationships, theorems, and some essential laws.

2.2 ENERGY SOURCES

A **source** is a device which converts mechanical, chemical, thermal or some other form of energy into electrical energy. In other words, the source is an active network element meant for generating electrical energy. The various types of sources available in the electrical network are voltage source and current sources. A voltage source has a forcing function of e.m.f whereas the current source has a forcing function of current.

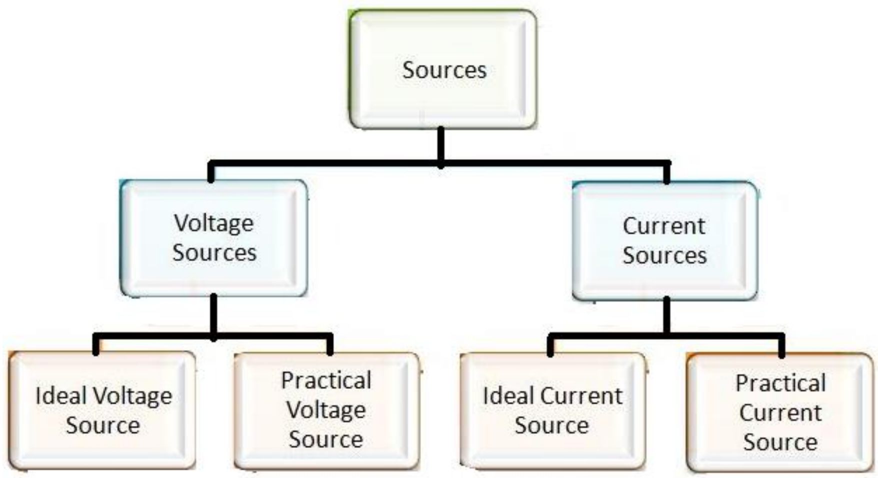


Fig. 2.1 : Energy Sources

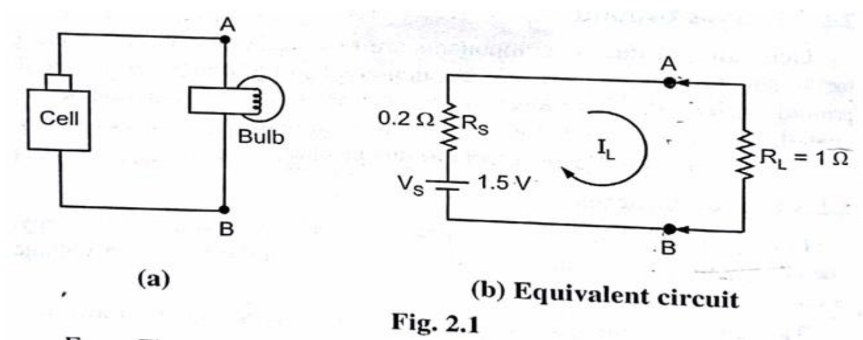
2.3 INTERNAL IMPEDANCE OF A SOURCE

Every energy source possesses some internal impedance (or resistance). The value of this impedance is few ohms. Due to this internal impedance, energy sources do not behave as ideal sources. Thus, when the voltage source provides the energy to the load, its terminal voltage drops. This voltage is low as compared to its open-circuit voltage.

For example, a dry cell used in a torch has a voltage of 1.5 V across its terminals when it is opened circuited (i.e. without load). But when it is connected to a bulb, its voltage reduces below 1.5 V. This reduction in terminal voltage of the source (cell) is due to the internal impedance of the cell.

The concept of internal impedance can be understood from following diagrams.

Fig. 2.2 (a) show a dry cell of 1.5 V connected to a bulb. In Fig. 2.2 (b), bulb is replaced by a load resistance R_L and the cell is replaced by a constant voltage source of 1.5 V in series with internal resistance R_S as given below.



(a) **(b) Equivalent circuit**

Fig. 2.2

From Fig. 2.2 (b), it is seen that the current flowing through the circuit (i.e. load R_L) is given by,

$$I_L = \frac{\text{Voltage}}{\text{Total ressistance in the circuit}}$$

$$I_L = \frac{1.5}{R_S + R_L} = \frac{1.5\text{ V}}{1.2\ \Omega} = 1.25\text{ A}$$

Thus, the terminal voltage, V_{AB} is given as

$$V_{AB} = I_L \times R_L = 1.25\text{ A} \times 1\ \Omega = 1.25\text{ V}$$

The voltage drop due to internal impedance R_1 of the cell is

$$= 1.5\text{ V} - 1.25\text{ V} = 0.25\text{ V}$$

If load current I_L is large than voltage drop is large. This means terminal voltage is small.

2.4 CONCEPT OF VOLTAGE SOURCE

A voltage source is a two-terminal device whose voltage at any instant of time is constant and is independent of the current drawn from it. Such voltage source is called an ideal Voltage source and has zero internal resistance. Practically an ideal voltage source cannot be obtained.

Sources having some amount of internal resistances are known as Practical Voltage Source. Due to this internal resistance; voltage drop takes place, and it causes the terminal voltage to reduce. The smaller is the internal resistance (r) of a voltage source, the more closely it is to an Ideal Source.

Consider a D.C. source V_S is connected across a load R_L as shown in Fig. 2.3

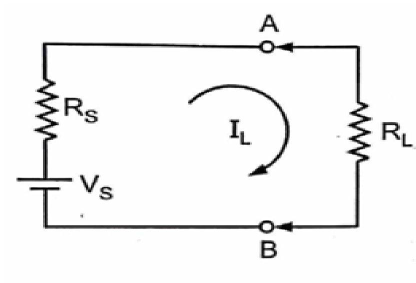


Fig. 2.3

Let V_S = Open-circuit voltage i.e. the voltage between A and B when R_L is disconnected (i.e. $R_L = \infty$).

R_S = Internal impedance of the source

and R_L = Load resistance

Then, from Fig. 2.3, we can write

$$\text{Current, } I_L = \frac{V_S}{R_S + R_L}$$

$\therefore$ Terminal voltage, V_{AB} is given as

$$V_{AB} = \text{Voltage across the load } R_L$$

$$= I_L \cdot R_L$$

$$V_{AB} = \frac{V_S}{R_S + R_L} \times R_L$$

$$V_{AB} = \frac{V_S}{1 + \frac{R_S}{R_L}}$$

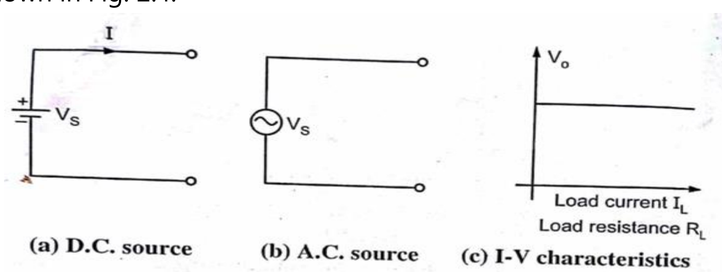
From this equation, it is seen that if $R_S = 0$, then $V_{AB} = V_S$. But it is not possible ideally. Hence, R_S must have very small value. Thus, if R_S is small then terminal voltage is approximately constant.

2.4.1 Constant Voltage Source

The voltage source which provides constant output voltage irrespective of the load R_L is known as 'constant voltage source'.

The ideal constant voltage source has zero internal impedance (i.e. $R_S = 0$).

Thus, the ideal constant voltage source and its I-V characteristics are as shown in Fig. 2.4.

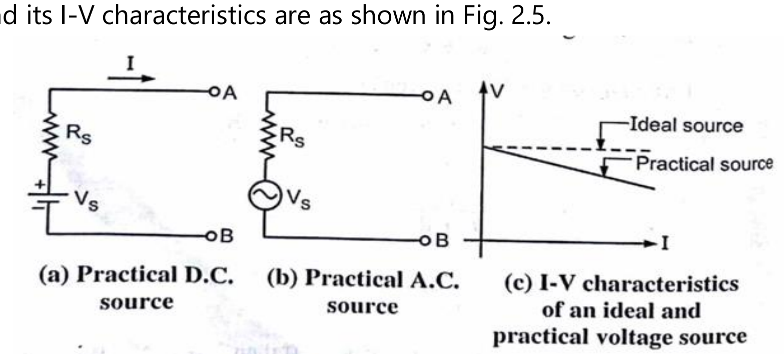


(a) D.C. source **(b) A.C. source** **(c) I-V characteristics**

Fig. 2.4

2.4.2 Practical Voltage Source

Practically, no any voltage source is ideal. Hence, every voltage source has some internal impedance. Smaller the value of R_S, then it gives approximately constant output voltage. Thus, practical voltage source and its I-V characteristics are as shown in Fig. 2.5.



(a) Practical D.C. source **(b) Practical A.C. source** **(c) I-V characteristics of an ideal and practical voltage source**

Fig. 2.5

In short, ideal voltage source has 'zero' input impedance, while practical voltage source has minimum (small) input impedance.

2.5 CONCEPT OF CURRENT SOURCE

The current sources are further categorized as Ideal and Practical current source. An Ideal current source is a two-terminal circuit element which supplies the same current to any load resistance connected across its terminals. It is important to keep in mind that the current supplied by the current source is independent of the voltage of source terminals. It has infinite resistance.

Let us consider the current source 'I' provides current to the load R_L as shown in Fig. 2.6. If R_S is the internal impedance (input impedance) of the source, then from Fig. 2.6, we can write according to Kirchhoff's current low.

$$I = I_S + I_L$$

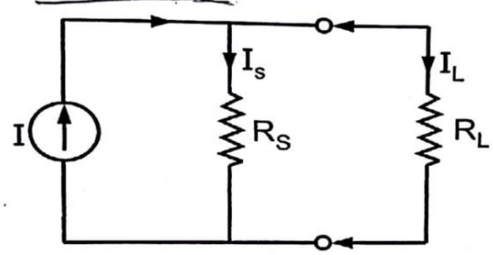


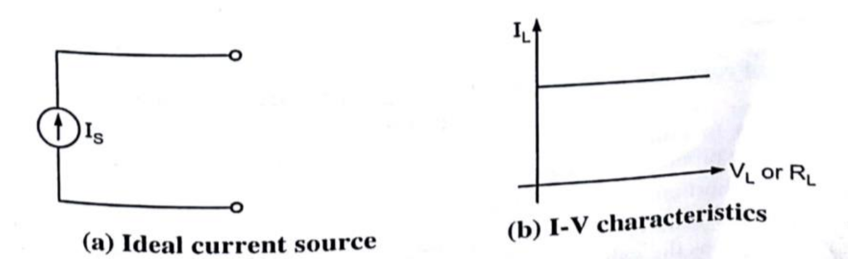
Fig. 2.6

Thus, in order to supply constant current to the load R_L, current I_S must be zero, this means input impedance of the source must be infinite. In short, input impedance of an ideal current source is "infinite (∞)". But practically no any source is ideal, therefore, practical current source must have "very large" input impedance.

2.5.1 Constant Current Source

The current source which supplies the constant current to the load R_L, irrespective of load R_L is known as constant current source.

Thus, in order to supply constant current to the load R_L, input impedance of the source must be infinite i.e. $R_S = \infty$. Thus, ideal constant current source and its load characteristics (I-V) are shown in Fig. 2.7.

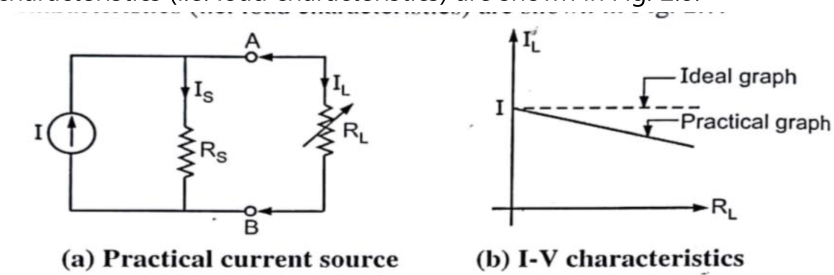


(a) Ideal current source **(b) I-V characteristics**

Fig. 2.7

2.5.2 PRACTICAL CURRENT SOURCE

In actual practice, no any source is ideal. Hence, practical current source does not have infinite input impedance but it has large value of internal impedance. Thus, larger the value of internal impedance, better is the current source. Therefore, practical current source and its I-V characteristics (i.e. load characteristics) are shown in Fig. 2.8.



(a) Practical current source **(b) I-V characteristics**

Fig. 2.8

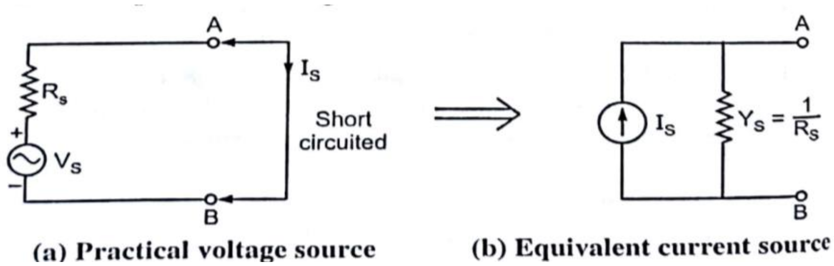
From Fig. 2.8 (a), we can write,

$$I = I_S + I_L$$

If I_S is negligible i.e. I_S is small then $I_L \approx I$. This means internal impedance of the current source must be very large.

2.6 CONVERSION OF VOLTAGE SOURCE TO CURRENT SOURCE

Let us consider practical voltage source V_S' with its internal resistance R_S as shown in Fig. 2.9.



(a) Practical voltage source **(b) Equivalent current source**

Fig. 2.9

Thus, to convert voltage source to its equivalent current source. Following procedure is used :

(i) Short circuit the output terminals.

(ii) Measure/calculate the value of short-circuit current I_S. This gives the value of current source I_S as shown in Fig. 2.9 (b).

(iii) Then open circuit the output terminals and short circuit the voltage source and calculate the value of internal impedance R_S between the output terminals.

(iv) Finally, calculate admittance Y_S by using $Y_S = \frac{1}{R_S}$.

(v) Connect this value of Y_S in parallel with I_S, we get equivalent circuit as shown in Fig. 2.9 (b) for the voltage source in Fig. 2.9 (a).

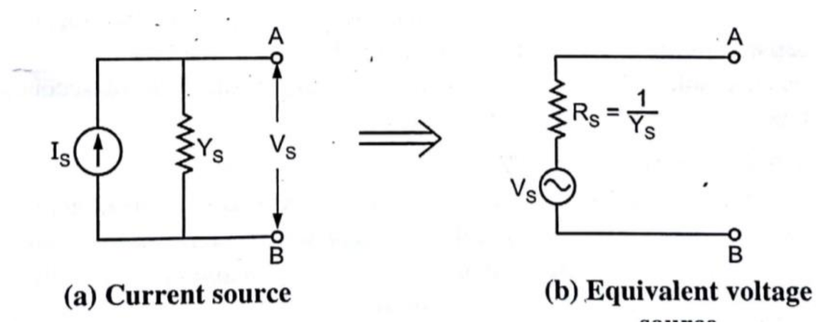
2.7 CONVERSION OF CURRENT SOURCE TO EQUIVALENT VOLTAGE SOURCE

For the conversion of current source to an equivalent voltage source the procedure used is as given below :

(i) Open circuit the output terminals (i.e. disconnect R_L if any).

(ii) Then measure the open-circuit voltage between the output terminals. This gives the value of V_S i.e. voltage source.

(iii) Then open circuit the current source I_S and calculate the impedance between the output terminals. This gives the value of source impedance R_S.



(a) Current source **(b) Equivalent Voltage**

Fig. 2.10

2.8 KIRCHHOFF'S LAWS

Network elements can be either of active or passive type. Any electrical circuit or network contains one of these two types of network elements or a combination of both.

Now, let us discuss about the following two laws, which are popularly known as Kirchhoff's laws.

- Kirchhoff's Current Law
- Kirchhoff's Voltage Law

1.8.1 Kirchhoff's Current Law:

Statement: "*Kirchhoff's Current Law (KCL) states that the algebraic sum of currents leaving (or entering) a node is equal to zero*".

A Node is a point where two or more circuit elements are connected to it. If only two circuit elements are connected to a node, then it is said to be simple node. If three or more circuit elements are connected to a node, then it is said to be principal Node.

Mathematically, KCL can be represented as

$$\sum_{m-1}^{M} I_m = 0$$

Where,

I_m is the m^{th} branch current leaving the node.

M is the number of branches that are connected to a node.

The above statement of KCL can also be expressed as "the algebraic sum of currents entering a node is equal to the algebraic sum of currents leaving a node". Let us verify this statement through the following example.

Example :

Write KCL equation at node A of the following figure.

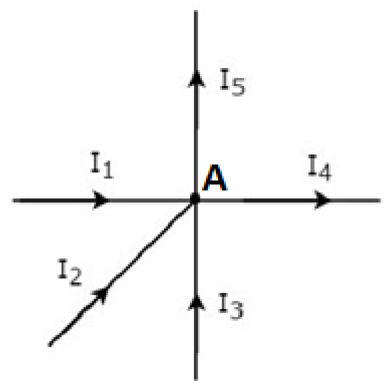


Fig. 2.11

In the above figure, the branch currents I_1, I_2 and I_3 are entering at node A. So, consider negative signs for these three currents.

In the above figure, the branch currents I_4 and I_5 are leaving from node A. So, consider positive signs for these two currents.

The KCL equation at node A will be

$$-I_1 - I_2 - I_3 + I_4 + I_5 = 0$$

$$I_1 + I_2 + I_3 = I_4 + I_5$$

In the above equation, the left-hand side represents the sum of entering currents, whereas the right-hand side represents the sum of leaving currents.

2.8.2 Kirchhoff's Voltage Law

Kirchhoff's Voltage Law (KVL) states that the algebraic sum of voltages around a loop or mesh is equal to zero.

A Loop is a path that terminates at the same node where it started from. In contrast, a Mesh is a loop that doesn't contain any other loops inside it.

Mathematically, KVL can be represented as

$$\sum_{n=1}^{N} V_n = 0$$

where,

V_n is the n^{th} element's voltage in a loop (mesh).

N is the number of network elements in the loop (mesh).

The above statement of KVL can also be expressed as "the algebraic sum of voltage sources is equal to the algebraic sum of voltage drops that are present in a loop." Let us verify this statement with the help of the following example.

Example :

Write KVL equation around the loop of the following circuit.

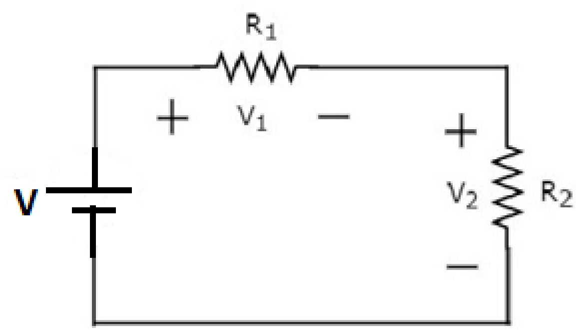


Fig. 2.12

The above circuit diagram consists of a voltage source, V in series with two resistors R_1 and R_2. The voltage drops across the resistors R_1 and R_2 are $V_1$and V_2 respectively.

Apply KVL around the loop.

$$V - V_1 - V_2 = 0$$

$$V = V_1 + V_2$$

In the above equation, the left-hand side term represents single voltage source V. whereas, the right-hand side represents the sum of voltage drops. In this example, we considered only one voltage source. That's why the left-hand side contains only one term. If we consider multiple voltage sources, then the left side contains sum of voltage sources.

2.9 NODAL ANALYSIS

There are two basic methods that are used for solving any electrical network: Nodal analysis and Mesh analysis. In this section, we discuss about the Nodal analysis method.

In Nodal analysis, we will consider the node voltages with respect to ground. Hence, Nodal analysis is also called as Node-voltage method.

2.9.1 Procedure of Nodal Analysis

Follow these steps while solving any electrical network or circuit using Nodal analysis.

- **Step 1 :** Identify the principal nodes and choose one of them as reference node. We will treat that reference node as the Ground.
- **Step 2 :** Label the node voltages with respect to Ground from all the principal nodes except the reference node.
- **Step 3 :** Write nodal equations at all the principal nodes except the reference node. Nodal equation is obtained by applying KCL first and then Ohm's law.
- **Step 4 :** Solve the nodal equations obtained in Step 3 in order to get the node voltages.

Now, we can find the current flowing through any element and the voltage across any element that is present in the given network by using node voltages.

Example :

Find the current flowing through 20 Ω resistor of the following circuit using Nodal analysis.

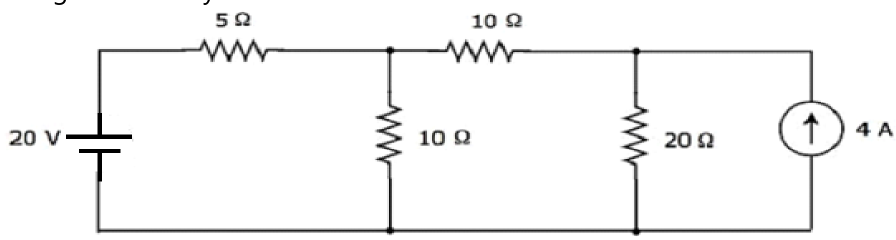


Fig. 2.13

Step 1 : There are three principle nodes in the above circuit. Those are labeled as 1, 2, and 3 in the following figure.

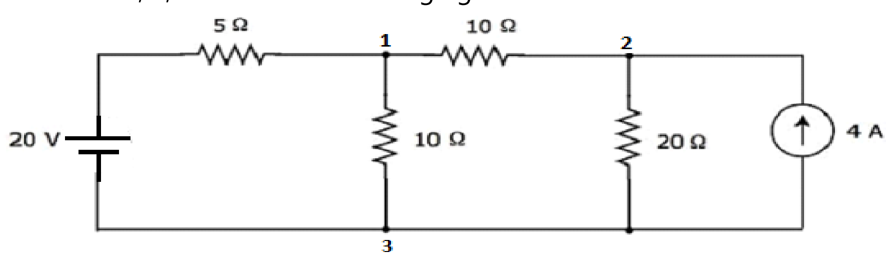


Fig. 2.14

In the above figure, consider node 3 as reference node (Ground).

Step 2 : The node voltages, V_1 and V_2, are labeled in the following figure.

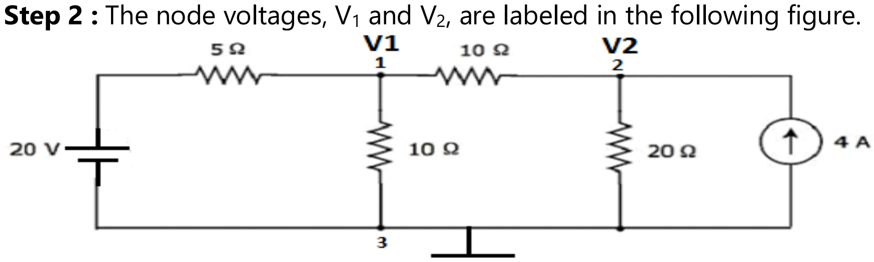


Fig. 2.15

In the above figure, V_1 is the voltage from node 1 with respect to ground and V_2 is the voltage from node 2 with respect to ground.

Step 3 : In this case, we will get two nodal equations, since there are two principal nodes, 1 and 2, other than Ground. When we write the nodal equations at a node, assume all the currents are leaving from the node for which the direction of current is not mentioned and that node's voltage as greater than other node voltages in the circuit.

The nodal equation at node 1 is

$$\frac{V_1 - 20}{5} + \frac{V_1}{10} + \frac{V_1 - V_2}{10} = 0$$

$$\frac{2V_1 - 40 + V + V_1 - V_2}{10} = 0$$

$$4V_1 - 40 - V_2 = 0$$

$$V_2 = 4V_1 - 40 \qquad \dots (1)$$

The nodal equation at node 2 is

$$-4 + \frac{V_2}{20} + \frac{V_2 - V_1}{10} = 0$$

$$\frac{-80 + V_2 + 2V - 2V_2}{20} = 0$$

$$3V_2 - 2V_1 = 80 \qquad \dots (2)$$

Step 4 : Finding node voltages, V_1 and V_2 by solving Equation 1 and Equation 2. Substitute Equation 1 in Equation 2.

$$3\,(3V_1 - 40) - 2V_1 = 80$$

$$12V_1 - 120 - 2V_1 = 80$$

$$10V_1 = 200$$

$$V_1 = 20V$$

Substitute V_1 = 20 V in equation (1)

$$V_2 = 4\,(20) - 40$$

$$V_2 = 40V$$

So, we got the node voltages V_1 and V_2 as 20V and 40V respectively.

Step 5 : The voltage across 20 Ω resistor is nothing but the node voltage V_2 and it is equal to 40 V. Now, we can find the current flowing through 20 Ω resistor by using Ohm's law.

$$I_{20\Omega} = \frac{V_2}{R}$$

Substitute the values of V_2 and R in the above equation.

$$I_{20\Omega} = \frac{40}{20}$$

$$I_{20\Omega} = 2A$$

Therefore, the current flowing through 20 Ω resistor of given circuit is 2 A.

Note : From the above example, we can conclude that we have to solve 'n' nodal equations; if the electric circuit has 'n' principal nodes (except the reference node). Therefore, we can choose Nodal analysis when the number of principal nodes (except reference node) is less than the number of meshes of any electrical circuit.

2.10 MESH ANALYSIS

In Mesh analysis, we will consider the currents flowing through each mesh. Hence, Mesh analysis is also called as Mesh-current method.

A **branch** is a path that joins two nodes and it contains a circuit element. If a branch belongs to only one mesh, then the branch current will be equal to mesh current.

If a branch is common to two meshes, then the branch current will be equal to the sum (or difference) of two mesh currents, when they are in same (or opposite) direction.

2.10.1 Procedure of Mesh Analysis

Follow these steps while solving any electrical network or circuit using Mesh analysis.

- Step 1 : Identify the meshes and label the mesh currents in either clockwise or anti-clockwise direction.
- Step 2 : Observe the amount of current that flows through each element in terms of mesh currents.
- Step 3 : Write mesh equations to all meshes. Mesh equation is obtained by applying KVL first and then Ohm's law.
- Step 4 : Solve the mesh equations obtained in Step 3 in order to get the mesh currents.

Now, we can find the current flowing through any element and the voltage across any element that is present in the given network by using mesh currents.

Example :

Find the voltage across 30 Ω resistor using Mesh analysis.

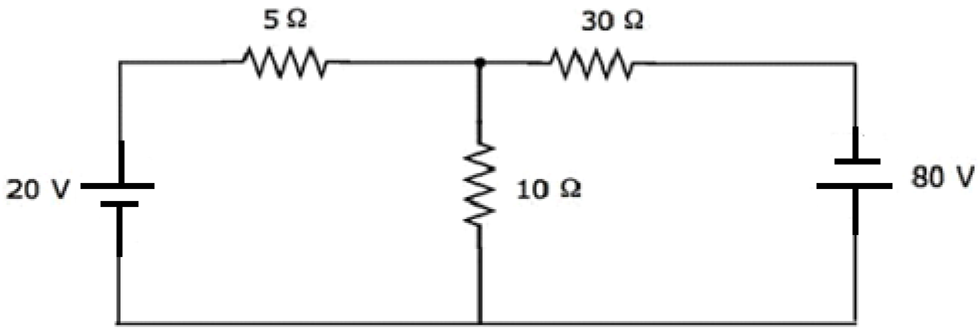


Fig. 2.16

Step 1 : There are two meshes in the above circuit. The mesh currents I_1 and I_2 are considered in clockwise direction. These mesh currents are shown in the following figure.

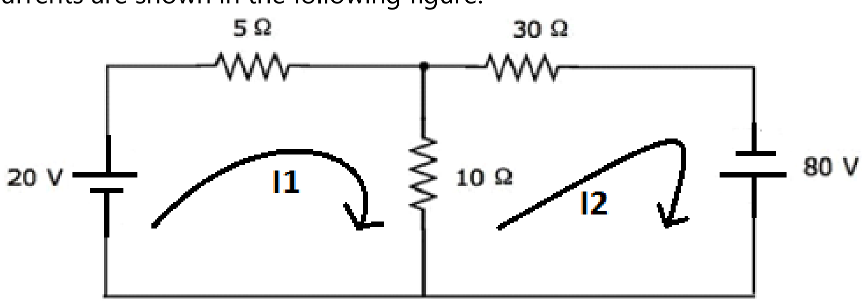


Fig. 2.17

Step 2 : The mesh current I_1 flows through 20V voltage source and 5Ω resistor. Similarly, the mesh current I_2 flows through 30 Ω resistor and −80 V voltage source. But, the difference of two mesh currents, I_1 and I_2, flows through 10 Ω resistor, since it is the common branch of two meshes.

Step 3 : In this case, we will get two mesh equations since there are two meshes in the given circuit. When we write the mesh equations, assume the mesh current of that particular mesh as greater than all other mesh currents of the circuit.

The mesh equation of first mesh is

$$20 - I_1 - 10\,(I_1 - I_2) = 0$$

$$20 - 15I_1 + 10I_2 = 0$$

$$10I_2 = 15I_1 - 20$$

Divide the above equation with 5.

$$2I_2 = 3I_1 - 4$$

Multiply the above equation with 2

$$4I_2 = 6I_1 - 8 \qquad \ldots (1)$$

The mesh equation of second mesh is

$$-10\,(I_2 - I_1) - 30I_2 + 80 = 0$$

Divide the above equation with 10.

$$-(I_2 - I_1) - 3I_2 + 8 = 0$$

$$-4I_2 + I_1 + 8 = 0$$

$$4I_2 = I_1 + 8 \qquad \ldots (2)$$

Step 4 : Finding mesh currents I_1 and I_2 by solving Equation (1) and Equation (2).

The left-hand side terms of Equation (1) and Equation (2) are the same. Hence, equate the right-hand side terms of Equation (1) and Equation (2) in order find the value of I_1.

$$6I_1 - 8 = I_1 + 8$$

$$5I_1 = 16$$

$$I_1 = \frac{16}{5}\ \text{A}$$

Substitute I_1 value in equation (2)

$$4I_2 = \frac{16}{5} + 8$$

$$4I_2 = \frac{56}{5}$$

$$I_2 = \frac{14}{5}\ \text{A}$$

So, we got the mesh currents I_1 and I_2 as $\frac{16}{5}$ A and $\frac{14}{5}$ A respectively.

Step 5 : The current flowing through 30 Ω resistor is nothing but the mesh current I_2 and it is equal to $\frac{14}{5}$ A. Now, we can find the voltage across 30 Ω resistor by using Ohm's law.

$$V_{30\Omega} = I_2 R$$

Substitute the values of I_2 and R in the above equation.

$$V_{30\Omega} = \left(\frac{14}{5}\right) 3-$$

$$\Rightarrow \quad V_{30\Omega} = 84 \text{ V}$$

Therefore, the voltage across 30 Ω resistor of the given circuit is 84 V.

Note 1 : From the above example, we can conclude that we have to solve ‘m’ mesh equations, if the electric circuit is having ‘m’ meshes. That’s why we can choose Mesh analysis when the number of meshes is less than the number of principal nodes (except the reference node) of any electrical circuit.

Note 2 : We can choose either Nodal analysis or Mesh analysis, when the number of meshes is equal to the number of principal nodes (except the reference node) in any electric circuit.

2.11 SUPERPOSITION THEOREM

Superposition theorem states that “*In any linear, active, bilateral network having more than one source, the response across any element is the sum of the responses obtained from each source considered separately and all other sources are replaced by their internal resistance.*”

2.11.1 Steps for Applying Superposition Theorem to Network

Considering the circuit diagram, let us see the various steps to solve the superposition theorem.

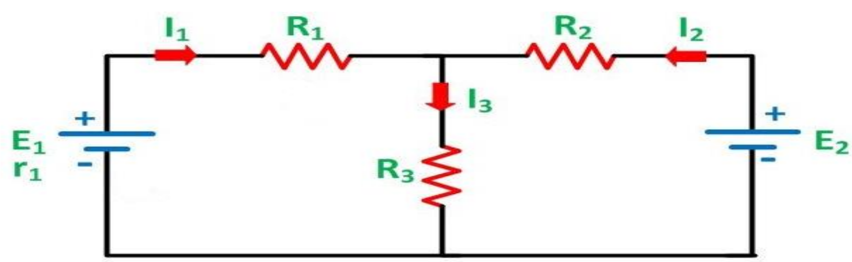


Fig. 2.18

Step 1 : Take only one independent source of voltage or current and deactivate the other source.

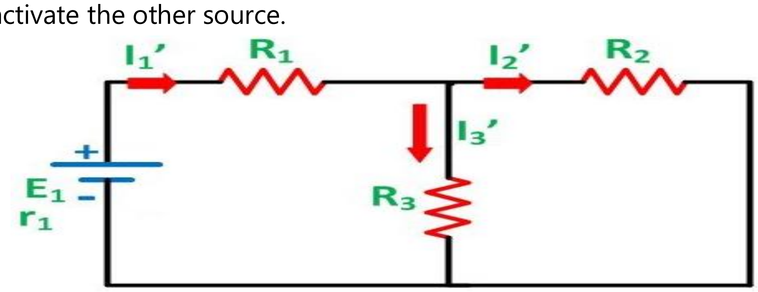


Fig. 2.19

Step 2 : In the circuit diagram shown above, consider the source E_1 and replace the other source E_2 by its internal resistance. If its internal resistance is not given, then it is taken as zero and the source is short circuited.

Step 3 : If there is a voltage source than short circuit it and if there is a current source than just open circuit it.

Step 4 : Find the current in each branch of the network. find the current I_1', $I_2'$and I_3'.

Here, the value of current flowing in each branch, i.e. I_1', I_2' and I_3' is calculated by the following equations.

$$I_1' = \frac{V_1}{\frac{r_2 r_3}{r_2 + r_3} + r_1} \quad \ldots (1)$$

$$I_2' = i_1' \frac{r_3}{r_2 + r_3} \quad \ldots (2)$$

$$i_3' = i_1' - i_2'$$

Step 5 : Now consider the other source E_2 and replace the source E_1 by its internal resistance r_1 as shown in the circuit diagram.

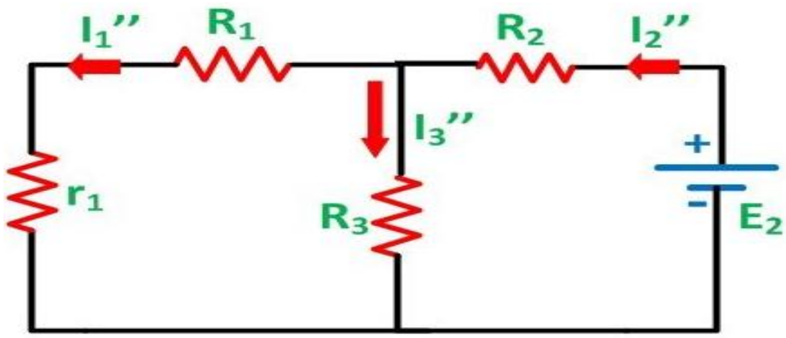


Fig. 2.20

Step 6 : Determine the current in various sections, I_1'', I_2'' and I_3''.

$$i_2'' = \frac{V_2}{\frac{r_1 r_3}{r_1 + r_3} + r_2}$$

and

$$i_2'' = i_2'' \frac{r_3}{r_1 + r_3}$$

$$i_3'' = i_2'' - i_1''$$

Step 7 : Now to determine the net branch current utilizing the superposition theorem, add the currents obtained from each individual source for each branch.

Step 8 : If the current obtained by each branch is in the same direction than add them and if it is in the opposite direction, subtract them to obtain the net current in each branch.

The actual flow of current in the circuit will be given by the equations shown below.

$$I_1 = I_1^{'} - I_1^{''}$$

$$I_2 = I_2^{'} - I_2^{''}$$

$$I_3 = I_3^{'} + I_3^{''}$$

SOLVED EXAMPLE

Example 2.1 : Using the superposition theorem, determine the voltage drop and current across the resistor 3.3K as shown in the figure below.

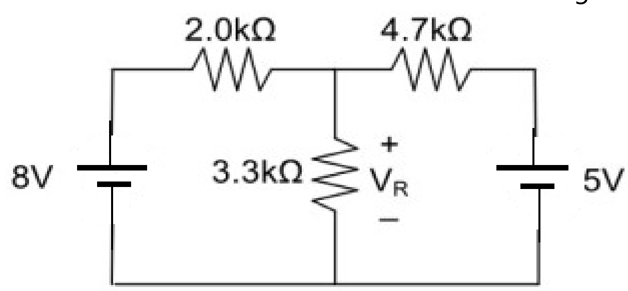


Fig. 2.21

Solution :

Step 1 : Remove the 8V source from circuit, such that the new circuit becomes as the following and then measure the voltage across a resistor.

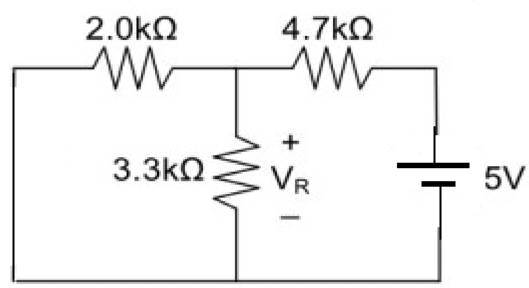


Fig. 2.22

Here 3.3 K and 2 K are in parallel; therefore resultant resistance will be 1.245 K.

Using voltage divider rule voltage across 1.245 K will be

$$V_1 = \left[\frac{1.245}{(1.245 + 4.7)}\right] \times 5 = 1.047 \text{ V}$$

Step 2 : Remove the 5 V power supply from the original circuit such that the new circuit becomes as the following and then measure the voltage across a resistor.

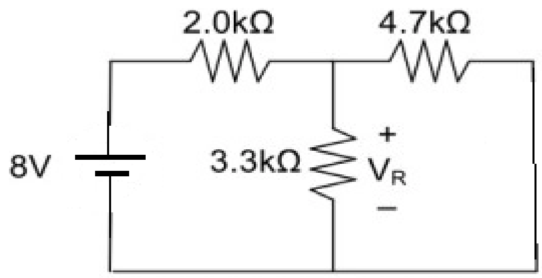


Fig. 2.23

Here 3.3 K and 4.7 K are in parallel; therefore resultant resistance will be 1.938 K.

Using voltage divider rule voltage across 1.938 K will be

$$V_2 = \left[\frac{1.938}{1.938 + 2}\right] \times 8 = 3.9377 \text{ V}$$

Therefore voltage drop across a 3.3K resistor is

$$V = V_1 + V_2 = 1.047 + 3.9377 = 4.9847$$

Therefore current through a 3.3K resistor is

$$I = \frac{V}{R} = \frac{4.9847}{3.3 \text{ K}} = 1.510 \text{ mA}$$

2.12 THEVENIN'S THEOREM

Statement: "*Thevenin's theorem* *states that any two terminal linear network or circuit can be replaced with an equivalent network or circuit, which consists of a voltage source (**V_{Th}** – Thevenin's voltage source) in series with a resistor (R_{Th} – Thevenin's resistance). It is known as Thevenin's equivalent circuit. Where V_{Th} is the voltage measured across the two terminals of network and R_{Th} is the resistance seen between terminals of network when all internal energy sources are replaced by their internal resistances".*

2.12.1 Steps for Applying Thevenin's Theorem to Network

Let us consider a simple DC circuit as shown in the figure above, where we have to find the load current I_L by the Thevenin's theorem.

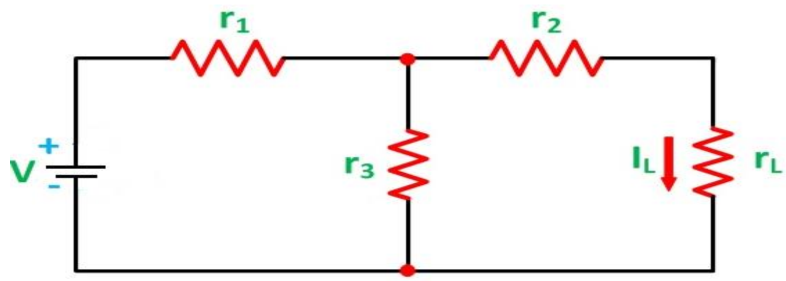
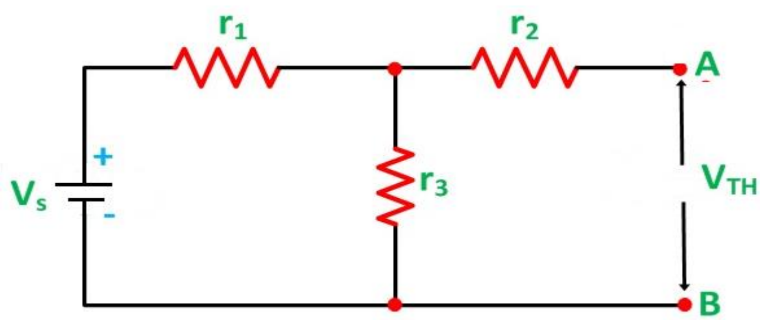


Fig. 2.24

Step 1 : First of all remove the load resistance $\mathbf{r_L}$ of the given circuit.

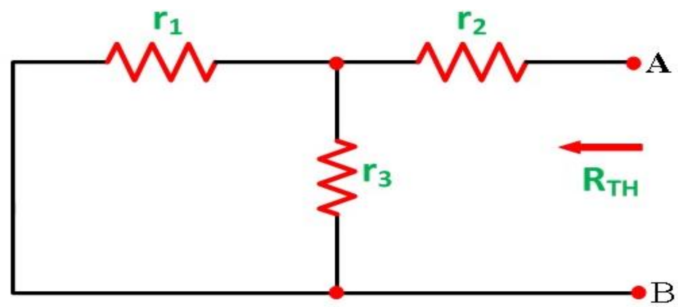
Step 2 : Measure the open terminal voltage across the points A and B. i.e. V^{th}.



$$V_{TH} = Ir_3 = \frac{V_S}{r_1 + r_3} r_3$$

Fig. 2.25

Step 3 : Now find the equivalent resistance at the load terminals A and B known as Thevenin's Resistance (R_{TH}), by Replacing the energy source by their internal resistance.



$$R_{TH} = r_2 + \frac{r_1 r_3}{r_1 + r_3}$$

Fig. 2.26

Step 4 : Draw the Thevenin's equivalent circuit by connecting the load resistance and after that determine the desired response.

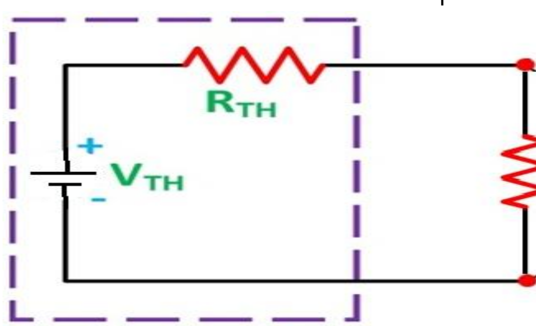


Fig. 2.27

Load current is determined from the equivalent Thevenin's circuit. The Load current I_L is given as

$$I_L + \frac{V_{TH}}{R_{TH} + r_L}$$

SOLVED EXAMPLE

Example 2.1 : Using the Thevenin's theorem, determine the voltage drop and current across the resistor 40 Ω as shown in the figure below.

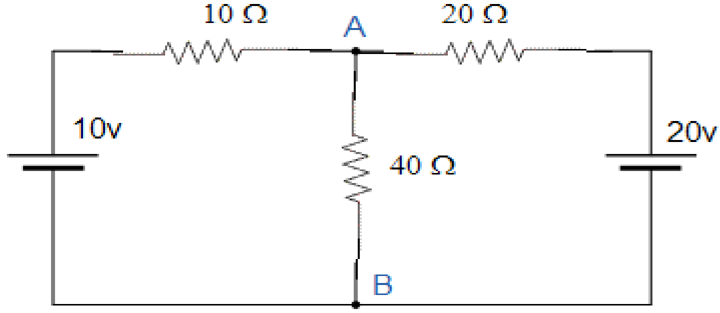


Fig. 2.28

Solution :

Step 1 : Find the Equivalent Resistance or Thevenin's resistance (R_{TH}):

The value of the equivalent resistance, R_{TH} is found by calculating the total resistance looking back from the terminals A and B with all the voltage sources shorted. We then get the following circuit.

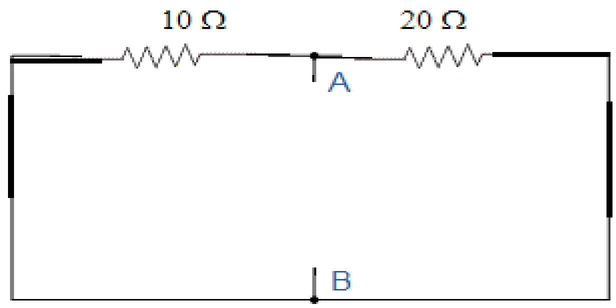


Fig. 2.29

10 Ω resistor in parallel with the 20 Ω resistor

$$R_{Th} = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{20 \times 10}{20 + 10} = 6.67\ \Omega$$

Step 2 : Find Thevenin's Voltage (V_{Th}) :

The voltage V_{Th} is defined as the total voltage across the terminals A and B when there is an open circuit between them.

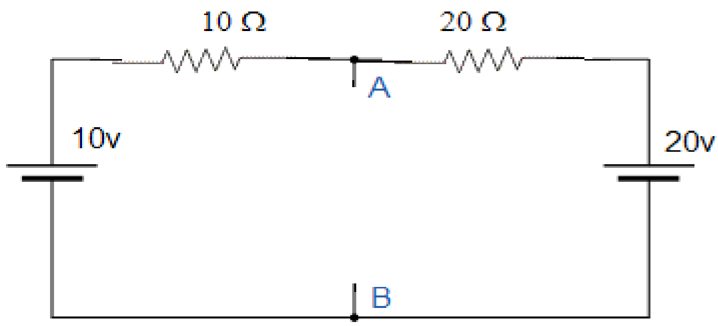


Fig. 2.30

From the circuit, the total current flowing around the loop is calculated as

$$I = \frac{V}{R} = \frac{20\text{ v} - 10\text{ v}}{20\ \Omega + 10\ \Omega} = 0.33\text{ amps}$$

This current of 0.33 amperes (330 mA) is common to both resistors so the voltage drop across the 20 Ω resistor or the 10 Ω resistor can be calculated as

$$V_{TH} = VAB = 20 - (20\ \Omega \times 0.33\text{ amps}) = 13.33\text{ volts}$$

or

$$V_{TH} = VAB = 10 + (10\ \Omega \times 0.33\text{ amps}) = 13.33\text{ volts}$$

Step 3 : Then the Thevenin's Equivalent circuit is

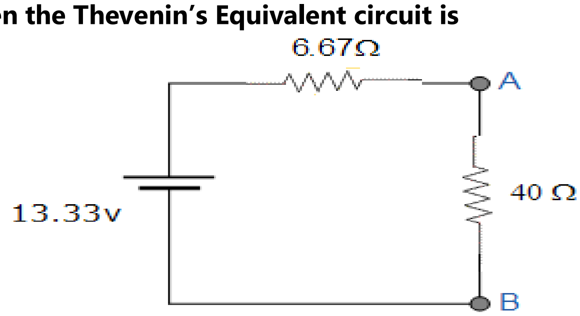


Fig. 2.31

From the Thevenin's Equivalent circuit, current through 40 Ω resistor is

$$I = \frac{V}{R} = \frac{13.33\text{ v}}{6.67\ \Omega + 40\ \Omega} = 0.286\text{ amps}$$

and voltage across 40Ω resistor is

$$V = I \times R = 0.286 \times 40 = 11.44 \text{ V}$$

2.13 NORTON'S THEOREM

Statement : "*Norton's theorem* *states that any two terminal linear network or circuit can be replaced with an equivalent network or circuit, which consists of a current source (I_N – Norton current source) in parallel with a resistor (R_N – Norton's resistance). It is known as Norton's equivalent circuit. Where I_N is the short circuit current measured between the two terminals of network and R_N is the resistance seen between terminals of network when all internal energy sources are replaced by their internal resistances".*

2.13.1 Steps for applying Norton's Theorem to Network

To understand Norton's Theorem in detail, let us consider a circuit diagram given below.

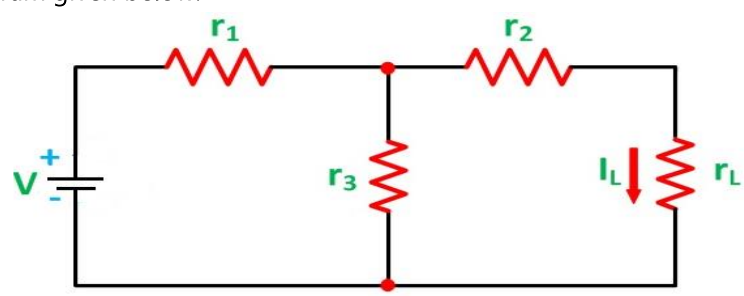


Fig. 2.32

Step 1 : Remove the load resistance of the circuit.

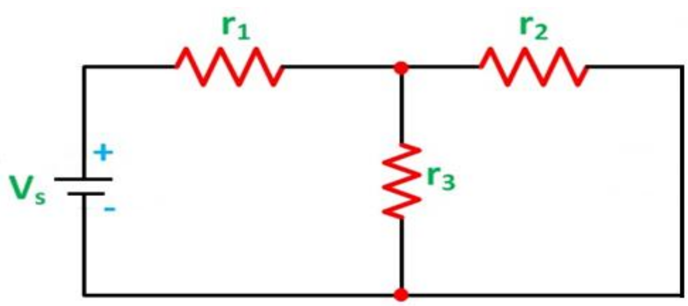


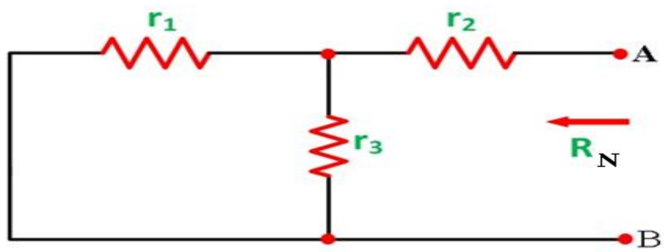
Fig. 2.33

Step 2 : Now find the Norton's current IN (short circuit current).

$$I = \frac{V_S}{r_1 + \frac{r_2 r_3}{r_2 + r_3}}$$

$$I_N = I_{sc} = I\frac{r_3}{r_3 + r_2}$$

Step 3 : Now find the equivalent resistance at the load terminals A and B known as Norton's Resistance (R_N), by Replacing the energy source by their internal resistance.



$$R_N = r_2 + \frac{r_1 r_3}{r_1 + r_3}$$

Fig. 2.34

Step 4 : Draw the Norton's equivalent circuit by connecting the load resistance and after that determine the desired response.

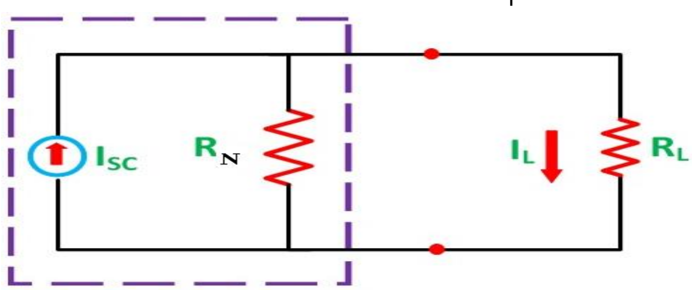


Fig. 2.35

Finally the load current I_L calculated by the equation shown below.

$$I_L + I_N\frac{R_N}{R_N + R_L}$$

SOLVED EXAMPLE

Example 2.1 : Using the Norton's theorem, determine the voltage drop and current across the resistor 40 Ω as shown in the figure below.

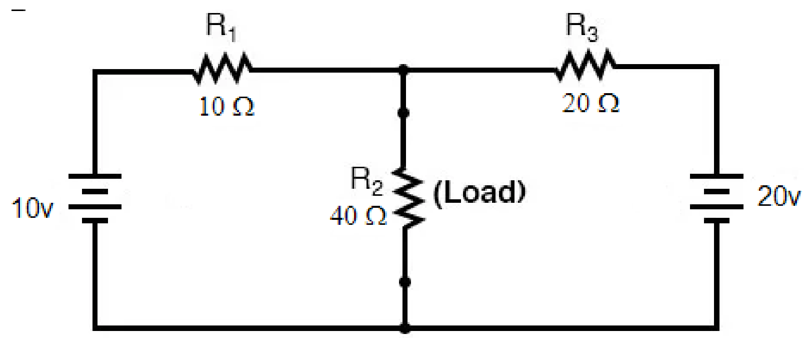
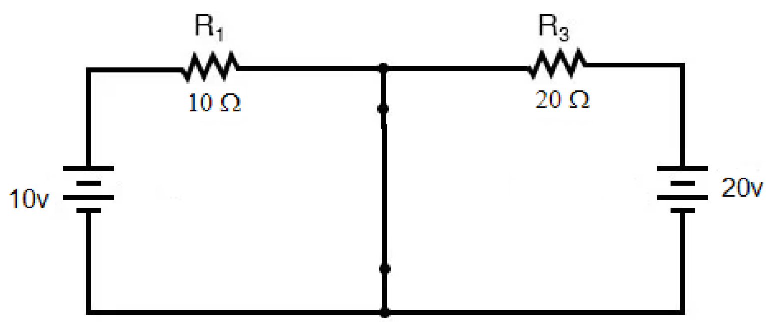


Fig. 2.36

Step 1 : To find the **Nortons current (I_N)** value from the above circuit, we firstly have to remove the 40Ω load resistor and short out the terminals A and B.



$$I_1 = \frac{10\text{ v}}{10\ \Omega} = \text{Lamp},\ I_2 = \frac{20\text{ v}}{20\ \Omega} = \text{lamp}$$

Therefore, $I_{short\text{-}circuit} = I_N = I_1 + I_2 = 2$ amps

Fig. 2.37

Step 2 : Find the Equivalent Resistance or Norton's resistance (RN) : The value of the equivalent resistance, R_{TH} is found by calculating the total resistance looking back from the terminals A and B with all the voltage sources shorted. We then get the following circuit.

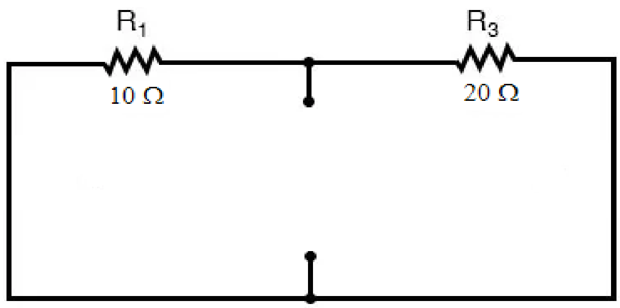


Fig. 2.38

10 Ω resistor in parallel with the 20 Ω resistor

$$R_N = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{20 \times 10}{20 + 10} = 6.67\ \Omega$$

Step 3 : Then the Norton's Equivalent circuit is

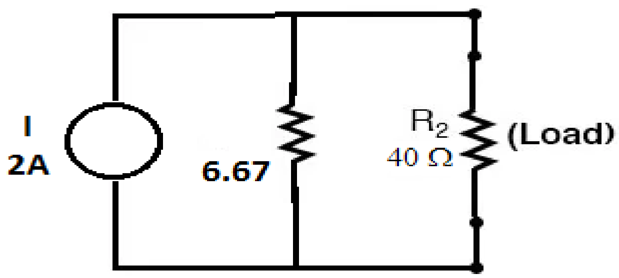


Fig. 2.39

From Norton's equivalent circuit it is seen that the two resistors are connected in parallel therefore a total resistance is,

$$R_T = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{6.67 \times 40}{6.67 + 40} = 5.72\ \Omega$$

Voltage across R_T is given by

$$V = I \times R = 2 \times 5.72 = 11.44\ v$$

Then the current flowing in the 40 Ω load resistor can be found as

$$I = \frac{V}{R} = \frac{11.44}{40} = 0.286\ \text{amps}$$

2.14 MAXIMUM POWER TRANSFER THEOREM

Maximum Power Transfer Theorem can be stated as, a resistive load, being connected to a DC network, receives maximum power when the load resistance is equal to the internal resistance known as (Thevenin's equivalent resistance) of the source network as seen from the load terminals. The Maximum Power Transfer theorem is used to find the load resistance for which there would be the maximum amount of power transfer from the source to the load.

2.14.1 Explanation of Maximum Power Transfer Theorem

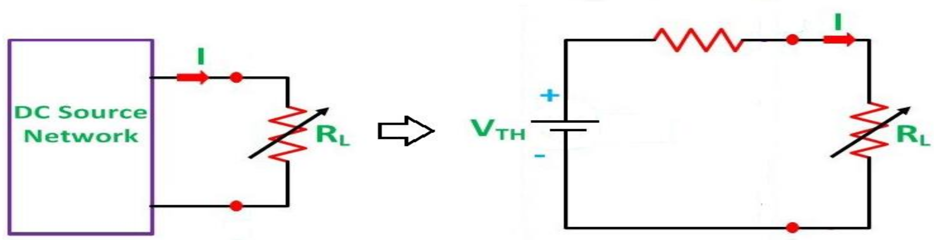


Fig. 2.40

A variable resistance RL is connected to a DC source network as shown in the circuit diagram in figure 1 and the figure 2 represents the Thevenin's voltage V_{TH} and Thevenin's resistance R_{TH} of the source network. The aim of the Maximum Power Transfer theorem is to determine the value of load resistance R_L, such that it receives maximum power from the DC source.

Considering Fig. 2.40 the value of current will be calculated by the equation shown below.

$$I = \frac{V_{TH}}{R_{TH} + R_L} \quad \dots (1)$$

While the power delivered to the resistive load is given by the equation

$$P_L = I^2 R_L \quad \dots (2)$$

Putting the value of I from the equation (1) in the equation (2) we will get,

$$P_L = \left(\frac{V_{TH}}{R_{TH} + R_L}\right)^2 \times R_L$$

P_L can be maximized by varying R_L and hence, maximum power can be delivered when $(dP_L/dR_L) = 0$.

However,

$$\frac{dP_L}{dR_L} = \frac{1}{[(R_{TH} + R_L)^2]^2}\left[(R_{TH} + R_L)^2 \frac{d}{dR_L}(V_{TH}^2 R_L) - V_{TH}^2 R_L \frac{d}{dR_L}(R_{TH} + R_L)^2\right]$$

$$\frac{dP_L}{dR_L} = \frac{1}{(R_{TH} + R_L)^4}\left[(R_{tH} + R_L)^2 V_{TH}^2 - V_{TH}^2 R_L \times 2\,(R_{TH} + R_L)\right]$$

$$\frac{dP_L}{dR_L} = \frac{V_{TH}^2 (R_{TH} + R_L - 2R_L)}{(R_{TH} + R_L)^3} = \frac{V_{TH}^2 (R_{TH} - R_L)}{(R_{TH} + R_L)^2}$$

But as we know, $\left(\frac{dP_L}{dR_L}\right) = 0$

Therefore,

$$\frac{V_{TH}^2 (R_{TH} - R_L)}{(R_{TH} + R_L)^2} = 0$$

which gives

$$(R_{TH} - R_L) = 0 \text{ or } R_{TH} = R_L$$

Hence, it is proved that power transfer from a DC source network to a resistive network is maximum when the internal resistance of the DC source network is equal to the load resistance.

Again, with $R_{TH} = R_L$, the system being perfectly matched to the load and the source, thus, the power transfer becomes maximum, and this amount of power P_{max} can be obtained by the equation shown below.

$$P_{max} = \frac{V_{TH}^2 R_{TH}}{(R_{TH} + R_{TH})^2} = \frac{V_{TH}^2}{4R_{TH}} \quad \dots (3)$$

Equation (3) gives the power which is consumed by the load. The power transfer by the source will also be same as the power consumed by the load, i.e. equation (3), as the load power and the source power being the same.

Thus, the total power supplied is given by the equation

$$P = 2\frac{V_{TH}^2}{4R_{TH}} = \frac{V_{TH}^2}{2R_{TH}}$$

During Maximum Power Transfer the efficiency η becomes

$$\eta = \left(\frac{P_{max}}{P}\right) \times 100 = 50\%$$

Example 2.4 : Find the maximum power that can be delivered to the load resistor R_L of the circuit shown in the following figure.

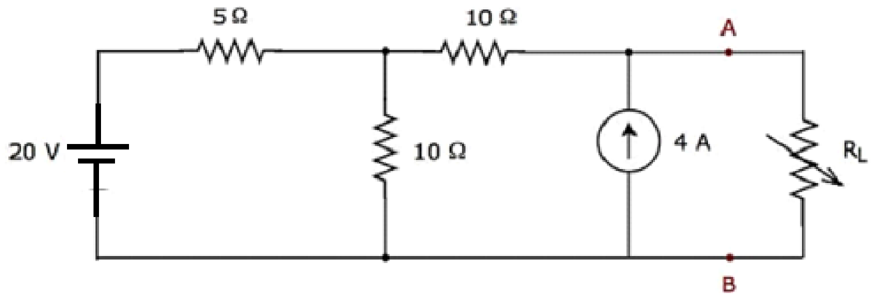


Fig. 2.41

Step 1 : From above circuit we find out the Thevenin's equivalent circuit seen between terminals A and B. It is shown in the following figure.

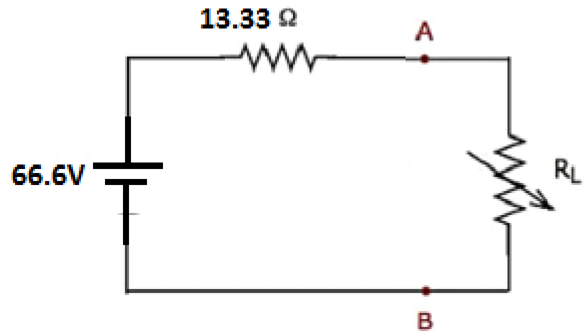


Fig. 2.42

Here, Thevenin's voltage V_{TH} = 66.6V and R_{TH} = 13.33 Ω.

Step 3 : We can find the maximum power that will be delivered to the load resistor, R_L by using the following formula.

$$P_{max} = \left[\frac{V_{Th}}{R_{Th} + R_L}\right] \times R_L \Big|_{R_L = R_{Th}}$$

$$= \frac{V_{Th}^2}{4R_{Th}}$$

Substituting V_{TH} = 66.6 V and R_{TH} = 13.33 Ω in above formula

$$P_{L,\ Max} = \frac{(6.66)^2}{4\ (13.33)}$$

$$P_{L,\,Max} = \frac{250}{3}\,W$$

Therefore, the **maximum power** that will be delivered to the load resistor R_L of the given circuit is 83.33 W.

2.15 MILLMAN'S THEOREM

The Millman's Theorem states that – when a number of voltage sources (V_1, V_2, V_3 … V_n) are in parallel having internal resistance (R_1, R_2, R_3 … R_n) respectively, the arrangement can replace by a single equivalent voltage source V in series with an equivalent series resistance R. In other words; it determines the voltage across the parallel branches of the circuit, which have more than one voltage sources, i.e. reduces the complexity of the electrical circuit.

This Theorem is given by Jacob Millman. The utility of Millman's Theorem is that the number of parallel voltage sources can be reduced to one equivalent source. It is applicable only to solve the parallel branch with one resistance connected to one voltage source or current source. It is also used in solving network having an unbalanced bridge circuit.

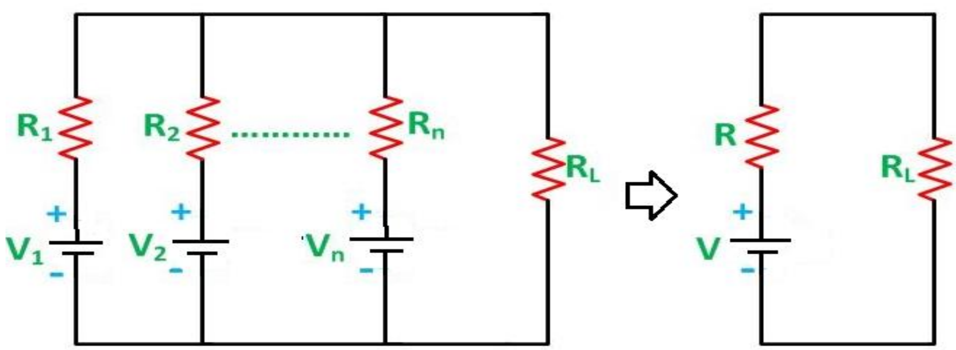


Fig. 2.43

As per Millman's Theorem

$$V = \frac{\pm V_1G_1 \pm V_2G_2 \pm \ldots\ldots \pm V_nG_n}{G_1 + G_2 + \ldots\ldots + G_n} \text{ and}$$

$$R = \frac{1}{G} = \frac{1}{G_1 + G_2 + \ldots\ldots + G_n}$$

2.15.1 Explanation of Millman's Theorem

Assuming a DC network of numerous parallel voltage sources with internal resistances supplying power to a load resistance RL as shown in the figure below.

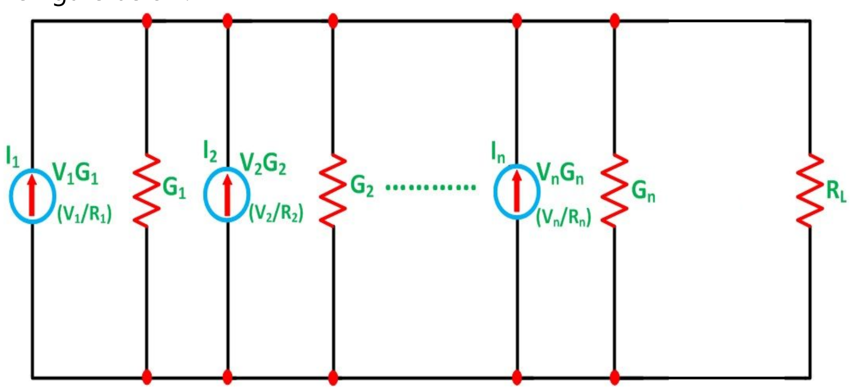


Fig. 2.44

Let I represent the resultant current of the parallel current sources while G the equivalent conductance as shown in the figure below

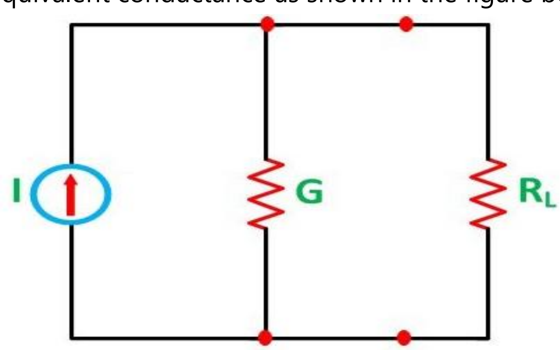


Fig. 2.45

$$I = I_1 + I_2 + I_3 + \ldots\ldots \text{ and}$$

$$G = G_1 + G_2 + G_3 + \ldots\ldots$$

Next, the resulting current source is converted to an equivalent voltage source as shown in the figure below

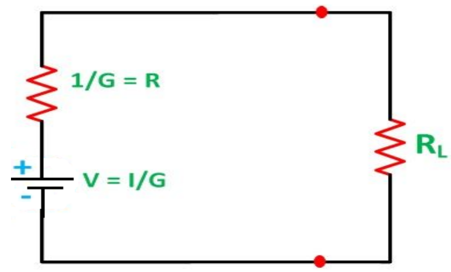


Fig. 2.46

Thus, $$V = \frac{1}{G} = \frac{\pm I_1 \pm I_2 \pm \ldots\ldots \pm I_n}{G_1 + G_2 + \ldots\ldots + G_n}$$

Positive (+) and negative (-) sign appeared to include the cases where the sources may not be supplying current in the same direction. Also,

$$R = \frac{1}{G} = \frac{1}{G_1 + G_2 + \ldots\ldots + G_n}$$

And as we know,

$I = \frac{V}{R}$, and we can also write $R = \frac{I}{G}$ as $G = \frac{I}{R}$

So the equation can be written as

$$V = \frac{\pm \frac{V_1}{R_1} \pm \frac{V_2}{R_2} \pm \ldots\ldots \pm \frac{V_n}{R_n}}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots + \frac{1}{R_n}}$$

Where R is the equivalent resistance connected to the equivalent voltage source in series. Thus, the final equation becomes

$$V = \frac{\pm V_1G_1 \pm V_2G_2 \pm \ldots\ldots \pm V_nG_n}{G_1 + G_2 + \ldots\ldots + G_n}$$

2.15.2 Steps for Solving Millman's Theorem

Following steps are used to solve the network by Millman's Theorem.

Step 1 : Obtain the conductance (G_1, G_2, ……) of each voltage source (V_1, V_2, ……).

Step 2 : Find the value of equivalent conductance G by removing the load from the network.

Step 3 : Now, apply Millman's Theorem to find the equivalent voltage source V by the equation shown below.

$$V = \frac{\pm V_1G_1 \pm V_2G_2 \pm \ldots\ldots \pm V_nG_n}{G_1 + G_2 + \ldots\ldots + G_n}$$

Step 4 : Determine the equivalent series resistance (R) with the equivalent voltage sources (V) by the equation

$$R = \frac{1}{G}$$

Step 5 : Find the current IL flowing in the circuit across the load resistance R_L by the equation

$$I_L = \frac{V}{R + R_L}$$

SOLVED EXAMPLE

Example 2.1 : Find the load current using Millman's theorem.

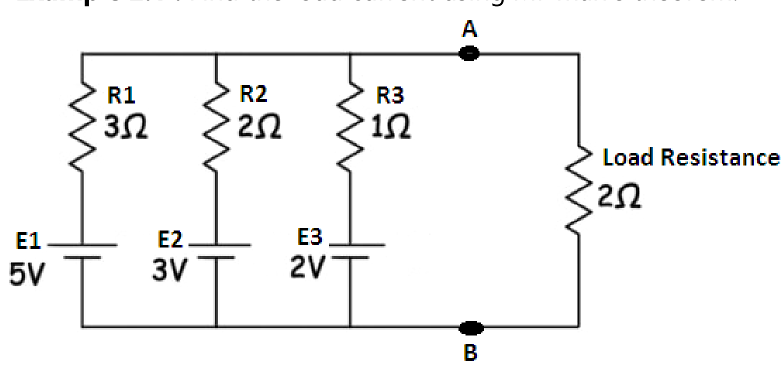


Fig. 2.47

Solution : Here, $E_1 = 5V$, $E_2 = 3V$, $E_3 = 2V$

$R_1 = 3\Omega$, $R_2 = 2\Omega$, $R_3 = 1\Omega$

By using Millman's theorem, the voltage across A and B is given by,

$$V_{AB} = \frac{\frac{E_1}{R_1} + \frac{E_2}{R_2} + \frac{E_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{\frac{5}{3} + \frac{3}{2} + \frac{2}{1}}{\frac{1}{3} + \frac{1}{2} + \frac{1}{1}}$$

$$= \frac{\frac{10 + 9 + 2}{6}}{\frac{2 + 3 + 6}{6}}$$

$$= \frac{31}{11}$$

The Thevenin equivalent resistance of the circuit across node A and B is

$$R_{Th} = \left(\frac{1}{3} + \frac{1}{2} + \frac{1}{1}\right)^{-1} = \frac{6}{11}$$

Now, the current through the load resistance is,

$$I_L = \frac{V_{AB}}{R_{Th} + R_2} = \frac{\frac{31}{11}}{\frac{6}{11} + 2} = \frac{31}{28}$$

$$I_L = 1.107 \text{ A}$$

EXERCISES

(A) Multiple Choice Questions (MCQs) :

1. In an electrical network to neglect a current source the current source is______.
   (a) open circuited (b) **short circuited**
   (c) replaced by a capacitor (d) replaced by an inductor
2. Which law plays a significant role in the loop analysis of the network?
   (a) KCL **(b) KVL**
   (c) Law of Superposition Theorem (d) None of the above
3. Kirchhoff`s current law is applicable to only _______.
   (a) junction in a network (b) closed loops in a network
   (c) electric circuits (d) electronic circuits
4. Superposition theorem can be applied only to circuits having ________.
   (a) resistive elements (b) passive elements

(c) non-linear elements **(d) linear bilateral elements**

5. In Thevenin's Theorem, Thevenin resistance R_{th} is found _______.

   (a) by removing voltage sources along with their internal resistances

   (b) by short-circuiting the given two terminals

   (c) between any two 'open' terminals

   (d) between same open terminals as for V_{th}, when sources are replaced by their internal resistances

6. An ideal voltage source should have __________.

   (a) large value of e.m.f. (b) small value of e.m.f.

   (c) zero source resistance (d) infinite source resistance

7. For maximum transfer of power, internal resistance of the source should be _______.

   (a) equal to load resistance (b) less than the load resistance

   (c) greater than the load resistance (d) none of the above

8. An ideal current source has ________.

   (a) large value of e.m.f. (b) small value of e.m.f.

   (c) zero source resistance **(d) infinite source resistance**

9. Millman's theorem yields _______.

   (a) equivalent resistance **(b) equivalent impedance**

   (c) equivalent voltage (d) infinite source resistance

(B) Short Answer Type Questions :

1. State and explain Kirchhoff's laws.
2. Describe nodal analysis method for finding solutions for network.
3. Describe mesh analysis method for finding solutions for network.
4. State and explain Thevenin's theorem.
5. State and explain Norton's theorem.
6. State and explain superposition theorem.
7. State and explain maximum power transfer theorem.
8. Explain concept of internal impedance of a source.

9. Write a note ideal and practical voltage source.
10. Write a note ideal and practical current source.
11. State and explain Reciprocity Theorem.
12. State and explain Milliman's Theorem.

(C) Long Answer Type Questions :

1. State and explain superposition Theorem. By using superposition theorem find the current through resistor R_2.

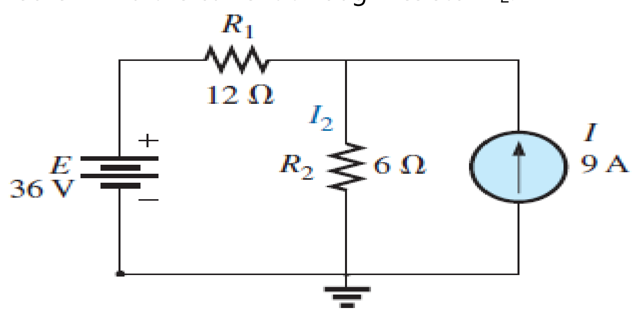


Fig. 2.48

2. By using superposition theorem, find the current through resistor R_2.

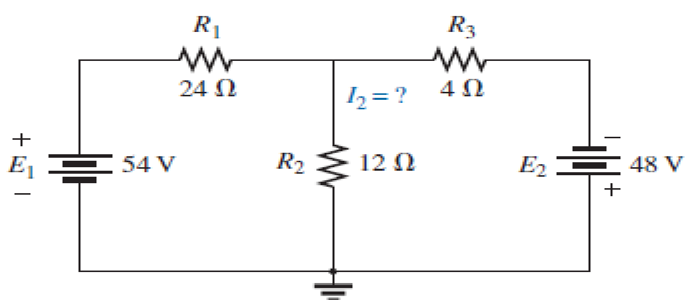


Fig. 2.49

3. By using Thevenin's theorem find the voltage and current through load resistor.

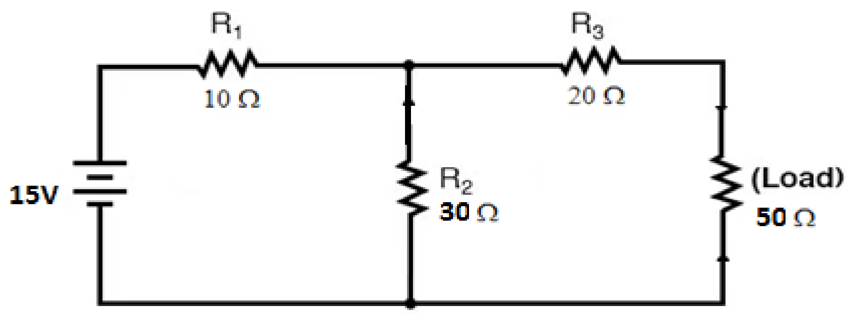


Fig. 2.50

4. By using Norton's theorem find the voltage and current through load resistor.

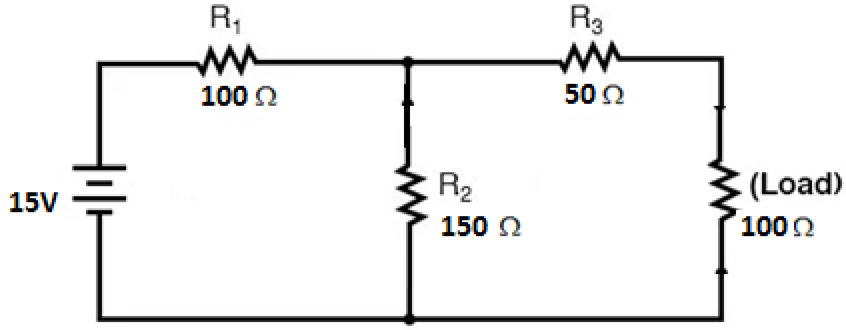


Fig. 2.51

5. By using maximum power transfer theorem. Find the maximum power that can be delivered to the load resistor R_L of the circuit shown in the following figure.

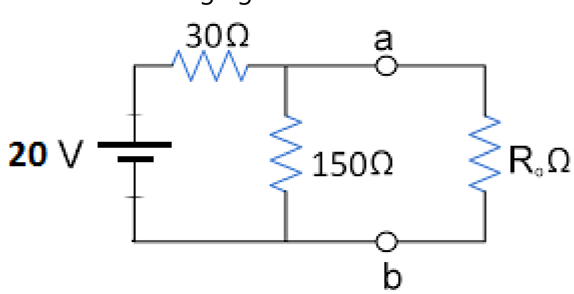


Fig. 2.52

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